ADVANCED MATHEMATICS FOR ECONOMICS – 2014/2015

Sheet 2. Difference Equations

2-1. Classify the following difference equations
(a) $x_{t+1} = x_t^2 - e^t$;
(b) $x_{t+1} = x_t - e^t$;
(c) $x_{t+1} = 3.2x_t(1 - 0.25x_t)$;
(d) $x_{t+1} - x_t = -\frac{4}{3}x_t$;
(e) $x_{t+1}(2 + 3x_t) = 4x_t$;
(f) $x_{t+2} = 3x_{t+1} - x_t + t$;
(g) $x_{t+4} - x_{t+3} = \sqrt[3]{x_{t+1}}$.

2-2. Check that the following sequences are solution of the corresponding difference equation
(a) $x_t = 2^t$; $x_{t+2} = x_{t+1} + 2x_t$;
(b) $x_t = \frac{t(t+1)}{2}$; $x_{t+1} = x_t + t + 1$;
(c) $x_t = \cos \pi t$; $x_{t+1} = -x_t$.

2-3. Consider the difference equation $x_{t+1} = \sqrt{x_t - 1}$ with $x_0 = 5$. Compute x_1, x_2. and x_3. What about x_4?

2-4. Find the solutions of the following difference equations with the given values of x_0:
(a) $x_{t+1} = 2x_t + 4$, $x_0 = 1$;
(b) $x_{t+1} = -0.5x_t + 3$, $x_0 = 1$;
(c) $2x_{t+1} + 3x_t + 2 = 0$, $x_0 = -1$;
(d) $x_{t+1} - x_t = -\frac{4}{3}x_t$, $x_0 = 3$.
Study the long run behavior of the solutions.

2-5. The income Y_t evolves according to the equation

$$Y_{t+1} = C_t + I_t,$$

where I_t denotes investment and C_t is consumption. Supposing that $C_t = mY_t + c$, with $0 \le m < 1$, $c > 0$, and that $I_t = I$ is constant, find a difference equation for income Y_t, solve it, and study the long run behavior of the solution.

2-6. Let S_0 denotes an initial sum of money. There are two basic methods for computing the interest earned in a period, for example, one year:
(a) S_0 earns *simple interest* at rate r if each period the interest equals a fraction r of S_0.
(b) S_0 earns *compound interest* at rate r if each period the interest equals a fraction r of the sum accumulated at the beginning of that period.
Find a difference equation for the two models above, and find the solution.

2-7. Given demand and supply for the cobweb model as follows, find the intertemporal equilibrium price, and determine whether the equilibrium is stable:
(a) $Q_d = 18 - 3P$, $Q_s = -3 + 4P$;
(b) $Q_d = 22 - 3P$, $Q_s = -2 + P$;
(c) $Q_d = 16 - 6P$, $Q_s = 6P - 5$;

2-8. In the cobweb model, suppose that the market clearance still holds in each period, $Q_{d,t} = Q_{s,t}$, but that the the supply function is determined not by the price at the previous period, $Q_{s,t} = S(P_{t-1})$, but by the *expected price* at period t:

$$Q_{s,t} = -\gamma + \delta P_t^*.$$

Sellers form expectations about the price according to the following adaptive rule:

$$P_{t+1}^* = P_t^* + \eta(P_t - P_t^*), \qquad 0 < \eta \leq 1,$$

where η is an expectation–adjustment parameter.

(a) Give an economic interpretation to the preceding equation;
(b) Is the cobweb model a particular case of the present model?
(c) Find a difference equation for this model;
(d) Find the trajectory of price. Is this path necessarily oscillatory? Can it be oscillatory? Under what circumstances?
(e) Show that the time path P_t, if oscillatory, will converge only if $1 - 2/\eta < -\delta/\beta$. As compared with the cobweb model without adaptive expectations, does the present model have a wider or narrower range for the stability–inducing values of $-\delta/\beta$?