This paper shows how competition generates reputation-building behavior in repeated interactions when the product quality observed by consumers is a noisy signal of firms' effort level. There are two types of firms and "good" firms try to distinguish themselves from "bad" firms. Although consumers get convinced that firms which are repeatedly successful in providing high quality are good firms, competition endogenously generates the outside option inducing disappointed consumers to leave firms. This threat of exit induces good firms to choose high effort, allowing good reputations to be valuable, but its uncompromising execution forces good firms out of the market. (JEL C7, D8)

We are what we repeatedly do. Excellence, then, is not an art, but a habit.
—Aristotle, *Nicomachean Ethics*

In traditional competitive theory, economists assume that market participants have complete knowledge of all relevant factors. This assumption has long been criticized as limiting the applicability of the theory, especially when competition is thought of as a dynamic process. (See, e.g., Friedrich A. Hayek, 1946; Joseph A. Schumpeter, 1950.) Indeed, the shortcomings of this approach are particularly clear when we acknowledge that competition is in a large measure competition for reputation and consumer good will. The costly and apparently unwavering efforts many firms make in order to establish and maintain a reputation for excellence are difficult to account for in the traditional framework.

Consider for instance the markets in which customers can only assess the quality of a seller's product by purchasing and consuming it.

Examples of such “experience good” markets include nondurables such as wine, durables such as appliances and cars, and most service providers such as lawyers, mechanics, and airlines.¹ In these settings, a consumer’s experience with a particular product becomes a precious guide—providing imperfect information about a combination of the seller’s efforts, ability, and luck. In these markets a seller’s reputation for quality therefore becomes a valuable asset.

Since the seminal work of Benjamin Klein and Keith B. Leffler (1981), several authors have shown formally how firms in such markets may be induced to exert costly effort when the fear of losing their reputation exceeds the temporary advantage of cheating their customers. (See Carl Shapiro, 1983; Russell Cooper and Thomas W. Ross, 1984; Bengt Holmstrom, 1999.) These theories of rational reputation building must contend with two fundamental problems, as described by Joseph E. Stiglitz (1989). First, a consumer’s refusal to purchase from a firm that has sold her a low-quality good must also be rational. In particular, it must be optimal for a consumer to end a long relationship with a firm she had considered trustworthy after perhaps just a short string of bad experiences. Second, as a reputation is only valuable if success brings profits, how could those possibly be driven down to zero in a competitive environment?

¹ Phillip Nelson (1974) introduces the concept of an experience good.
However, the most formidable obstacle that a theory of rational reputation building faces remains the argument developed by Holmstrom (1999), in the framework of a monopoly, which will be examined shortly. His reasoning shows why there exists no equilibrium in which a single firm repeatedly exerts high effort.

This paper suggests how these three problems may be simultaneously overcome and theories of competition and reputation reconciled. It offers a model in which many consumers and firms repeatedly trade. In each period, consumers pay up front for a good the expected quality of which increases with the effort exerted by the firm. Some firms, however, are inherently lazy (inept) and always exert low effort. Consumers cannot distinguish inept firms from opportunistic firms, do not observe the effort level exerted, and are thus confronted with both adverse selection and moral hazard problems. On the other hand, consumers do observe the prices posted by firms and (the existence of) their customer bases. Moreover, for those trades in which they were personally involved, consumers of course realize the quality obtained (but since the outcome reflects both the luck and the effort of the firm, this monitoring is imperfect). Consumers may freely switch firms at any time.

I show in this setting how competition supports the existence of equilibria in which opportunistic firms always exert high effort—a result that contrasts sharply with the results obtained for a monopoly. To illustrate the importance of competition for maintaining effort, consider the following version of Holmstrom’s argument (see also George J. Mailath and Larry Samuelson, 1998a). Suppose that in equilibrium a monopolistic (opportunistic) firm chose to exert high effort always, and that its revenues varied continuously with consumers’ expectations about the efforts of the firm. As the firm’s average product quality converges almost surely to the expected quality associated with high effort, consumers become convinced that this firm is opportunistic rather than inept. Since this type of firm is always supposed to exert high effort, over time consumers’ expectations—and hence the firm’s revenues—become inelastic to a bad outcome. In these circumstances, low-quality outcomes will simply be attributed to bad luck. But then, of course, the firm has an incentive to slack off and this proposed equilibrium with high effort unravels.

Under persistent competition, however, firms’ revenues do not vary continuously with consumers’ expectations. Because any consumer can break off her relationship with a firm and take some other preferred option, it does not matter how good a firm is thought to be, but rather whether it is thought to be better than its rivals. This outside option, endogenously generated by competition, is precisely what is required for consumers’ behavior to exert effective discipline over sellers.

This crucial outside option will exist as long as operating rivals with worse, similar, or better reputations charge appropriate prices. In view of the price dispersion that these conditions imply, it may seem that a firm could profitably attract consumers with a slight price decrease. This need not be so, however, if consumers believe that such a firm is bound to exert low effort as a result. Indeed this consumer concern is justified if the sequence of equilibrium prices posted by a firm over time is so low that any further decrease in price necessarily violates its incentives to exert high effort.

In this uncertain environment consumers are able to identify preferred options because they observe both the prices and the customer bases of other firms. Importantly, it is the behavior of the other consumers that ensures that the information conveyed by prices is dependable. In particular, consumer knowledge of customer bases prevents firms from raising their prices in order to mimic rivals with better reputations. If a firm tried to do so, the flight of its old customers would effectively deter any potential new ones.

