WORKSHEET 2: Limits and Continuity

1. (*)Calculate:

a) $\lim_{x\to 0} \frac{4x^3+2x^2-x}{5x^2+2x}$ b) $\lim_{x\to 2} \frac{x^3-x^2-x-2}{x-2}$ c) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

d) $\lim_{x\to \infty} \frac{x^2-\sqrt{x}}{\sqrt{x^3+3x^4}}$ e) $\lim_{x\to \infty} \frac{senx}{x}$ f) $\lim_{x\to -\infty} \frac{x^2cosx+1}{x^2+1}$

g) $\lim_{x\to -\infty} \frac{3x^3+2x^2+x+2}{x^2-7x+1}$ h) $\lim_{x\to -\infty} \frac{x^4-ax^3}{x^2+1}$ i) $\lim_{x\to 0} \frac{x^4-x^3}{x^2+b}$

2. Find all discontinuities (if there are any) of the following functions:

a)(*)$f(x) = \frac{|x-3|}{x-3}$ b) (*)$f(x) = \begin{cases} \frac{2x}{x+1} & if \quad x < -1. \\ e^{1/x} & if \quad -1 \le x < 0 \\ \pi & if \quad x = 0 \\ 1/x & if \quad 0 < x \end{cases}$

3. (*)Calculate:

a) $\lim_{x\to -3^+} \frac{x^2}{x^2-9}$ b) $\lim_{x\to -3^-} \frac{x^2}{x^2-9}$ c) $\lim_{x\to 0^+} \frac{2}{senx}$ d) $\lim_{x\to 0^-} (1-1/x)^{\frac{1}{x}}$ e) $\lim_{x\to 0^-} \frac{x^2-2x}{x^3}$

4. Find every asymptote of the following functions:

(*)$f(x) = \frac{x^3}{x^2-1}$ $g(x) = \frac{x^2-1}{x}$ (*)$h(x) = \sqrt{x^2-1}$ (*)$m(x) = \frac{1}{\ln x}$ (*)$n(x) = e^{\frac{1}{x}}$

5. Prove that every odd-degree polynomial has at least one root (zero).

6. (a) (*) Use the Intermediate Value Theorem to prove that the following functions have a zero in the given interval.
i)$f(x) = x^2 - 4x + 3$ in $[2, 4]$; ii)$f(x) = x^3 + 3x - 2$ in $[0, 1]$.

(b) (*) Use the Bisection Method (applying Bolzano's Theorem repeatedly), to obtain the zero with an error of ± 0.25.

7. (*)Demonstrate that the equations $x^4 - 11x + 7 = 0$ and $2^x - 4x = 0$ have at least two different solutions each.

8. (*)Prove that the equation $x^7 + 3x + 3 = 0$ has only one solution. Round the solution to the nearest whole number.

9. Let f and g be continuous functions on $[a, b]$ and $f(a) < g(a)$, $f(b) > g(b)$, prove that $x_0 \in (a, b)$ exists such that $f(x_0) = g(x_0)$

10. (*) Discuss in the following cases if the functions attain their global and/or local extrema in the given interval:
a) $f(x) = x^2 \quad x \in [-1, 1]$ b) $f(x) = x^3 \quad x \in [-1, 1]$

11. Change the given interval in the previous exercise by $[0, \infty)$ firstly and then by $\mathbb{R}$.