

Exercise List 5: Auctions

Exercise 1. Consider a second-price sealed-bid auction where there are n bidders whose values are iid according to a cdf F with support on $[0, 1]$. For each of the following scenarios calculate the seller's revenue, the bidders' payoff and the gross surplus in the equilibrium of the auction where bidders bid their value:

(2.1) $n = 3$ and $F(x) = x$.

(2.2) $n = 2$ and $F(x) = x^2$.

(2.3) $n = 2$ and $F(x) = 2x - x^2$.

Exercise 2. Consider a first-price sealed-bid auction (*FPA*) in which there are n bidders whose values are iid according to a cdf F with support on $[0, 1]$. For each of the following scenarios calculate the bidding strategy, the seller revenue, the bidders payoff and the gross surplus in the unique symmetric increasing differentiable equilibrium of the auction.

(3.1) $n = 3$ and $F(x) = x$.

(3.2) $n = 2$ and $F(x) = x^2$.

(3.3) $n = 2$ and $F(x) = 2x - x^2$.

Exercise 3. Consider the equilibria described in exercises 2 and 3 for the case $n = 2$ and $F(x) = x$. Verify that the bidders' expected payoff is the same in a *FPA* and a *SPA*. Also, graph the cdfs of the seller's revenue in these auctions, R_1 and R_2 , and verify that $E(R_1) = E(R_2)$, but $E(R_1^2) < E(R_2^2)$, and therefore that the variance of R_2 greater than that of R_1 . Which auctions would be preferred by a risk-averse seller?