The model sketched above has many equilibria distinguished by the degree of patience shown by consumers when they experience low quality. In some equilibria, consumers may be willing to stick with a firm in spite of a low-quality experience, provided that the price decreases sufficiently. This paper focuses instead on the simpler, more dramatic case in which consumers are so exacting that they leave a firm as soon as it disappoints them with a low-quality experience. In these equilibria, the firms that retain loyal customers are therefore those that always provide high quality; and the reputations of these firms increase with their age.
Along with the zero-profit assumption, the conditions described above uniquely identify the equilibrium prices and market shares. Prices are shown to rise with a firm’s reputation, a result that accords with the findings of the empirical literature on reputations (see, for instance, Allen T. Craswell et al., 1995). In addition, prices are initially low enough to deter fly-by-night attempts, and eventually reflect a premium for quality.

In the related literature, the exit option mechanism of this paper is close to that of Albert O. Hirschman (1970), though its role is rather complementary. Hirschman discusses how consumers’ exit may constitute a mechanism of recuperation for firms that accidentally fell behind. This paper shows instead why the exit option keeps a firm “on its toes” in the first place. The literature on the market for reputations which focuses on the economics of name trading and on the incentives such markets may generate is also germane (see David M. Kreps, 1990; Mailath and Samuelson, 1998b; Steven Tadelis, 1999, 2000). Research in credit markets is also related (see Douglas W. Diamond, 1989; Harold L. Cole et al., 1995; Mitchell A. Petersen and Raghuram G. Rajan, 1995; Cesar Martinelli, 1997). Diamond (1989), for instance, investigates reputation formation and the evolution of reputation’s mitigating effects on the conflicts of interest between borrowers and lenders. Diamond shows that, with sufficient adverse selection, reputation does not initially provide adequate incentives to borrowers with short credit histories. Over time, however, when a good reputation is acquired, reputation provides improved incentives.

In Section I, I develop a basic model in which the role of competition in easing the problem of preserving reputations appears very clearly. This simple game focuses on the mechanisms of the exit option, their generality and importance, and on the relationship between prices, incentive constraints, and the zero-profit condition. Section II then develops a richer framework, building on the basic model to show how its insights extend to a world with both entry and exit of firms, and in which consumers may choose among firms with diverse reputations. Section III discusses the role of the assumptions and the robustness of the results. Section IV concludes.

I. The Basic Model

This section outlines a model which isolates the role of competition in providing incentives. For simplicity, it does not allow for entry of firms after the beginning of the game and postulates an exit behavior for firms that is justified in the richer model of Section II.

A. The Market

Consider a market in which firms and consumers repeatedly trade. Time is discrete, indexed by \( t = 0, 1, \ldots \) and the horizon is infinite. In each period, firms and consumers may trade. In this event, a consumer pays the firm up front and enjoys either a good outcome or a bad outcome, depending on the unobserved effort level exerted by her chosen firm. Firms are of two types, which is private information. Good firms choose either high or low effort, while bad firms only exert low effort. Normalize the cost of high and low effort to \( c \in (0, 1) \) and 0 respectively. High effort generates a good outcome, or success, with probability \( \alpha \) which is larger than the corresponding probability \( \beta \) generated by low effort. Specifically, \( 0 < \beta < \alpha < 1 \), so that a bad outcome, or failure, may occur even under high effort (all the results also hold for \( \alpha = 1 \)). Firms maximize their payoff, that is, the expected discounted sum of profits. In the initial period, the measure of firms is one, and good firms account for a fraction \( \varphi_0 \in (0, 1) \) of the market.

Consumers (also referred to as clients or customers) are infinitely-lived, and their measure is one. In each period, every firm can serve a continuum of consumers, while each consumer may pick only a single firm.\(^2\) All the consumers of a given firm experience the same outcome. (This simplifying assumption is not necessary for the results, as discussed in Section III). Consumers are Bayesian: they have beliefs over firms’ types and use all the available information to update their beliefs according to Bayes’ rule. They know \( \varphi_0 \), but do not know the type of a particular firm. A consumer maximizes the

\(^2\) Specifically, if the unit measure of consumers was divided equally among the unit measure of firms, each firm would serve a unit measure of consumers.
expected discounted sum of utilities, and prefers good outcomes to bad ones. The utilities associated with a good and a bad outcome are normalized to 1 and 0, respectively. In each period, consumers may either trade or take their outside option, which provides a utility of $\gamma$. The outside option can be thought of as the value of their next best alternative. For instance, $\gamma = 0$ is the outside option if the alternative consists of a separate competitive sector composed only of bad firms. At the end of each period, consumers may freely leave (or quit) their firm and switch to another one, as explained below. The discount rate, $\delta \in [0, 1)$, is common to both firms and consumers.

The game proceeds as follows. In the initial period, consumers observe prices which have been simultaneously set by firms and either take their outside option or trade with a firm randomly selected from the firms posting the price they choose. Firms then simultaneously and independently exert an effort level which generates an outcome for each firm. Consumers then decide whether to stay or leave. More generally, at the end of period $t = 0, 1, \ldots$, after trading with a firm, consumers decide whether to be loyal and remain with their firm, or to quit and become a switching consumer. Quitting is costless. At the beginning of the next period, firms simultaneously set prices or exit. Consumers then simultaneously decide whether to trade or to take their outside option. The loyal consumers only consider trading with the firm previously chosen, while consumers who quit may also choose at that point with which firm to trade. Finally, firms simultaneously exert an effort level which generates an outcome. The timing of moves is summarized in Figure 1.

The following five assumptions about the information structure are important. First, firms observe whether they have loyal consumers before choosing a price. Second, consumers who previously quit observe the price distribution set by firms. Third, loyal consumers observe their own firm’s new price. Fourth, firms observe whether they have consumers willing to trade. Fifth, consumers do not observe the effort level exerted by firms and only observe the outcome generated by the firm they chose.

To complete the description of the information structure, the following assumptions are made, but do not affect the results. They are chosen both because they are simple and minimal, and because they make consumers’ switching less attractive, and therefore reputations harder to preserve.

