ADVANCED MATHEMATICS FOR ECONOMICS – 2014/2015

Sheet 1. Diagonalization

1-1. Given the matrix
$$A = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$$
compute its eigenvalues, eigenvectors and diagonalize A.

1-2. Given the following matrices
$$A = \begin{pmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix} \qquad C = \begin{pmatrix} 4 & 5 & -2 \\ -2 & -2 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$
(a) Compute its eigenvalues, eigenvectors and the eigenspaces.
(b) Diagonalize them, whenever possible.

1-3. What are the values of a for which the matrix
$$A = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$
is diagonalizable?

1-4. Show that
(a) If A is a diagonalizable matrix, so is A^n for each $n \in \mathbb{N}$.
(b) A diagonalizable matrix A is regular if and only if none of its eigenvalues vanishes.
(c) If A has an inverse, then both A and A^{-1} have the same eigenvectors and the eigenvalues of A are the reciprocal of the eigenvalues of A^{-1}.
(d) A and A^t have the same eigenvalues.

1-5. Study for which values of a and b the matrix $A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & -1 & a \\ 3 & 0 & b \end{pmatrix}$ is diagonalizable.

1-6. Which of the following matrices are diagonalizable?
$$A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \qquad C \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

1-7. The matrix $\begin{pmatrix} 1 & 0 & 0 \\ \alpha+1 & 2 & 0 \\ 0 & \alpha+1 & 1 \end{pmatrix}$ is diagonalizable if and only α is...

1-8. Consider the matrices
$$A = \begin{pmatrix} 3 & 2 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$
Find whether they are diagonalizable and, whenever they are, compute their n-th power.

1-9. The following are the characteristic polynomials of some square matrices. Determine which of them correspond to diagonalizable matrices.
$p(\lambda) = \lambda^2 + 1$ $\quad p(\lambda) = \lambda^2 - 1$
$p(\lambda) = \lambda^2 + \alpha$ $\quad p(\lambda) = \lambda^2 + 2\alpha\lambda + 1$
$p(\lambda) = \lambda^2 + 2\lambda + 1$ $\quad p(\lambda) = (\lambda - 1)^3$
$p(\lambda) = \lambda^3 - 1$

1-10. Determine whether the following matrices are diagonalizable. Compute the n-th power whenever they are diagonalizable.

$$A = \begin{pmatrix} \alpha & 0 \\ 1 & \alpha \end{pmatrix} \quad B = \begin{pmatrix} \alpha & 1 \\ 1 & \alpha \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

1-11. Study for what values of the parameters the following matrices are diagonalizable. Find the eigenvalues and eigenvectors.

$$A = \begin{pmatrix} a & b & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 & -2-\alpha \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}$$

1-12. The matrix

$$\begin{pmatrix} a & 1 & p \\ b & 2 & q \\ c & -1 & r \end{pmatrix}$$

has $(1,1,0)$, $(-1,0,2)$ and $(0,1,-1)$ as eigenvectors. Compute its eigenvalues.

1-13. Determine whether the following matrices are diagonalizable. If possible, write their diagonal form.

$$A = \begin{pmatrix} 5 & 4 & 3 \\ -1 & 0 & -3 \\ 1 & -2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -2 & -1 & -1 \\ 1 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 5 & 7 & 5 \\ -6 & -5 & -3 \\ 4 & 1 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 3 & 2 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad E = \begin{pmatrix} -1 & 2 & -2 \\ 0 & 2 & 0 \\ 0 & 3 & -2 \end{pmatrix} \quad F = \begin{pmatrix} 5 & -10 & 8 \\ -10 & 2 & 2 \\ 8 & 2 & 11 \end{pmatrix}$$

$$G = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 2 \\ 0 & 1 & 4 \end{pmatrix} \quad H = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ -1 & 0 & -2 \end{pmatrix} \quad I = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \quad K = \begin{pmatrix} -1 & 2 & -2 \\ 0 & 2 & 0 \\ 0 & 3 & -2 \end{pmatrix} \quad L = \begin{pmatrix} -9 & 1 & 1 \\ -18 & 0 & 3 \\ -21 & 4 & 0 \end{pmatrix}$$