## THEORY OF THE FIRM

## A. Production.

1. Answer the questions of parts (a)-(e) for firms whose production functions are: (1.1) $F(L, K)=\sqrt{L K}$; (1.2) $F(L, K)=L+4 K$, and (3.3) $F(L, K)=2 \min \{2 L, K\}$.
(a) Compute and graph the isoquants $Q=2$ and $Q=10$.
(b) Calculate the $\operatorname{RMST}(L, K)$ and evaluate it at $(L, K)=(4,1)$ and $(L, K)=$ $(10,10)$.
(c) Determine and graph the short run production functions $f(L)=F(\bar{K}, L)$ for $\bar{K}=4$ and for $\bar{K}=10$.
(d) Determine and graph the marginal productivity of labor $F_{L}(L, K)=\partial F(L, K) / \partial L$ for $\bar{K}=4$ and for $\bar{K}=10$.
(e) What are the firms' returns to scale in the short and long run?
2. How are the answers to the questions in exercise 1 to parts (a)-(e) altered if the functions in (1.1), (1.2) and (1,3) $\hat{F}(L, K)=(F(L, K))^{2}$ ?
3. A famous bakery's production capacity of bread loaves, given its capital equipment, depends on the number of workers as described by the following table:

| Number of Workers | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Bread Loaves (thousands) | 1 | 1.8 | 2.4 | 2.8 | 3 | 2.8 | 2.5 |

(a) Calculate the marginal and average productivity of each worker.
(b) Determine the types of returns to scale the bakery has.
(c) Discuss the reasons why the marginal productivity of a worker may be negative. What are the implications of this fact over the slope of the isoquants?

## A. Multiple Choice Questions

1. Lolita, the competitive cow in all markets of Holstein, produces milk ( $L$ ) using oats $(A)$ and hay $(H)$ according to the production function $L=A^{2}+H$. Hence, as a milk producer Lolita's returns to scale areincreasingconstantdecreasingundetermined.
2. If a firm's production function is $F(L, K)=\min \{2 L, \sqrt{K}\}$, then the firm's returns to scale are$\square$ increasingconstantdecreasingundetermined.

## B. Costs

3. A certain firm produces a good using two factors of production: energy and "other inputs".
(a) Assume that the energy price, controlled by an international cartel, rises by $100 \%$. Determine how the long-run expansion path of the firm varies, and how its long-run total, average and marginal-cost curves are affected.
(b) Assume now that the government imposes an upper bound to the amount of energy firms can import. Determine the effects of this additional restriction on the expansion path and the cost curves.
4. The production function of a firm is $Q=F(K, L)=\sqrt{L K}$.
(a) Derive the conditional demands for the factors of production.
(b) Derive the long-run total-cost function for $w=r=1$.
(c) Obtain the long-run marginal-cost and average-cost functions for $w=r=1$.
(d) Obtain the average-cost, marginal-cost and variable-average-cost curves in the short run for $K=25$.
5. A firm produces a good with labor $L$ and capital $K$. The prices of labor and capital are $w$ and $r$, respectively. For each of the production functions given below, calculate the demands of inputs. Calculate the total, average and marginal cost functions when the input prices are $w=r=1$.
(a) $F(L, K)=\sqrt{L+2 K}$.
(b) $F(L, K)=(L-1)^{\frac{1}{4}} K^{\frac{1}{4}}$.
(c) $F(L, K)=2(\min \{2 L, K\})^{2}$.