A client of firm $j$ knows the sequence of prices and outcomes of this firm since she began trading with it. However, she does not observe prices or outcomes of any other firm during that relationship. If she quits firm $j$, she is henceforth unable to distinguish it from other firms. That is, while she recalls all the prices she paid and the outcomes she experienced, she only distinguishes between two kinds of firms at the end of a period: the one with which she just traded, if any, and all the others for which, in the event she quits, she only observes the posted prices. This means that a switching consumer only observes the price distribution and chooses a price rather than a particular firm. It is essential but convenient to assume that a firm posting such a price has then a probability of obtaining her patronage proportional to its actual

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\[FIGURE 1. TIMING OF MOVES IN A PERIOD.\]
size. That is, the probability of obtaining her patronage is proportional to the mass of its loyal consumers, as a fraction of the total size of firms posting such a price. Finally, a consumer who chose her outside option in a period starts the next one as a switching consumer would. Firms observe their own outcomes as well as the price distribution, but not the outcomes of other firms. Furthermore, firms observe the actual measure of their loyal consumers and of those consumers who agree to trade with them. Formal definitions of histories are given in Appendix A.

It is further assumed, in this section only, that firms exit if and only if all their consumers quit (that is, if and only if the measure of remaining consumers is zero). While it is innocuous to assume that a firm does not exit otherwise (after all, it may always choose low effort and set positive prices, which guarantees nonnegative profits), forcing a firm to exit as soon as all its consumers quit is stringent at this point. It captures the intuition, formalized as a result in Section II, that such a firm, having no customer base, is indistinguishable from entrants, and therefore realizes zero profits in a competitive equilibrium.

An equilibrium of this game refers to a Perfect Bayesian Equilibrium in symmetric, pure, Markovian strategies. Markovian strategies are strategies in which consumers’ decisions only depend on firms’ prices and on their own beliefs about the firms’ types, while firms decisions only depend on consumers’ beliefs and willingness to trade. These strategies only depend on payoff-relevant variables and are therefore particularly simple. In each period, consumers decide whether to trade (and possibly with which firm to trade) depending on the prices they observe and the beliefs that they associate with those prices. Their belief about their particular firm is then updated according to the outcome which they observe, given the equilibrium strategies (as are their beliefs about other firms, on the basis of which they decide whether to stay or quit). Similarly, a firm sets its price as a function of the measure of its consumers and their beliefs. Their updated beliefs and measure then determine the effort level chosen by a good firm. It is assumed that the equilibrium actions of each firm are constant on histories in which the set of consumers taking a decision at each point in time differ at most by sets of consumers per firm of measure zero. Equilibrium strategies are symmetric, that is, consumers choose identical strategies, as do firms of a given type. Strategies are formally defined and discussed in Appendix A.

The focus is on establishing conditions under which high-effort equilibria exist; i.e., equilibria in which good firms always exert high effort on the equilibrium path. Of particular interest are nonrevealing high-effort equilibria, which are defined to be high-effort equilibria in which all operating firms’ strategies specify identical prices in each period on the equilibrium path (but that price may of course vary over time), and consumers always trade rather than use their outside option (on the equilibrium path). Although focusing on such symmetric strategies is certainly restrictive at this point, it permits a simple illustration of the impact of competition on incentives.

B. From Competition to Selection

Consider then a nonrevealing, high-effort equilibrium. At the end of each period, partition the consumers into groups according to the history of outcomes and quitting decisions which they have experienced. Since a consumer may follow the strategy of switching in every period \( t' \geq t \), her total equilibrium payoff at the end of period \( t \) is at least the weighted average of the groups’ payoffs. This simply means that, by switching in every period, a consumer can ensure herself a payoff of:

\[
\sum_{s=t+1}^{*} \delta^{s-t} \int_{[0,1]} u'_i di = \int_{[0,1]} \sum_{s=t+1}^{*} \delta^{s-t} u'_i di
\]

where \( u'_i \) is the utility of consumer \( i \) in period \( s \). This implies that all consumers have the same total expected continuation payoff at the end of each period, regardless of her history, for otherwise a consumer of the least favored group would gain from deviating. This in turn implies that all consumers have the same
expected utility per period, and therefore the same beliefs about their current firm. In a high-effort equilibrium, customers of firms that have been repeatedly successful certainly do not leave (these firms would not exert high effort otherwise). Therefore, all consumers must have such a maximal belief about their firm, and thus they must quit as soon as they experience a bad outcome. This establishes:

PROPOSITION 1: In any nonrevealing, high-effort equilibrium, the only firms operating in a given period are those firms which have not experienced a single bad outcome up to then.

Observe that the argument, and therefore the result, do not require strategies to be Markovian or symmetric, and all a consumer needs to observe are the outcomes she experiences. It is certainly not surprising that it is a (symmetric) equilibrium strategy for consumers to quit as soon as they are disappointed, that is, as soon as they experience a bad outcome. It may be more surprising that there are no other outcomes induced by equilibrium strategies. Indeed, if some consumers were willing to show more patience with disappointing firms, one might expect that others would optimally choose to do so too, since they may prefer the potentially valuable information gathered during their relationship to the uncertainty associated with an unknown firm. A “market for lemons” logic is at work here: among those consumers who are supposed to be loyal, those with the most pessimistic beliefs only gain from quitting instead, so that only the most optimistic ones remain loyal. In particular, if there were more than two outcomes, and more desirable outcomes were also more likely to be generated by high effort, then the operating firms would be precisely those which have produced nothing but the best possible outcome in every period.

Notice that in such an equilibrium, by pursuing their own myopic interest, consumers provide firms with the strongest possible incentives to exert high effort. That is, given the equilibrium prices, the worst punishment that can be inflicted upon disappointing firms is the consumers’ best alternative, and the patronage of switching consumers represents the best possible reward for successful firms. As time passes, the dynamics of competition generate the outside option required for consumers to optimally quit after a bad outcome, but not after a good one.

