Universidad Carlos III de Madrid
Economics of Information

Final exam with solutions

No calculators, computers, cell phones or other electronic devices are allowed.

1. (25 points) There are two types of consumers in an economy, the rich and the richer. Rich consumers have income $I_R = 50$, while richer consumers have $I_{RR} = 100$. In addition to their income, richer consumers enjoy being recognized as richer by everyone, but the income is not observable. However, a consumer can buy conspicuous, luxury goods that will be observable. The utility function of a consumer is given by $u_i = I_i\left(1+\frac{L_i}{2}\right) - X_i$, where I_i is the income (50 or 100), X_i is the amount spent on conspicuous consumption (observable), and L_i takes the value 1 if the individual is labeled as "richer", and zero if she is not.

(a) **(5 points)** Interpret the utility function. In particular, interpret the trade-off between L_i and X_i for rich and richer consumers.
(b) **(10 points)** Show for which values of X^* there is a separating equilibrium such that individuals spending $X_i \geq X^*$ or more will be labeled as "richer", and individuals spending $X_i < X^*$ will not.
(c) **(5 points)** Explain the role of X_i as a signal.
(d) **(5 points)** Is there an adverse selection problem in this economy?

(a) $\frac{\partial u_i}{\partial L_i} > 0, \frac{\partial u_i}{\partial X_i} < 0.$
$\frac{\partial u_{RR}}{\partial L_{RR}} > \frac{\partial u_R}{\partial L_R}, \frac{\partial u_{RR}}{\partial X_{RR}} = \frac{\partial u_R}{\partial X_R}.$

(b) $100\left(1+\frac{1}{2}\right) - X \geq 100,$
$50 \geq 50\left(1+\frac{1}{2}\right) - X.$

Which means that any $25 \leq X^* \leq 50$ serves for a separating equilibrium: Rich indiv, choose $X_i = 0$, richer individuals choose $X_i = X^*$, believes: $p(\text{richer} \mid X_i < X^*) = 0, p(\text{richer} \mid X_i \geq X^*) = 1.$

(c) X does not provide utility, it is costly, but it is more costly to the rich than to the richer. Levels $25 \leq X^* \leq 50$ can be afforded by the richer, but not by the rich.

(d) In the absence of X there is no adverse selection. Both rich and richer are there in the society minding their own business.

2. (25 points + 5 extra points) In a given economy all individuals have a unit of wealth and face a risk of losing it. In particular there are three equally abundant types of consumers, A, B and C for which this probability is 0.8, 0.5 and 0.2, respectively. All types have utility function $u_i = \sqrt{x_i}$, where x_i is wealth of individual i.

(a) **(5 points)** Assume that there is a perfectly competitive insurance market and that types are observed. What contracts will be offered? Compute the utility for each type.

In the following assume types are not observed.

(b) **(5 points)** What would be the best pooling insurance policy that a firm can offer if all types subscribed it? Which types will it attract?
(c) **(5 points)** What would be the best partially pooling policy that a firm can offer if types A and B subscribed it? Will it attract them?
(d) **(10 points)** Assume that by law firms must pool types A and B, but are free to offer a contract that may be attractive to type C. Which is this policy? You can just justify and show the equation that must be satisfied, without solving it.

(e) **(5 extra points)** What is the condition you need to add to make sure that (c) and (d) constitute an equilibrium? (Recall that (c) and (d) constitute an equilibrium only under the provision than contracts have to be separating.)

(a) $(I_A, \pi_A) = (1, 0.8),\ u_A(1, 0.8) = \sqrt{0.2} = 0.447,$
$(I_B, \pi_B) = (1, 0.5),\ u_B(1, 0.5) = \sqrt{0.5} = 0.707,$
$(I_C, \pi_C) = (1, 0.2),\ u_C(1, 0.2) = \sqrt{0.8} = 0.894.$

(b) $(I, \pi) = \left(1, \frac{0.8+0.5+0.2}{3}\right) = (1, 0.5).$

$u_i(1, 0.5) = \sqrt{0.5} = 0.707.$
$u_A(\text{no insurance}) = 0.2\sqrt{1} + 0.8\sqrt{0} = 0.2 < 0.707,$
$u_B(\text{no insurance}) = 0.5 < 0.707,$
$u_C(\text{no insurance}) = 0.8 > 0.707.$

Only A and B will subscribe the contract.