## B. Multiple Choice Questions

1. If a firm has constant returns to scale, thenits total cost function is strictly concaveits total cost function is strictly convexits marginal cost is less than its average costits average cost is constant.
2. Lolita, the competitive cow in all markets of Holstein, produces milk ( $L$ ) using oats $(A)$ and hay $(H)$ according to the production function $L=\sqrt{A+2 H}$. Hence, as a milk producer Lolita has

## C. The Competitive Firm

1. The production function of a firm is $F(K, L)=4 K L^{\alpha}$, where $\alpha \in(0,1), K$ is the amount of capital, and $L$ is the amount of labor. The wage is $w=2$ and the price of capital is $r=4$. Calculate the firm's total-cost, average-cost and marginal-cost functions, and determine whether the firm has economies or diseconomies of scale.
2. The production function of a firm is $F(L, K)=2 \sqrt[3]{L K}$. The wage is $w=1 / 4$ and the price of capital is $r=1$ euros/unit. Calculate the firm's total-cost, averagecost and marginal-cost functions, and determine whether the firm has economies or diseconomies of scale.
3. For the production functions given in exercise 3 of section B , calculate the short run total cost, average cost and marginal cost functions assuming that in the short run the capital is fixed and equal to $\bar{K}=4$, and the only variable input is labor. Determine whether the firm has economies or diseconomies of scale.
4. Graph the total cost functions (a) $C(Q)=100+Q^{2}$, and (b) $C(Q)=Q^{3}-4 Q^{2}+$ $\frac{37}{3} Q$. In each case, calculate graph the average and marginal cost functions. For which output levels does the firm has economies or diseconomies of scale?

## C. Multiple Choice Questions

1. If a firm has economies of scale, then its average cost isdecreasing and smaller than its marginal costdecreasing and larger than its marginal cost increasing and larger than its marginal costincreasing and smaller than its marginal cost.
2. A firm whose total cost function is $C(Q)=\frac{Q^{2}}{2}+Q$ has
$\square$ economies of scale $\quad \square$ $\square$ decreasing returns to scalediseconomies of scaleincreasing returns to scale.
3. A firm whose total cost function is $C(Q)=5 Q+7$ haseconomies of scaleconstant returns to scalediseconomies of scaleincreasing returns to scale.
4. If a firm has diseconomies of scale, then itsmarginal cost is decreasingaverage cost is decreasingaverage cost is less than its marginal costtotal cost function is concave.
5. Lolita is a competitive cow that produces milk using oats $(O)$ and hay $(H)$ according to the production function $F(O, H)=\min \left\{2 O^{2}, H^{2}\right\}$. Therefore, as a milk producer Lolita hasdiseconomies of scaleconstant returns to scale economies of scalea convex total cost function.