While best for incentives, these equilibria leave little hope of survival for a particular firm, whether good or bad. Since high effort generates a bad outcome with positive probability, a constant fraction of good firms is compelled to exit in every period. This is true even though consumers know that, eventually, almost all firms which operate are of the good type. From this perspective, it may seem that modeling firms as a continuum is more than a technically convenient assumption. The model of Section II shows why this concern is unfounded, and also shows that the unbounded features which this basic model exhibits are not necessary for the argument to hold. However, the reasoning and the result break down if there are switching costs or if the average belief over the effort level of the remaining firms is noisy. This is because, for any given cost, there is a time at which beliefs of consumers are so close to one that the second best outcome does not induce them to quit any more, since posteriors do not change in response to different outcomes.

The focus on nonrevealing equilibria is essential for Proposition 1 to hold. More sophisticated equilibria exist, in which firms with different histories charge different prices and consumers are willing to forgive a firm for a bad outcome. This issue is tackled in Section III. First however, it remains to be established under which conditions nonrevealing, high-effort equilibria do indeed exist, and what the features of equilibrium prices are.

C. The Equilibrium with Reputation

In any nonrevealing, high-effort equilibrium, operating firms are precisely those which have been repeatedly successful. Therefore, the probability assigned by consumer \( i \) in period \( t \) that her current firm is good, consumer \( i \)'s belief \( q_i^t \), is obtained by \( t \) successive applications of Bayes’ rule, conditional on the occurrence of a
good outcome, given that good firms exert high effort, starting with \( \varphi_0 \). This number is denoted by \( \varphi_\ast \). More precisely,

\[
\varphi_i = \alpha \varphi_{i-1} / [\alpha \varphi_{i-1} + \beta (1 - \varphi_{i-1})] \quad t \geq 1.
\]

Because only a fraction \( \alpha \varphi_{i-1} + \beta (1 - \varphi_{i-1}) \) of firms operating in period \( t-1 \) still operates in period \( t \), the mass of consumers per operating firm in period \( t \), or market size, \( n_t \), satisfies:

\[
n_t = n_{t-1} / (\alpha \varphi_{t-1} + \beta (1 - \varphi_{t-1}))
\]

\[
t \geq 1 \quad n_0 = 1.
\]

Three conditions are necessary for a nonrevealing, high-effort equilibrium to exist. First, the equilibrium value of a good firm per consumer in period \( t \), \( V_t \), must exceed the payoff from a one-shot deviation to low effort. That is,

\[
V_t = p_t - c + \alpha \delta n_{t+1} V_{t+1} / n_t
\]

\[
\geq p_t + \beta \delta n_{t+1} V_{t+1} / n_t \quad \forall t \geq 0
\]

where \( p_t \) is the common price charged in period \( t \). Second, consumers must prefer trade to the outside option:

\[
u_t \triangleq \alpha \varphi_t + \beta (1 - \varphi_t) - p_t \geq \gamma \quad \forall t \geq 0.
\]

Finally, in any period, the equilibrium price must be optimal for the firm given consumers’ beliefs, and in particular, given how consumers interpret deviations to out-of-equilibrium prices. If any such deviation is perceived as certainly coming from a bad firm, then there may be many equilibrium price sequences, but all these sequences and associated belief functions need not be equally reasonable.

Equilibria in which all firms, whether good or bad, realize zero profits overall are of particular interest, both because they satisfy a standard assumption of competitive theory that could certainly be derived as a result in a larger game allowing for free entry, and because this condition is rarely compatible with the emergence of reputations in the earlier literature (with the notable exception of Shapiro, 1983). Now good firms are only willing to exert high effort in the initial period if the discounted continuation value per consumer exceeds the cost of effort [as can be seen from equation (4) with \( r = 0 \)]. As a consequence, the initial price \( p_0 \) must be negative if good firms realize zero profits overall. Intuitively, this negative price prevents the outbreak of “fly-by-night” firms that could make a profit by entering the market, exerting low effort, and leaving immediately (as in Shapiro, 1983). With a positive initial price and costless low effort, bad firms would clearly realize zero profits, as they could always choose positive prices and low effort thereafter. Since good firms exert high effort persistently in a high-effort equilibrium and charge a negative price initially, some prices must necessarily exceed cost later on, reflecting a premium for quality. In particular, these premia must keep on being collected frequently enough.

It is instructive to further restrict attention to the special case in which good firms are precisely indifferent between high and low effort in every period. This state of affairs can be thought of as the outcome of Bertrand competition which drives prices down to a level below which it is common knowledge that incentives cannot be maintained if market operations keep on taking place according to the equilibrium outcome. More precisely, such a sequence satisfies the property that no price can be lowered without violating some incentive constraint of the good firm, holding both its other prices and market sizes fixed at their equilibrium levels. It is true that a lower price could be interpreted by sophisticated consumers, in the spirit of forward induction, as a signal from a firm that its future prices and market sizes will also differ from their equilibrium values, possibly in a way such that high effort was and remains persistently optimal. On the other hand, in view of a price cut, consumers’ skepticism may never be more legitimate than when prices are supposed to be at the minimal level compatible with the good firms’ incentives. This motivates the following definition.

**Definition 1:** A competitive equilibrium with reputation, or competitive equilibrium, is a nonrevealing equilibrium in which good firms exert high effort on the equilibrium path, the incentive constraint of equation (4) holds with equal-
ity for any $t \geq 0$, and firms' payoffs are zero in the initial period.

Notice that, since the incentive constraint of the initial period binds in a competitive equilibrium, bad firms realize zero profits overall as long as good firms do, so that one of those constraints is redundant. More importantly, prices are uniquely determined.