(c) $(I_{AB}, \pi_{AB}) = \left(1, \frac{0.8+0.5}{2}\right) = (1, 0.65).$

$u_i(1, 0.5) = \sqrt{0.35} = 0.59.$
$u_A(\text{no insurance}) = 0.2 < 0.59,$
$u_B(\text{no insurance}) = 0.5 < 0.59,$
$u_C(\text{no insurance}) = 0.8 > 0.59.$

Both A and B will subscribe it.

(d) $\max_{I_C, \pi_C} 0.8\sqrt{1-\pi_C} + 0.2\sqrt{I_C - \pi_C}$

s.t. $0.8\sqrt{1-\pi_C} + 0.2\sqrt{I_C - \pi_C} \geq 0.8$ (C's participation),
$0.8\sqrt{1-\pi_C} + 0.2\sqrt{I_C - \pi_C} \geq 0.59$ (C's incentive compatibility),
$0.59 \geq 0.2\sqrt{1-\pi_C} + 0.8\sqrt{I_C - \pi_C}$ (A's incentive compatibility),
$0.59 \geq 0.5\sqrt{1-\pi_C} + 0.5\sqrt{I_C - \pi_C}$ (B's incentive compatibility),
$\pi_C = 0.2 I_C$ (zero profits).

Either A's or B's incentive compatibility constraint is binding. B's is more restrictive ($I_C < 1$, which means that $0.5\sqrt{1-\pi_C} + 0.5\sqrt{I_C - \pi_C} > 0.2\sqrt{1-\pi_C} + 0.8\sqrt{I_C - \pi_C}$) so B's it is. Together with zero profits we obtain the equation to be solved.

$$0.5\sqrt{1 - 0.2 I_C} + 0.5\sqrt{0.8 I_C} = 0.59.$$

After solving for I_C, we need to check the non-binding constraints.

(e) If there was a solution to (d) We need to check that a pooling contract can not be better for all A, B and C, and also that a partially pooling contract can not be better for B and C.

A, B and C:
Non zero profits: $\pi_{ABC} \geq 0.5 I_{ABC},$
C's want it: $0.8\sqrt{1-\pi_{ABC}} + 0.2\sqrt{I_{ABC} - \pi_{ABC}} \geq 0.8\sqrt{1-\pi_C} + 0.2\sqrt{I_C - \pi_C},$
(If C's want it, so do A and B).

B and C:
Non zero profits: $\pi_{BC} \geq 0.35 I_{BC},$
C's want it: $0.8\sqrt{1-\pi_{BC}} + 0.2\sqrt{I_{BC} - \pi_{BC}} \geq 0.8\sqrt{1-\pi_C} + 0.2\sqrt{I_C - \pi_C},$

(If C's want it, so do A and B).

3. (25 points) The outcome that a principal can obtain from of a work made by an agent has three possible results, 500, 300 and 0 with probabilities 1/2, 1/4 and 1/4, respectively, if the agent makes his work diligently, and with probabilities 1/2, 1/8 and 3/8 if the agent shirks. The utility function of the agent is $u(w,e) = \sqrt{w} - e$, where $e = 1$ if the agent works diligently and $e = 0$, if the agent shirks. The principal is a perfectly discriminating monopolist in the labor market for agents. The agent has a reservation utility $u^R = 2$. The result is observable, but not the effort of the agent.

(a) **(20 points)** Compute the wage scheme $(w_0, w_{300}, w_{500}) \geq (0,0,0)$ offered by the principal. (You may assume $w_0 = 0$.)
(b) **(5 points)** Is the wage increasing in the result? Why?

Done in class.

4. (25 points) A consumer has a unit of wealth, but can lose half of it with probability 1/3, and all of it also with probability 1/3. However, if he exerts effort $e = 0.1$ the probabilities change to 1/5 and 1/5, respectively. Write the problem that one needs to solve to compute the best perfectly competitive insurance policy when the level of effort is not observable in the following three scenarios (i.e., just write the objective function and the restrictions.)

(a) **(6 points)** The standard scenario: policies can specify an unrestricted indemnity.
(b) **(6 points)** The indemnity has to be a proportion α of the lost.
(c) **(6 points)** The consumer must pay the first δ units of the lost, and the firm will pay the rest up to the total lost.
(d) **(7 points)** Without performing any calculations, can you tell whether (b) or (c) will result in a policy that is better, equal or worse for the consumer compared to (a)?