## D. Competitive Markets

1. The production and sale of a certain good is legally forbidden, though that good is illegally traded in a market whose demand function is $D(p)=540 / p$. There are only six firms operating in this market, all of whom are identical and price-taking, using land $(T)$ and labor $(L)$ to manufacture the good according to the following production function $F(L, T)=\sqrt{T L}$.
(a) Given the legal conditions related to this good, the land available to every firm is limited to $T=10$ hectares, being $p(T)=4$ the price of every hectare. The labor price in this market is $p(L)=2$, which doubles the "normal" price because of the risks associated with the manufacturing and distribution of the product. Compute the price and the quantity of the good traded in the market, the quantity sold by every firm and the profits obtained by the group of all owners of the firms.
(b) The legalization of the good is currently under discussion. It is known that if the good were legal - the wage would fall to $p(L)=1$, the amount of land available to firms would not be limited, there would be free entry into the market and the consumers would be better informed. Because of the last fact, the demand would turn out to be $D(p)=720 / p$. Those opposed to the legalization argue that such a good is damaging to human health and legalization would increase its consumption. Compute the increase in consumption following the legalization, and determine how the profits of the whole set of firms would vary.
(c) Those in favor of legalization argue that a tax could be established per unit sold, in such a way that after-legalization consumption would not vary with respect to the current demand. Compute the amount of such a tax and determine the corresponding revenue obtained by the state.
2. The aggregate demand of a good is $D(p)=\max \{150-2 p, 0\}$. The market is supplied by 4 price-taking firms whose average costs are $C M_{e}(q)=\frac{100}{q}-5+q$.
(a) Determine the short-run equilibrium in this industry, specifying the aggregate quantity traded in the market and the profits for every firm.
(b) The total costs of a foreign firm are $C(Q)=8 Q$. Under the assumptions that this industry is opening to foreign trade and that the foreign firm operates as a pricetaker, determine the total quantity sold, the amount of imports, the price and the equilibrium profits (or losses) in the short-run equilibrium for both kinds of firms.
(c) Assume that the authority establishes a price equal to the minimum average cost of the firms from the first group, so that they can break even. Determine the equilibrium in that case. Which is the variation in the consumer surplus?.
(d) How much would it cost to make up for the losses of the firms from the first group without altering the market price? Which of both policies would be preferred by the foreign firm? And by the consumers?.
3. The inverse demand function for a good in a country of the European Union where all the agents are price-takers is given by $P_{E U}(q)=\max \{500-4 q, 0\}$, whereas the national inverse supply function is $P_{N}^{S}(q)=5(1+q)$.
(a) Determine the market equilibrium in the absence of international trade.
(b) Suppose that now the country is open to international trade and that the exchange rate euro/dollar is equal to unity. The inverse supply function by the rest of the world (measured in the national currency unit) is given by $p(Q)=2+20 Q$. Determine the equilibrium situation after aggregating both supply curves. Who gains and who loses in this situation? Do you think that this country should open up to international trade?
(c) Assume that the euro depreciates, so that for one euro you only obtain 0.8 dollars. Determine how would change the supply curve from the rest of the world and the new equilibrium. Who gains and who loses in this situation?.
4. Suppose that, in some industry producing a good, there are n identical firms with a technology characterized by increasing returns to scale up to a certain level of output, say $Q=100$, and decreasing returns to scale from that quantity onwards.
(a) Represent in the same graph the average-cost and marginal-cost curves, and the supply curve of one of these firms, under the assumption that it behaves as a price-taker.
(b) Assume that there are $m$ firms with a technology characterized by decreasing returns to scale from the origin. Assume also that this technology is less efficient than that of the previous group, so that - for any level of production - its total costs are higher. Represent in the same graph the average-cost, marginal-cost and supply curves of any of these firms under the price-taking assumption.
(c) Derive industry's supply curve graphically, and represent the equilibrium industry when the market demand is such that only firms of the first type produce.
(d) Represent a governmental policy of rising the price of the product - with the commitment to finance the generated excess supply - so that there is also a positive production by the $m$ firms of the second group. Determine who gains and who loses with this policy.
5. Assume that there are only two kinds of suppliers of ethanol in the US market: the Brazilian producers and the national ones.
(a) Suppose that the ethanol market is perfectly competitive and, for whatever reason, the Brazilian producers are considerably more efficient. Represent in three different graphs the Brazilian supply curve, the national supply curve and the aggregate supply curve in the US.
(b) Represent an equilibrium situation in which all the traded quantity corresponds to Brazilian producers.
(c) Suppose that a tariff is imposed on the ethanol imports from Brazil. Show a new equilibrium situation in which the traded quantity still corresponds to the Brazilian producers.
(d) Represent a third equilibrium in which, due to a subsidy, the national producers are now able to sell a positive quantity of the total product.
6. Ethanol is obtained from corn. There are two kinds of corn buyers: ethanol producers and corn direct consumers.
(a) Assume that the corn market is perfectly competitive, and represent in three different graphs the demand for corn by both groups of buyers and the aggregate
demand for this product. Represent an equilibrium situation as well.
(b) Due to the subsidy they receive, the corn demand curve by ethanol producers shifts to the right. Analyze the consequences of this policy measure on the corn market. Who gains and who loses in such a complex scenario?.
7. The long-run total-cost function of firm that acts as a price-taker in all markets is $C(Q)=Q^{3}-8 Q^{2}+30 Q$.
(a) Compute and represent the functions of marginal and average costs.
(b) Which is the price under which the firm would choose a zero production level?.
(c) Which is the price for which the firm supplies 6 units of the product?.