PROPOSITION 2: If a competitive equilibrium exists, the equilibrium prices \{p^*_i\}_{i=0}^\infty are given by

\begin{equation}
(6)
    p^*_i = \frac{c}{\delta(\alpha - \beta)} \left( \frac{\varphi t - 1}{\alpha \varphi t - \beta \delta} \right) \forall t \geq 1.
\end{equation}

They are strictly increasing over time and converge to

\begin{equation}
    p_\infty = \left( 1 + \frac{(1 - \delta)/\delta}{(\alpha - \beta)/\alpha} \right) c.
\end{equation}

The proof is in Appendix B. As expected, the initial price is negative, to ensure that firms realize zero profits overall. Prices gradually rise, slowly early on if the fraction $\varphi_i$ is low enough, and eventually converge to a level which reflects a premium for quality. The eventual premium, $(p^*_\infty - c)$, is simply the cost times the ratio of the interest rate $(1 - \delta)/\delta$ over the quality of monitoring, as measured by $(\alpha - \beta)/\alpha$. Perhaps more surprising, prices increase with the initial fraction of good firms $\varphi_i$. If this fraction is high, fewer firms fail in each period, and thus successful firms attract fewer additional consumers, reducing their incentives to exert high effort, which must therefore be conveyed by higher prices. In every period, a fraction of good firms is forced to exit, even though it is common knowledge that the fraction of bad firms vanishes over time. However, good firms expect to operate longer than bad firms, so that the future higher prices provide incentives to good firms without generating rents for bad ones.

Of course, consumers must be willing to trade at those prices. The next theorem, proved in Appendix B, states that this willingness is also sufficient for existence.

THEOREM 1: A competitive equilibrium exists if and only if

\begin{equation}
(7)
    \min\{\alpha \varphi_i + \beta (1 - \varphi_i) - p^*_i\} \equiv \gamma.
\end{equation}

This condition is obviously satisfied whenever the cost of effort is low enough and the outside option $\gamma$ is hardly better than the utility $\beta$ generated by low effort. On the other hand, it is violated if firms are very impatient (small $\delta$) or if monitoring is of bad quality [small $(\alpha - \beta)/\alpha$]. This condition is clearly necessary for existence, as consumers would use their outside option at some point otherwise. If this condition is satisfied, consumers are willing to trade, and as by construction good firms prefer to exert high effort, it only remains to check that firms choose to post the equilibrium prices, given consumers' beliefs. Lower prices are unattractive for firms if such price cuts are interpreted as certainly coming from bad firms, an interpretation which has been motivated before. Higher prices are hardly attractive either, provided that consumers do not believe that such higher prices are more likely to come from a good firm than the equilibrium price is.

Among the unsatisfactory features of this simple model, notice that the equilibrium market size (per firm) increases at a geometric rate. As a consequence, the total value of a firm, $n_i V_i = n_i c/\delta(\alpha - \beta)$, diverges over time. Moreover, the whole argument seems to rely on the persistence of competition among firms which are repeatedly successful. These are inexorable consequences of a model with exit but no entry, but they do not challenge the robustness of the main insights of the paper, as the next section shows.

II. A Richer Model and its Stationary Equilibrium

This section extends the previous model to an economy in which both entry and exit of firms occur. As before, on the equilibrium path, good firms always exert high effort because of their reputation, and consumers quit a firm as soon as
they experience a bad outcome. Moreover, the firms’ decision to exit after a bad outcome is now voluntary, and consumers freely choose among the offerings of firms with various reputations. This global competition determines the distribution of market sizes, in addition to the equilibrium prices.

The previous framework is enriched as follows. Time extends into the infinite past as well as into the infinite future. The total mass of (doubly infinitely lived) consumers is normalized to one, as is the mass of firms entering the market in every period. Because in the stationary model analyzed below, an equal mass of firms exits, the measure of operating firms remains constant over time. A fraction \( \varphi_0 \in (0, 1) \) of entrants are of the good type.

The assumption previously made about exit is dropped. Instead, firms freely decide whether to exit. However, the timing and information structure are modified as follows. As before, consumers decide whether to stay or to go at the end of a period, on the basis of which prices. Then, the measure of operating firms remains constant over time. A fraction \( \varphi_0 \in (0, 1) \) of entrants are of the good type.

The assumption previously made about exit is dropped. Instead, firms freely decide whether to exit. However, the timing and information structure are modified as follows. As before, consumers decide whether to stay or to go at the end of a period, on the basis of which firms then choose a price or exit. Loyal consumers, that is, consumers who chose to stay decide then whether to trade. Those (loyal) consumers who agree to trade form the customer base. Switching consumers, that is, consumers who quit previously, observe the joint distribution of prices and of the existence of a customer base, and decide then with which firm to trade, if with any. Hence, rather than only observing prices, they also observe whether a given firm, as identified by its price, has a customer base or not. (Assuming instead that they observe the actual size of the customer base would only reinforce the results.) This enables switching consumers to make inferences from the price about a firm’s reputation on the basis of possibly dependable information. In view of the well-known attempts made by various suppliers to appear popular, there is little doubt that such information, if available, is indeed a determining factor in consumers’ decisions. No other change is made to the basic model. The timing of moves is summarized in Figure 2.

Although this market cannot be formally modeled as a game, Markovian strategies are still well defined. A nonrevealing, high-effort equilibrium is then a family of Markovian strategies and of consumers’ beliefs such that beliefs are correct, strategies are mutual best responses, and (i) consumers (mixed) strategies are identical, as is the restriction to prices of firms’ pure strategies, and, on the equilibrium path, (ii) good firms always exert high effort, (iii) consumers always trade and quit as soon as they experience a bad outcome, and (iv) firms optimally exit when no loyal consumer remains.

