Exercise List 4: Signaling

Exercise 1. In a competitive labor half of the workers are high skilled (H) and the other half are low skilled (L). A firm (principal) that hires a high (low) skilled worker obtains an expected revenue of $\bar{x}_H = 12$ (respectively, $\bar{x}_L = 4$) euros. The reservation utilities of high (low) skilled workers $u_H = 6$ (respectively, $u_L = 0$). Workers may signal their skill by taking an action $y \in \mathbb{R}_+$ at at cost $c(y, H) = y/4$ and $c(y, L) = y$, respectively.

(a) Compute the pooling PBNE (that is, the PBNE in which y is not used as a signal).

(b) Compute the most efficient separating PBNE (that is, the PBNE in which an worker's choice of y signals her type). Which of the

two PBNE would each type of worker prefer?

Exercise 2. In a competitive labor there are three types of workers present in equal proportions, whose productivities and reservation utilities are $(a_1, u_1) = (1, 1/2)$, $(a_2, u_2) = (2, 1)$ and $(a_3, u_3) = (3, 22/10)$, respectively. A worker's productivity is his private information. Before entering the market each work chooses its level of education, which is observed by the firms. The cost of acquiring a level of education y for a worker of type i is $C_i(y) = y/a_i^2$. The (expected) revenue depend on it labor force according to the formula

$$\bar{x}(L_1, L_2, L_3) = a_1 L_1 + a_2 L_2 + a_3 L_3,$$

where L_i is the number of workers of type $i \in \{1, 2, 3\}$ in the firm's labor force.

(a) Determine whether there is a *fully separating* PBNE (that is, a PBNE in which each type chooses a different level of education), and if so identify the PBNE most favorable to the workers. (Hint: start by arguing that $y_1 = 0$, and find the levels of education $0 < y_2 < y_3$ that will form a PBNE. Make explicit the beliefs of the firm.)

(b) Find, if possible, a PBNE in which $y_1 = y_2 = 0$, and $y_3 > 0$, that is, in which the firm treats differently only the type 3 workers. Compute the most favorable PBNE for the workers of type 3. Write explicitly the beliefs of the company. Compare the workers' utilities with those of part (a).

Exercise 3. A worker's output depends of her ability $t \in \{H, L\}$, and her level of education $y \in \mathbb{R}_+$, according to the function $f(y, t) = \alpha_t + \beta\sqrt{y}$, where $\alpha_H = 2$, $\alpha_L = 1$, and $\beta \in \mathbb{R}_+$. The worker's cost of education also depends of her ability and

is given by the function $c(y, t) = c_t y$, where $c_H = 1$ and $c_L = 4$. The worker, upon observing her ability, must decide her level of education. Firms observe workers level of education (but their abilities), and offer salaries $w \in \mathbb{R}_+$ that may depend on it. For $y, w \in \mathbb{R}_+$ the payoffs of the worker and the firm are given by $u(y, w, t) = w - c(y, t)$, and $v(y, w, t) = w - f(y, t)$, respectively. Assume that under competitive pressure the firm always offers the worker a wage equal to her expected output. In a separating (pooling) PBNE high and low ability workers choose different (the same) levels of education. Identify the pooling and separating PBNE for $\beta = 0$ and $\beta = 1$.