(d) Represent graphically the profits that the firm would obtain if the price of the product were 25 monetary units.
(e) Suppose now that, from the situation in which the product price is equal to 25 , the government establishes a tax of 2 monetary units for unit of product. How would affect this measure to the production level and the equilibrium price if all firms were price-takers and had the same technology?. How would affect the tax the firm production level and the firm profits? Represent graphically your answer.
8. The Town Hall of a certain city is studying the economic convenience of building a school in the place that is currently occupied by a sport complex. It is known that the demand curves for "school services" $(x)$ and "sport services" $(y)$ are, respectively, $x=\max \left\{3.000-10 p_{x}, 0\right\}$, and $y=\max \left\{3.000-10 p_{y}, 0\right\}$. It is also known that the consumers have identical preferences. At present $p_{y}=0$, and there is complete free entry in the market for sport services. However, if the school is built, the capacity of the sport complex would decrease until 600 "sport services" per year. Therefore, the Town Hall decides that the most convenient decision would be charging a price for the use of the sport complex, so that the excess demand would be exactly eliminated. Besides, the revenue obtained with this measure could be devoted to financing the school. Suppose that the "school services" are free ( $p_{x}=0$ ).
(a) Compute the surplus obtained by consumers as a consequence of the use of the school.
(b) Compute the loss in consumer surplus subsequent to charging a positive price at the entrance of the sport complex.
(c) Provided that the annual cost of the school is 400.000 monetary units, should it be built according to the implied economic costs and benefits?.
9. Consider an industry where 20 identical firms are operating, each of which has the following production function: $F_{i}(L, K)=\sqrt[4]{L K}$. Assume that the prices of both factors are equal to one and the firm is a price-taker in the factor markets.
(a) Determine the function of total costs of any of these firms.
(b) Suppose that the industry demand function is $D(p)=\max \{100-5 p, 0\}$. Compute the market equilibrium if all 20 firms behave as price takers.
(c) Consider that the state is willing to finance part of its public expenditure by means of a tax per unit sold of the good. Determine the necessary amount of the tax if the desired revenue is 2.5 monetary units.
10. Consider a country whose demand and supply curves of a product in a competitive market are, respectively, $P_{N}^{d}(q)=\max \{400-10 q, 0\}$, and $P_{N}^{s}(q)=10+20 q$.
(a) Determine the equilibrium in the market.
(b) Assume that the country opens up to international trade and that the supply curve of the rest of the world is infinitely elastic at the price of 190 monetary units. Determine the total quantity demanded, the quantity supplied by national firms and the amount of imports in the new equilibrium.
(c) Compute the consumer surplus and the producer surplus, and determine whether the opening up to international trade is beneficial for the country as a whole.
(d) Suppose that the government imposes a quota and a system of import licenses with the purpose of reducing the amount of imports by one half. Which would be the price, the quantity demanded and the quantity supplied by national producers in the new equilibrium?
(e) Which would be the consumer surplus, the producer surplus and the government revenue obtained through import licenses? Is the quota advantageous for the country as a whole?
11. Recently, the government of a certain country has bestowed two licenses of cellphones for the exploitation of this communication system. The concessions are identical and each of them uses two factors to establish telephone communications. Those factors are, first, electricity $(E)$ and, on the other hand, some physical structures we will call "towers" $(T)$. Both variable factors are combined according to the production function $F(E, R)=\sqrt[3]{(L-1) R}$. The prices of the productive factors are equal to one. Assume that the market aggregate demand is $D(p)=\max \{1000-p, 0\}$.
(a) Compute and represent graphically the total, average and marginal-cost curves, and also the supply curve for one of the firms.
(b) The concession of the licenses has been made by means of a contest, and each firm has paid 100 monetary units as a canon to obtain the license. The idea of the government is simple: " If the licenses are cheap, firms will increase their production and will sell their products at lower prices." Compute the price and the equilibrium quantity in the market with 2 price-taking firms.
(c) Compute which is the maximum canon that the government could have charged the firms without forcing them to exit the market, and establish the equilibrium price and quantity under such a canon. Comment the idea expressed by the government in the light of the results.
(d) Assume that the government tries an alternative policy awarding free licenses to the maximum number of firms (identical to those described above) that the market can admit. Compute the price, the equilibrium quantity and the number of firms in the market.
(e) Let us define the total surplus as the sum of the consumer surplus, firms' profits and government revenue obtained through the canon. Order the solutions to questions b), c) and d) in terms of the total surplus.
12. In a market there are 3 firms that behave in a perfectly competitive fashion (that is, they are price-takers). The total-cost functions for each one of them are $C_{1}(Q)=Q^{2}+2 Q+36, C_{2}(Q)=2 Q^{2}+2 Q+10$ and $C_{3}(Q)=Q^{2}+6 Q+6$. We know
that, for the current price existing in the market, the first firm is producing where average cost is minimum.
(a) Calculate the market equilibrium price, and each firm's production and profits?
(b) Calculate the long run market equilibrium price. Which firms survive?
13. The supply and demand schedules for residential housing in Getafe are $S(p)=\frac{p}{2}$ and $D(p)=\max \{600-2 p, 0\}$, respectively, where $p$ thousand euros/unit.
(a) Calculate the equilibrium price and quantity traded (built).
(b) Determine the effect 50 thousand euros subsidy on the number of houses built. Calculate the change in total surplus (make sure to account for the government expenditure).
14. Consider a competitive labor market in which the supply and demand schedules are $L^{S}=5 w$, and $L^{D}=\max \{12-w, 0\}$, respectively, where $w$ is given in euros per hour, and $L$ is given in million hours.
(a) Represent the supply and demand in a diagram and calculate the equilibrium age and employment, and the surpluses of workers and firms.
(b) Determine the effect on the wage and employment of a social security tax of 1 euro per hour paid by firms.