In such an equilibrium, firms of a given age, that is, firms that have been operating for a given number of periods, post the same price. However, firms of different ages may charge different prices. Hence, a variety of alternatives is available to consumers, and it is therefore convenient to allow them to independently randomize, both when they decide whether to stay or to quit, and when they choose among the different offerings. (The measurability issues raised by mixed strategies in such a context, discussed in Robert J. Aumann, 1964, for instance, could be avoided at the expense of expository simplicity.) Observe that, unless consumers leave with probability one even after a good outcome in some circumstances, successful firms always retain some loyal consumers. The logic underlying Proposition 1 applies here too: if any kind of firm of age \( i \) retains loyal consumers, it must be those firms which have been repeatedly successful, and consumers, who could as well trade with the successful ones, must therefore have quit the other unlucky firms from the same generation.\(^5\)

\(^5\) As a referee pointed out, because the equilibria of interest are stationary ones, the formal analysis should be restricted to a generic subset of \([0,1]\) for the probabilities \( \alpha \) and \( \beta \), because, in equilibrium, the exit decision of a
Notice that, for consumers to be indifferent among alternatives, older firms, which in equilibrium have better reputations, also charge higher prices. Hence, the prices set by a firm increase with its age. Suppose furthermore that entrants realize zero profits. First, as before, it implies that the initial price must be negative, since the value per consumer of a firm of age one must exceed the cost of effort. Second, as a firm whose consumers have all quit is indistinguishable from an entrant, such a firm must also realize zero profits and may then just as well exit.

Attention is focused, for the same reasons as before, on a particular class of equilibria.

**Definition 2:** A stationary competitive equilibrium with reputation, or stationary equilibrium, is a nonrevealing, high-effort equilibrium in which the payoff of firms entering the market is zero, and good firms are always indifferent between high and low effort on the equilibrium path.

In a stationary equilibrium, in each period, a measure \(\lambda_i\) of firms of age \(i\) charge price \(p_i\) and each serves a mass \(n_i\) of consumers. Their value per consumer, \(V_i\), does not depend on their type, and therefore satisfies \(V_0 = 0\), as well as \((n_{i+1}/n_i)V_{i+1} = c/\hat{b}(\alpha - \beta)\), all \(i\). Moreover, since consumers must be indifferent among the offerings of firms of different ages, both the equilibrium sequence of prices and market sizes are determinate. Recall that \(\varphi_i\) is simply the probability that a firm is good, given that good firms exert high effort and a string of \(i\) good outcomes occurs.

**Proposition 3:** In any stationary equilibrium, prices \(p_i\) are given by

\[
(8) \quad p_i = (\alpha - \beta)(\varphi_i - \varphi_0) - \frac{\beta}{\alpha - \beta} c \quad i \geq 1
\]

and the mass of firms of age \(i\), \(\lambda_i\), satisfies

\[
(9) \quad \lambda_i = [\alpha \varphi_{i-1} + \beta (1 - \varphi_{i-1})] \lambda_{i-1}
\]

\[i \geq 1 \quad \lambda_0 = 1\]

while the market size of a firm of age \(i\), \(n_i\), satisfies

\[
(10) \quad n_i = \frac{c}{\hat{b}(\alpha - \beta) \varphi_i (\varphi_i - \varphi_0)} n_{i-1}
\]

\[i \geq 1 \quad \sum_{i=0}^{\infty} n_i \lambda_i = 1.
\]

As expected, prices rise over time, from an initial negative level to an asymptotic level which is larger than the cost of effort under the conditions given below guaranteeing existence of a stationary equilibrium. It may appear counterintuitive that prices decrease with the cost of effort, as well as, for \(i\) large enough, with the prior \(\varphi_0\). If the cost of effort increases, for instance, the continuation value per consumer of a good firm of age one must increase, so as to preserve incentives of entrants. This increase is exacerbated by the imperfect monitoring, as measured by \((\alpha - \beta)/(\alpha \varphi_i)\), and therefore exceeds the increase in cost, so that, for entrants to realize zero profits, the initial price must decrease. But older firms have then to decrease their price so as to remain competitive. The growth rate of a firm of age \(i\), \((n_{i+1} - n_i)/n_i\), decreases with its age, but increases with the prior \(\varphi_i\) for \(i\) sufficiently large. Indeed, when the fraction of good firms among entrants increases, the utility derived by consumers from trading with those firms grows, as their price does not depend on \(\varphi_i\). Older firms must then lower their price. Since incentives are conveyed either through future premia or through growth perspectives, lower prices then imply higher growth rates. This is not true on the other hand for younger firms if \(\varphi_0\) is sufficiently low, for then an increase of the prior actually enhances the reputational advantage of young firms over entrants.

It is then easy to understand why consumers may randomize their decisions, to ensure that the prescribed equilibrium distribution of consumers...
across firms obtains. As a consequence, the measure of consumers who quit exceeds the size of the customer bases of unlucky firms.

Two conditions are necessary for the existence of a stationary equilibrium. First, prices must be low enough so that consumers agree to trade. Second, as the measure of consumers is finite, the eventual growth rate of a firm must be accordingly bounded. Specifically, for any choice of (positive) \( n_0 \), the series \( \sum_{i=0}^{\infty} n_i \lambda_i \) must converge, where the terms are recursively defined as in Proposition 3. These conditions are also sufficient.

**THEOREM 2:** A stationary equilibrium exists if and only if

(i) \[ c < \alpha \delta \left( \frac{\alpha - \beta}{\alpha} \right)^2 (1 - \varphi_0) \]

and

(ii) \[ \gamma \leq \alpha \varphi_0 + \beta (1 - \varphi_0) + \frac{\beta}{\alpha - \beta} c. \]

Condition (ii), which ensures that consumers are willing to trade, is satisfied whenever the outside option \( \gamma \) is sufficiently close to \( \beta \). Condition (i), which is equivalent to the convergence of the series aforementioned, gives an upper bound on the cost of effort, equal to the product of the effective discount rate of good firms (that is, the product of the discount and the hazard rates), the (square of) quality of monitoring \( (\alpha - \beta)/\alpha \), and the initial fraction of bad firms. It is intuitive that reputations are easier to preserve when success is statistically more informative, or good firms more patient. With many entrants of the good type, prices of older firms must be set low enough to remain competitive, and the resulting rents from success may be too low to provide incentives. Notice also that this condition implies that \( c < \alpha \beta \). That is, a stationary equilibrium exists only if high effort is the efficient action.

