# WORKSHEET 1: The real line. Functions

**1.** Solve the following inequalities, determine the set of real numbers that satisfies them. Represent its solution using a real line.

a)(*) $|9-2x|<1$ b)(*) $-5|x+3|<4x-5$ c)(*) $\dfrac{|x|}{3}+2<|x|$

d)(*) $1<|3-2x|$ e)(*) $\dfrac{(x^2-16)(x-1)}{x-3}\geq 0$ f) $|x-3|+|x+3|<10$

**2.** (*)Interpret the previous inequalities a), b), c) and d) geometrically using functions.

a) $y=|9-2x|$ ; $y=1$ b) $y=-5|x+3|$ ; $y=4x-5$

c) $y=\dfrac{|x|}{3}+2$ ; $y=|x|$ d) $y=1$ ; $y=|3-2x|$

**3.** Discuss if the following inequalities are satisfied:

a) (*) $|x+y|\leq|x|+|y|$ b)(*) $|x|+|y|\leq|x+y|$ c)(*) $|x-y|\leq|x|+|y|$ d)(*) $|x|-|y|\leq|x-y|$

**4.** Discuss if the following implications are true or false.

a) $x<y\Rightarrow x^2<y^2$ b) $|x|<|y|\Rightarrow x^2<y^2$

c) $x^2<y^2\Rightarrow x<y$ d) $x^2<y^2\Rightarrow|x|<|y|$

**5.** Obtain for the following sets of real numbers, $A\subset\mathbb{R}$, their maximum and minimum elements whenever they exist. Calculate them for $\alpha=-1$, $\alpha=0$ and $\alpha=1$

a) $A=\{x:e^x\leq\alpha\}$ b) $A=\{x:e^x\geq\alpha\}$ c) $A=\{x:\ln x\leq\alpha\}$ d) $A=\{x:\ln x\geq\alpha\}$

**6.** (*)Given the functions $f(x)=1/x$ and $g(x)=x^2-1$.

a) Find their domain and range.
b) Calculate $f(g(2))$ and $g(f(2))$.
c) Obtain the functions $f(g(x))$ and $g(f(x))$.

**7.** Review the graphs of the following functions: a)(*) $f(x)=x^2$ b)(*)$f(x)=e^x$ c)(*)$f(x)=lnx$

In each case sketch the graph transformations of the previous ones, interpreting geometrically the results.

i) $g(x)=f(x+1)$ ii) $h(x)=-2f(x)$ iii) $p(x)=f(3x)$

iv) $s(x)=f(x)+1$ v) $r(x)=|f(x)|$ vi) $m(x)=f(|x|)$

**8.** (*)Find the domain and the range of the following functions:

a)$f(x)=ln(1+|x|)$ b)$g(x)=2-\sqrt{1-x^2}$ c) $h(x)=e^{\sqrt{1-x^2}}$

**9.** (*)Let $f$, $g:I\to\mathbb{R}$ be two increasing functions. Discuss if the following statements are true or false.

a) $f+g:I\to\mathbb{R}$ is an increasing function.

b) $f\cdot g:I\to\mathbb{R}$ is an increasing function.

**10.** Let $f$, $g:\mathbb{R}\to\mathbb{R}$ be monotonic functions. Discuss if the composition $g\circ f$ is increasing or decreasing depending on the monotonicity of $f$ and $g$. (There are four diferent cases)

**11.** Find the intervals $I, J$ for the following functions so that they are bijective. For example ($f:I\to J$ )

a) $f(x)=x^2$ b) $g(x)=\ln|x|$ c) $h(x)=e^{-x^2}$

**12.** Determine if the following functions are even, odd or neither:

a) $f(x)=\dfrac{x^2}{x^2+1}$ b) $g(x)=\dfrac{x^3}{x^4+1}$ c) $h(x)=\dfrac{x^3}{x^5+1}$

**13.** (*)Calculate the inverse function:

$f(x)=(x^3-5)^5$, $g(x)=(\sqrt[3]{x}-5)^5$

**14.** Let $f$ be an even function and $g$ an odd one. Prove that:

$|g|$ is even; $f\circ g$ is even; $g\circ f$ is even;

$f\cdot g$ is odd; $g^k$ is even (if $k$ is even); $g^k$ is odd (if $k$ is odd)