## D. Multiple Choice Questions

2. If a competitive firm produces a positive output, then the market price isequal to its marginal cost, and greater than or equal to its average costequal to its marginal cost, and greater than or equal to its average variable costequal to its average cost, and less than or equal to its marginal costequal to its average variable cost, and greater than or equal to its marginal cost.
3. If the total cost function of a competitive firm is $C(Q)=2 Q^{3}-12 Q^{2}+38 Q$, and the long run equilibrium price is $P=20$, then the firm
$\square$ obtains losses $\quad \square$ produces $Q^{*}=0$ units
$\square$ obtains profits $\quad \square$ produces $Q^{*}=3$ units.
4. There are two technologies for the production of a certain good which generate the cost functions $C_{A}(Q)=Q^{2}+3 Q+1$, and $C_{B}(Q)=2 Q^{2}+Q+6$. The good is traded in a competitive market where demand is $D(P)=\max \{10-P, 0\}$. If there is free entry and neither of these technologies is protected by patents, which of these technologies will survive in a the long run competitive equilibrium?

$$
\begin{array}{ll}
\text { Technology } A & \square \text { Both technologies } \\
\text { Technology } B & \square \text { Neither technology. }
\end{array}
$$

5. In a short run equilibrium of a competitive industry each firm's
$\square$ average cost is less than or equal to the market pricesurplus is greater than or equal to its profitsprofits are zeroprofits are positive.
6. If a competitive firm produces a positive output in the short run, then itsprofit is greater or equal to zeroaverage cost is decreasingtotal average cost is less than the market pricemarginal cost is greater or equal to its average variable cost.
7. If a competitive firm produces a good with total cost $C(q)=q^{3}-6 q^{2}+10 q$, then its supply at the price $p=10$ is$S(10)=6$
$S(10)=4$$S(10)=0$$S(10)=2$.