Theorem 2 is proved, and the equilibrium strategies described, in Appendix B. These strategies are the straightforward analogues of those detailed in the previous section and are therefore not discussed here.

**III. Discussion**

**A. Pricing**

In the equilibria examined so far, consumers have been extremely alert, quitting a firm as soon as...
as that firm failed to keep up with the highest standards. As Proposition 1 shows, consumers’ behavior is necessarily exacting in an equilibrium with reputation, as long as firms of a given age set identical prices in spite of possibly different histories. There is however no reason to focus exclusively on such pricing strategies. Firms may compensate consumers for a loss in reputation by an appropriate reduction in prices, so as to retain at least some of their clients. The pay of lawyers, physicians, consultants, or even professional athletes, for instance, typically reflects their recent performances. In such markets, the restriction to nonrevealing equilibria is importantly limiting. As consumers’ exit becomes less threatening, the scope for competitive equilibria with reputation is reduced for two reasons. First, the punishment for failing to provide high quality is alleviated, since unsuccessful firms may keep on operating profitably. Second, since some consumers remain loyal to such firms, successful ones fail to attract as many additional consumers as they would otherwise, reducing accordingly the reward from providing high quality. As a consequence, reputations are harder to preserve, and equilibria with reputation only exist for very low values of the cost of effort and of the outside option. It remains nevertheless possible to analyze, if not to solve explicitly, for such a stationary equilibrium, in which prices of unlucky firms reflect their previous performances, along the lines of the previous section. Prices are pinned down by consumers’ indifference among offerings, so that firms only exit if they experience histories such that their consumers’ beliefs fall below the prior $\varphi_0$, as the profits of firms with such beliefs must be negative, and market sizes are then determined by the binding incentive compatibility constraints (to preserve stationarity, it is however necessary to introduce an arbitrarily small exogenous hazard rate, as there would be otherwise a positive probability that a good firm never exits in equilibrium, while bad firms eventually exit for sure). No additional insights would be gained from such a model, and the degree of sophistication demanded from consumers in such an equilibrium would be tremendous.

**B. Idiosyncratic Outcomes**

Related issues arise if the assumption that all the clients of a given firm observe the same outcome is relaxed. While some industries display such characteristics (i.e., arts, entertainment, etc.), in many others, consumers’ latest experiences are only imperfectly correlated. Thus, suppose that, although the fraction of consumers enjoying a good outcome increases with the effort level, satisfied and dissatisfied consumers necessarily coexist, in sizes determined by the effort level in a deterministic fashion. Specifically, under high (respectively low) effort, a fraction $\alpha$ (respectively $\beta$) of consumers experience a good outcome (of course, a consumer only observes her own realization of the effort level). Assume, to ensure finiteness of the mass of firms operating, that firms may be exogenously forced to exit in any period with probability $\theta > 0$ which can be arbitrarily small. In such a modified framework, a stationary high-effort equilibrium in which entrants realize zero profits exists under conditions very similar to those of the equilibrium derived in the previous section. On the equilibrium path, consumers remain loyal if and only if they experience a good outcome. Firms of age $i$ set price $p_i = (\alpha - \beta) (\varphi_i - \varphi_0) - p_0$, leaving consumers with belief $\varphi_i$ indifferent between staying and switching to an entrant. All firms keep on operating, as some consumers necessarily stay. Thus, the fraction of good firms among firms of a given generation remains constant over time, at level $\varphi_0$, but the relative market share of good firms increases. Accordingly, loyal consumers believe that their firm is good with probability $\varphi_i$, and given the equilibrium prices, optimally quit as soon as a bad outcome occurs. Notice that the quitting decision does not depend here on out-of-equilibrium beliefs. Good firms find it optimal to exert high effort in period $i$ as long as the value per consumer in period $i + 1$ satisfies

$$n_i + 1 \frac{1}{n_i} V_{i+1} \geq c[(1 - \theta)\delta(\alpha - \beta)]$$

for all $i \geq 0$. In a stationary equilibrium then, these incentive constraints bind and are used to determine market sizes, and the zero-profit condition pins down the initial price. Obviously, for $\theta$ small enough, these conditions are similar to the ones of the richer model, and equilibrium exists under analogous restrictions.
The main insights of the paper are thus robust to more sophisticated pricing schemes, as well as to a relaxation of the correlation between quality levels experienced by clients. Reputations can be preserved whenever success guarantees sufficiently high future revenues, both with respect to current expenditures and to future revenues in case of failure. The rewards of success may be conveyed either through price increases, precisely sustainable under competition because of the reputation, or by a widening of the customer base, which presupposes however that potential customers may be able to identify such a firm. This in turn requires that consumers have reliable information about a firm’s reputation, either through the direct evidence of a firm’s popularity or indirectly through the advice of acquaintances. The availability of such information also alleviates the moral hazard problem because it backs up the threat of exit by dissatisfied consumers. The more a consumer knows about the reputation of her firm’s rivals, the more effective the exit option will be. Even if loyal consumers have “insider” information at their disposal, and do not observe other market activity during their relationship, as is assumed in the model developed in this paper, even then, the uncompromising behavior of consumers may disclose precisely the information required for this hard line to remain optimal later on.

\begin{equation}
\delta \frac{n_{i+1}}{n_i} V_i + 1 \\
\in \left[ \max_{0 \leq j \leq k-1} \frac{c_j - c_i}{\alpha_k - \alpha_j}, \min_{k+1 \leq j \leq n-1} \frac{c_j - c_k}{\alpha_j - \alpha_k} \right]
\end{equation}

where the interval is nonempty because action \( k \) maximizes \( \alpha_k - c_k \) by definition of efficiency. If in a competitive equilibrium with reputation, this discounted continuation value is defined to be equal to the lower extremity of the interval, as suggested by the earlier definitions, good firms will optimally be choosing the efficient action in equilibrium.