(a) Let I_A and I_B be the indemnities in case the consumer loses half and all of her wealth, respectively. To incentive $e = 0.1$:

$$\max_{I_A, I_B, \pi} \frac{3}{5}u(1-\pi) + \frac{1}{5}u(0.5 + I_A - \pi) + \frac{1}{5}u(I_B - \pi) - 0.1$$

s.t. $\frac{3}{5}u(1-\pi) + \frac{1}{5}u(0.5 + I_A - \pi) + \frac{1}{5}u(I_B - \pi) - 0.1 \geq \frac{1}{3}u(1) + \frac{1}{3}u(0.5) + \frac{1}{3}u(0)$,

$\frac{3}{5}u(1-\pi) + \frac{1}{5}u(0.5 + I_A - \pi) + \frac{1}{5}u(I_B - \pi) - 0.1 \geq \frac{1}{3}u(1-\pi) + \frac{1}{3}u(0.5 + I_A - \pi) + \frac{1}{3}u(I_B - \pi)$,

$\pi = \frac{1}{5}I_A + \frac{1}{5}I_B$.

To incentive $e = 0$:

$\pi = \frac{1}{3}I_A + \frac{1}{3}I_B$,

$I_A = 0.5,\ I_B = 1$.

(b) Now we have the additional restrictions $I_A = 0.5\alpha$, $I_b = \alpha$:

$$\max_{\alpha, \pi} \frac{3}{5}u(1-\pi) + \frac{1}{5}u(0.5 + 0.5\alpha - \pi) + \frac{1}{5}u(\alpha - \pi) - 0.1$$

s.t. $\frac{3}{5}u(1-\pi) + \frac{1}{5}u(0.5 + 0.5\alpha - \pi) + \frac{1}{5}u(\alpha - \pi) - 0.1 \geq \frac{1}{3}u(1) + \frac{1}{3}u(0.5) + \frac{1}{3}u(0)$,

$\frac{3}{5}u(1-\pi) + \frac{1}{5}u(0.5 + 0.5\alpha - \pi) + \frac{1}{5}u(\alpha - \pi) - 0.1 \geq \frac{1}{3}u(1-\pi) + \frac{1}{3}u(0.5 + 0.5\alpha - \pi) + \frac{1}{3}u(\alpha - \pi)$,

$\pi = \frac{1}{5}0.5\alpha + \frac{1}{5}\alpha$.

To incentive $e = 0$:

$$\max_{\alpha,\pi} \frac{1}{3}u(1-\pi) + \frac{1}{3}u(0.5 + 0.5\alpha - \pi) + \frac{1}{3}u(\alpha - \pi)$$

s.t. $\pi = \frac{1}{3}0.5\alpha + \frac{1}{3}\alpha.$

(c) Now we have the additional restrictions $I_A = 0.5 - \delta, I_B = 1 - \delta$:

$$\max_{\delta,\pi} \frac{3}{5}u(1-\pi) + \frac{1}{5}u(0.5 + 0.5 - \delta - \pi) + \frac{1}{5}u(1 - \delta - \pi) - 0.1$$

s.t. $\frac{3}{5}u(1-\pi) + \frac{1}{5}u(1-\delta-\pi) + \frac{1}{5}u(1-\delta-\pi) - 0.1 \geq \frac{1}{3}u(1) + \frac{1}{3}u(0.5) + \frac{1}{3}u(0),$

$\frac{3}{5}u(1-\pi) + \frac{1}{5}u(1-\delta-\pi) + \frac{1}{5}u(1-\delta-\pi) - 0.1 \geq \frac{1}{3}u(1-\pi) + \frac{1}{3}u(1-\delta-\pi) + \frac{1}{3}u(1-\delta-\pi),$

$\pi = \frac{1}{5}(0.5-\delta) + \frac{1}{5}(1-\delta),$

$\delta \leq 0.5.$

To incentive $e = 0$:

$$\max_{\delta,\pi} \frac{1}{3}u(1-\pi) + \frac{1}{3}u(1-\delta-\pi) + \frac{1}{3}u(1-\delta-\pi)$$

s.t. $\pi = \frac{1}{5}(0.5-\delta) + \frac{1}{5}(1-\delta),$

$\delta \leq 0.5.$

(d) Both (b) and (c) add more restrictions compared to (a), therefore the maximum will be equal or lower than in (a). Notice that in both (b) and (c) we have fewer degrees of freedom than (a).