## E. Monopolistic Firms and Markets

1. The demand curve of a good produced by a monopolist is $D(p)=\max \{20-p / 2,0\}$. Determine and represent graphically the inverse demand function, the total revenue curve and the marginal revenue curve.
2. Intel owns the monopoly in a country's market of microprocessors. During the year 2001 the inverse demand function for microprocessors in the country was $D(p)=$ $\max \{9-p, 0\}$, where quantity is in millions of microprocessors per year. Let us also suppose that nothing is known about the production costs of Intel. Do you think that Intel would sell 7 million microprocessors in such a country? (We assume that Intel is a profit-maximizing monopolist).
3. A monopoly sells a product with a total-cost curve $C(Q)=1200+Q^{2} / 2$. The market demand is given by the function $D(p)=\max \{300-p, 0\}$. Calculate the price and quantity that maximize profits for this monopolist and compute his Lerner Index.
4. A monopolist produces at constant marginal costs, $c=10$, a good with a marketdemand of $D(p)=\max \left\{\frac{1}{4}(210-p), 0\right\}$.
(a) Compute the price and quantity at monopoly equilibrium, and also those that maximize total revenue.
(b) Let us assume that the monopolist's marginal cost increases to $c^{\prime}=20$. Verify that the monopolist's total revenue would fall.
(c) Let us suppose that instead of a monopoly we have a perfectly competitive market in which all firms have a marginal cost $c=10$. Find out the long-run-equilibrium price and quantity in this industry.
(d) Let us assume now that, in the environment corresponding to the previous question, marginal costs for all firms increased and were $c=20$. Check that this rise of marginal costs makes the industry obtain a higher total revenue.
5. Let us imagine that United Airlines enjoyed from a monopoly on the route Chicago-Nebraska. The monthly market-demand on this route was given by $D(p)=$ $\max \{a(t)-b p, 0\}$. The constant $a(t)$ takes the value $a_{w}$ during the winter and $a_{s}$ in the summer, where $a_{w}>a_{s}$. The marginal costs of United are the same in both seasons and independent on the amount of tickets sold. Would United quote a higher price in the summer than in the winter?.
6. The market demand of a good is given by $D(p)=\frac{1}{2} \max \{100-p, 0\}$. The marginal cost of a monopolist in this market is $C^{\prime}(q)=\frac{q}{2}$.
(a) Compute the profit-maximizing quantity and price for the monopolist in this market.
(b) Compute the resulting price and quantity that would be obtained if the monopolist were a price-taker.
(c) Compute the efficiency loss due to the monopoly.
(d) If the demand curve were instead $D(p)=\max \left\{\frac{1}{4}(180-p), 0\right\}$, which would be the irreversible efficiency loss in this case? What is the reason for the difference with the loss in case c)?
7. Let us imagine a market in which a monopolist faces a demand curve given by $D(p)=\max \{I-p, 0\}$, where $I$ stand for the average income of consumers in this
market. We know that the marginal-cost curve of the monopolist is nondecreasing. If the average income of consumers increases, will the monopolist quote a higher price, a lower price or the same one?
8. Consider the case of a monopolistic firm that produces a good using labor and capital. The monopolist takes the factor prices as given.
(a) If at the current production level, the Marginal Rate of Technical Substitution is higher than the ratio wage/price of capital, can the monopolist take any action that raises profits for sure?
(b) Suppose that the monopolist is producing a quantity for which the price of the product is higher than the marginal cost, and the marginal cost is in turn higher than the marginal revenue. Can he take any action that raises profits for sure?
9. The market-demand function for a good is $D(p)=\max \{23-p, 0\}$. The market is served by a monopolist whose total costs function is $C(q)=3 q+q^{2}$.
(a) Determine the quantity traded, the price, the firm's profit and the consumer surplus at the equilibrium choice for the monopolist.
(b) The government decides to nationalize the monopoly and determine its price and production levels under the condition of never incurring losses. (The profits that these activity may generate would be used to reduce taxes.) Determine the optimal values of the previous variables according to that decision rule.
10. Assume we have a natural-monopoly industry, that is, an industry with increasing returns to scale for the relevant output levels, according to the market-demand function of the product.
(a) Represent the average-cost and marginal-cost curves, together with the aggregatedemand curve.
(b) The government wishes to regulate the workings of this industry. It is considering the following rules: i) making the price equal to the average cost; ii) making the price equal to the marginal cost; iii) making the price equal to zero. Which of these norms maximizes the aggregate surplus in this industry, that is, the sum of the consumer and the producer surplus?.
11. The market-demand curve for a good is $D(p)=\max \{700-p, 0\}$. This market is monopolized by a firm whose total-cost function is $C(q)=q^{2}+100 q$.
(a) Obtain the optimal production and the profit level for the monopolist.
(b) Suppose that the government establishes a fixed tax of 10000 monetary units. How would this tax affect the production and profit levels of the monopolist?
(c) Alternatively, suppose that it is established a tax of 20 monetary-units per unit sold of the product. How would this tax affect the production and profit levels of the monopolist?
(d) Finally, suppose that it is established a tax of $10 \%$ over the firm's revenue. How would this tax affect the production and profit levels of the monopolist?
(e) Which of the three previous taxes do you think it is more beneficial for consumers? Why?.
12. The market-demand for a good is $D(p)=\max \{100-p, 0\}$. This market is monopolized by a firm whose total-cost function is $C(q)=q^{2} / 2$.
(a) Determine the quantity produced, the price and the equilibrium profit when the firm behaves as a monopolist.
(b) Suppose now that the monopolist is awarded a subsidy of 50 euros per unit sold. Determine the new equilibrium and represent in the same graph these two initial situations.
