Exercise List 3: Adverse Selection

Exercise 1. A good of two qualities, high (H) and low (L), trades in competitive markets in which each seller has a single unit and each buyer wants to buy a single unit. There are n_H sellers with a unit of high quality whose opportunity cost is c_H euros, n_L sellers with a unit of low quality whose opportunity cost is c_L euros, and n buyers who value a unit of high quality in u_H euros and a unit of low quality in u_L euros. Assume that $u_H > c_H > u_L > c_L$.

(a) Suppose that quality is observable. Calculate the competitive prices for the cases $n > n^H + n^L$ and $n < n_H + n_L$. Discuss if these competitive equilibria generate the maximum surplus. (If you find it helpful, assume $u_H = 10$, $c_H = 7$, $u_L = 5$, $c_L = 0$, $n_H = 1$, $n_L = 1$ and $n \in \{1, 3\}$.)

(b) Now suppose that quality is not observable and both qualities trade in the same market. Also assume that $n = n_H + n_L$. Represent the supply and demand schedules in the plane (q, p) and calculate the competitive equilibria of this market when the expected value of a random unit,

$$u(n^H, n^L) = \frac{n_H}{n_H + n_L} u_H + \frac{n_L}{n_H + n_L} u_L,$$

is greater than c^H, and when it is less than c^H. (If you find it helpful, use the parameter values suggested in part (a), and consider the cases $n_L = 1$ and $n_L = 2$.)

Exercise 2. Consider a market for used cars whose qualities, indexed by the sellers' cost, are uniformly distributed in the interval $[2, 6]$. Buyers are risk-neutral and value each quality 20% more than sellers. Naturally, each seller knows the quality of the good he sells, but quality is not observable to buyers prior to purchase. Assume that there are more buyers than sellers.

(a) Determine the market supply and the average quality of the cars offered at each price.

(b) Calculate the market equilibrium.

Exercise 3. Consider an insurance market in which all individuals have the same initial wealth $W = 1$ and the same preferences, which are represented by the von Neumann-Morgenstern utility function $u(x) = \sqrt{x}$, where x is the individual's disposable income. Each individual faces the risk of having an accident resulting in losing his wealth. For a fraction $\lambda \in (0, 1)$ of individuals the probability of having this accident is $p^L = 1/2$ whereas for the remaining fraction $1 - \lambda$ this probability is

$p^H = 4/5$. Insurance companies know this information, but at the time of signing a policy do not observe whether the probability of having an accident for a particular individual is p^L or p^H.

(a) If it is mandatory that policies offer full insurance, which policies will companies offer for each value of λ? Which individuals would subscribe them?

(b) If companies are free to offer any policy, which policies will be offered for each value of λ? (Here you need to identify a separating equilibrium.)

Exercise 4. Consider an economy with three types of workers whose abilities and reservation utilities are $(a_1, a_2, a_3) = (1, 2, 3)$ and $(u_1, u_2, u_3) = (1/2, 1, 22/10)$, respectively. The probability of each type is 1/3. A perfectly competitive firm considers hiring the workers, but the productivity is private information only known to each worker. Once the workers are hired, the production function of the firm is $y = a_1L_1 + a_2L_2 + a_3L_3$, where L_i is the amount of work done by a worker of type a_i. The product y is sold at the price of 1.

(a) Would the three types of workers accept an average salary?

(b) Can the firm offer a salary that in equilibrium attracts only workers of types 1 and 2?

Exercise 5. In an insurance market there are two types of agents A and B, in equal proportions. Both types of agents have the same initial wealth $w = 1$ and the same preferences on money represented by the utility function $u = \sqrt{x}$, where x is money. However, their risk of loss is different. Agents of type A have a probability of loss of $0,5$, whereas the probability of loss for type B agents is $0,2$. The insurance companies can distinguish the types of the agents but the agents do not know their types.

(a) Compute the competitive equilibrium in this market.

(b) The government regulates the market and forces the companies to offer a unique full insurance policy and cannot reject any customer. Compute the new equilibrium and explain why this would not be an equilibrium if there were no regulation.

(c) Discuss the efficiency in the situation (a) and (b).

Exercise 6. The genetic advances allow the technology that determines the risk of acquiring a certain disease through a test. The government is considering the possibility of allowing the insurance companies to make the above test, before they offer the insurance policy to the buyers. What would you advise to the government? Assume that the cause of the disease is only genetic, so it is not determined by the habits of the consumer.

Exercise 7. A monopolist who produces with marginal cost 1 faces two consumers with demands $q_1 = 8 - p_1$ and $q_2 = 10 - p_2$, respectively. The monopolist cannot distinguish the demand of each consumer.

(a) Compute the price that the monopolist would choose and the profits it would obtain if it cannot discriminate consumers in any way.

(b) Suppose now that the monopolist can discriminate and it is thinking on a second degree price discrimination. This consists in a two-part tariff (Q, p) in which consumers pay a one-time access fee Q for the right to buy a product, and a per-unit price p for each unit they consume. In this manner, the monopolist obtains revenues from the access fee and from the sales of the good. Suppose the monopolist sets the tariff so that both consumers pay it. That is, it sets the access fee Q to be less or equal than the surplus of consumer 1 at the price p. Write the profits of the monopolist as a function of p and compute the optimal tariff for the monopolist and its profits under the optimal tariff. If the monopolist is not interested in attracting consumer 1, what is the optimal two-part tariff? Compare the profits of this case with the profits in part (b) above. Compute the surplus of each consumer.

The monopolist is considering the possibility of a menu of two-part tariffs (Q_1, p_1) and (Q_2, p_2). These tariffs are designed so that the consumers with demand $q_1 = 8 - p_1$ choose (Q_1, p_1) and the consumers with demand $q_2 = 10 - p_2$ choose (Q_2, p_2).

(c) What are the conditions under which the consumers accept to purchase the two-part tariffs?

(d) What are the conditions under which each type of consumers prefer the two-part tariff addressed to them?

(e) Write the optimization problem for the monopolist if it wants to find the optimal menu of two-part tariffs (Q_1, p_1) and (Q_2, p_2) that satisfies the above constraints and solve this problem.

(f) Compute the total surplus in the above equilibrium and compare it with the surplus obtained in part (b) of problem 1.

Exercise 8. Exercises 1, 2, 3, 4 and 6 in chapter 4 of the textbook by Macho and Perez Castrillo.