(c) Suppose now that there exist other 4 firms with the same total-cost function as the first one. Determine the supply curve of these five firms when all of them act as price-takers. Determine the aggregate production level, the price and the profit of every individual firm in this case.
(d) Which of the three described situations (monopoly, subsidized monopoly, competition between 5 firms) do you think it is more beneficial for consumers?
(e) Would there be any problem for the long-run workings of this market if the number of price-taking firms using that technology were not limited?.
13. An electricity-generating firm produces energy according to the cost function $C(Q)=30 Q$, where $Q$ represents the production measured in thousands of kilowatts per hour. The demand for energy for residential purposes is $D\left(p_{R}\right)=\max \left\{50-p_{R}, 0\right\}$, where $p_{R}$ is the price paid by consumers for 1000 kilowatts per hour. The demand for energy for industrial purposes is $D\left(p_{I}\right)=\max \left\{400-p_{I}, 0\right\}$, where $p_{I}$ is the price paid by industry for 1000 kilowatts per hour.
(a) Determine graphically and analytically the monopoly equilibrium with and without price-discrimination.
(b) Assume we force the monopolist to adopt the competitive solution. How much money should we give the monopolist for him to have the same profits as in the situation of monopoly without discrimination?
14. Individuals $\mathrm{A}, \mathrm{B}$ and C are running the only canteen in the village. Agent A wishes to sell the highest possible number of beer pints without losing money. Agent B wants the canteen to generate the highest possible revenue. Agent C wants to obtain the highest possible profit.
(a) Use a single graph to represent the combinations of prices and quantities defended by each one of the partners. Explain briefly your answer.
(b) Which of the 3 options would be chosen by the representative of consumers?. Why?.
15. Assume that a monopolist faces two types of consumers with demand curves $Q_{1}=D_{1}(p)$ and $Q_{2}=D_{2}(p)$ in a situation in which price discrimination is not possible. Suppose he is selling the quantities $Q_{1}^{*}$ and $Q_{2}^{*}$ to each group at a price $p^{*}$ for both.
(a) If he is informed that $I^{\prime}\left(Q_{1}^{*}\right)=7$ and $I^{\prime}\left(Q_{2}^{*}\right)=2$, what action would you recommend if price discrimination were possible?
(b) Assume that the monopolist finds out that $M C\left(Q_{1}^{*}+Q_{2}^{*}\right)=7$. What action would you recommend after inspecting the whole information?
16. Assume there are 2 groups of consumers whose demands are $D_{1}(p)=\max \{20-$ $p, 0\}$, and $D_{2}(p)=\max \{60-2 p, 0\}$. There exists an singly firm producing at constant marginal and average cost $c=4$.
(a) Represent graphically both demand curves and the corresponding marginalrevenue curves.
(b) We are informed that the monopolist is charging a price of 18 euros to both types of consumers. Compute the production level for both types, the monopoly profits and the surplus of both types of consumers.
(c) Suppose now that the monopolist can price-discriminate. Determine the choices he would make, the profits he would reach and the consumer surplus for both groups of consumers.
17. In a country the market for a good is monopolized by a firm whose total costs are $C(q)=\frac{q^{2}}{2}+10 q$. In that country the aggregate demand for that product is $D(p)=\max \{100-p, 0\}$.
(a) Determine the monopoly equilibrium.
(b) If the State regulated the monopoly with the purpose of maximizing the aggregate surplus (from consumers and the monopolist), with the restriction of imposing no losses to the monopolist, which should be the price and the quantity produced?
(c) Instead of regulating the monopolist, the State opens this market to trade with the rest of the world. The production level of the monopolist is so small in relation to the world market for the product that, once boundaries are open, the monopolist acts as a price-taking firm. The international supply is infinitely elastic at the price of 50 euros. Determine the new equilibrium and indicate whether the monopolist and consumers would be better or worse-off than in b).
(d) Suppose now that, due to the pressure from the monopolist, the government imposes a tariff of 15 euros per unit of product imported. Accordingly, the supply from the rest of the world is infinitely elastic at the price of 65 euros. Determine the new equilibrium. Which the production level of the monopolist? Are his profits higher or lower than those obtained in situations a) and c)?
18. A monopolist faces, in the national market demand of $D(p)=\max \{100-p, 0\}$. Its total cost function is $C(q)=\frac{3}{2} q^{2}$.
(a) Represent graphically the problem of the monopolist.
(b) Which would be his production level and prices if he was forced to behave competitively?
(c) Which is the optimal production and price level?
(d) Assume now that, at the international market price (which can not be affected by the monopolist's choices), the monopolist can export any quantity he wishes. Which quantity would he sell in the international and the interior market if the international price is $p_{I}=90$ ? Which would be his profits?
(e) Which quantity would he sell in the international and the interior market if the international price is $P_{I}=70$ ? Which would be his profits?
19. The national demand for a certain good is $D(p)=\max \left\{\frac{1}{10}(80-p), 0\right\}$. The market is monopolized by a firm whose production costs are $C(q)=3 q^{2}+2 q$.
(a) Determine the market equilibrium.
(b) Suppose now that people from the rest of the world are willing to acquire any quantity the firm sells at a price $p=40$. Compute the price, the quantity sold in the national market and the quantity exported in equilibrium under the assumption that
the firm can price-discriminate, that is, it can charge different prices in the national and international markets.
(c) Finally, suppose that it is forbidden to price discriminate, so that the firm must charge now the same price in both markets. Compute the price, the quantity sold in the national market and the quantity exported in equilibrium.

## D. Multiple Choice Questions

1. A monopolist's Lerner Index isinversely proportional to the absolute value of demand elasticitydirectly proportional to the absolute value of demand elasticitylarger the larger is the monopolist total costlarger the smaller is the monopolist profit.
2. If in a monopolized market it is eliminated a legislation that prohibits third degree price discrimination, thenboth the monopolist profits and total surplus decreasethe monopolist profits increase and total surplus decreasesthe monopolist profits decrease and total surplus increases $\square$ both the monopolist profits and total surplus increase.
3. If a market is monopolized by a firm that produces the good at zero cost, then in the market equilibrium theprice is equal to zeroLerner index is equal to oneconsumer surplus is equal to zeroproducer surplus is equal to zero.
4. With respect to the monopoly equilibrium without price discrimination, third degree price discrimination inducesa decrease of the level of outputan increase of the producer surplusan increase of the price to all consumersa decrease of the surplus of every consumer.
5. If a monopoly produces the good with total cost $C(q)=2 q$ and the market demand is $D(p)=\max \{10-p, 0\}$, then the monopoly's Lerner index is

$$
L=\frac{1}{3} \quad \square L=\frac{1}{2} \quad \square L=\frac{2}{3} \quad \square L=1 .
$$

6. If a monopoly produces the good with zero cost and the market demand is $D(p)=$ $\max \{10-p, 0\}$, then under first degree price discrimination
the deadweight loss is $25 / 2$the total surplus is $25 / 2$the producer surplus is 50 .