The limited applicability of the analysis should of course be borne in mind. Although competition may help preserve reputations, severe restrictions have been imposed on parameters to guarantee the existence of equilibria with reputation. When consumers’ information is poor and the control of firms over the quality of the product is limited and costly, competition per se is unlikely to make a difference.

IV. Conclusion

This paper explains why competition helps preserve reputations. Without competition, a firm cannot always work hard to provide a high-quality product, when the clients’ monitoring is imperfect and their up-front payment continuously depend on the firm’s reputation, because the firm’s temptation to shirk becomes irresistible as its reputation improves. Competition, on the other hand, endogenously generates the outside option for consumers that is necessary to keep firms on their toes, as it gives consumers the power of choosing between the offerings of rival suppliers whose prices adjust to their reputation. This threat of exit provides incentives for firms to try their best to keep up with the standards of the industry, but its uncompromising execution forces able but unlucky firms out of the market. As put by Andrew Carnegie, “while the law [of competition] may be...
sometimes hard for the individual, it is best for the race, because it ensures the survival of the fittest in every department.” This competitive pressure works best if the information about a firm’s reputation transmitted through prices is reliable, as is the case if it is backed up by the visible behavior or advice of an experienced customer base.

Now competition per se does not guarantee that reputations are valued and moral hazard problems thereby solved. On the contrary, the analysis emphasizes the conditions which are prerequisites for such a healthy competition. Customer competence, for instance, which affords many problems of policy and implementation, is an important ingredient. The main point is rather that a theory of reputations which fails to take competition into account neglects an important aspect of economic life. To paraphrase Schumpeter (1950), “even if correct in logic as well as in fact, it is like Hamlet without the Danish prince.”

Appendix A

This Appendix defines formally histories and strategies for the basic model.

Histories

Let $\mathcal{F}$ denote the set of Borel measures on $\mathbb{R}$, with total masses smaller than or equal to 1. Define:

$$R^S = \bigcup_{F \in \mathcal{F}} F \times (\{O\} \cup (\text{Supp } F \times \{G,B\} \times \{Q\}))$$

$$R^I(\sigma) = [ \bigcup_{F \in \mathcal{F}} F \times \text{Supp } F \times \{G,B\} \times \{S\} ]$$

$$\times [ \mathbb{R} \times \{T\} \times \{G,B\} \times \{S\} ]^t$$

$$\times [ \mathbb{R} \times \{O\} \cup (\{T\} \times \{G,B\} \times \{Q\}) ]$$

$$R^I = \{ R^I(\sigma) | \sigma \geq 0 \}.$$  

An element of $R^S$ describes the information collected during one period by a switching consumer who leaves in the following period, either because she chose her outside option ($\{O\}$), or after trading once with a firm [observing first a price distribution $F$, then, in case she accepts to trade, choosing a price in the support of the price distribution, Supp $F$, observing then an outcome, either good (G) or bad (B), and deciding then to quit ($Q$)]. An element of $R^I(\sigma)$ describes the information collected by a consumer during $\sigma + 2$ periods, from the point at which she initially chose a firm as a switching consumer, until she quits that particular firm or chooses her outside option, after having been loyal for $\sigma + 1$ consecutive periods ($T$ refers to her decision to accept to trade after observing a price in $R$, while $S$ refers to her decision to stay at the end of a period). To formalize duration, define $I(\cdot)$ by $I(r^S) = 1 \forall r^S \in R^S$, $I(r^I) = \sigma + 2 \forall r^I \in R^I(\sigma)$. Define finally

$$I(R^I)(\sigma) = \bigcup_{F \in \mathcal{F}} F \times \text{Supp } F \times \{G,B\}$$

$$\times \{S\} \times [ \mathbb{R} \times \{T\} \times \{G,B\} \times \{S\} ]^n$$

as the collection of elements describing the information collected during a relationship which has not been “ended” after $\sigma + 1$ periods and let $I(R^I) = \bigcup_{\sigma \geq 0} I(R^I)(\sigma)$, and for $i(R^I) \in I(R^I)$, $I(i(R^I)) = \sigma + 1$ whenever $i(R^I) \in I(R^I)(\sigma)$. Define then

$$H^C_0 = \emptyset$$

and, $\forall t \geq 1$,

$$H^C_t(n) = \bigcup_{i=1}^n E_i \in R^I \cup R^S, \forall i < n,$n

$$E_n \in R^I \cup R^S \cup I(R^I), \sum_{i=1}^n I(E_i) = t$$

$$H^C_t = \bigcup_i H^C_t(n)$$

Hence, a history $h^C_t$ for a consumer at the beginning of period $t$, or history of length $t$, consists of a succession of $n$ “relationships,” for some $n$, with firms, some to which she may have been loyal for a while, others which she immediately quit after joining them (and some “relationships” in which she preferred to use her outside option), and her “current” relationship, the $n$th one, which may not be over yet. Define $H^C_t(Q)$ as the set of histories $h^C_t \in H^C_t(n)$, for
some $n$, with $E_n \in R^t \cup R^\infty$, that is, as the set of histories of length $t$ such that the consumer starts period $t$ as a switching consumer, and define $H^t_0(S) = H^t_0 H^t(S)$ as the set of histories of length $t$ such that the consumers starts period $t$ as a loyal consumer. Histories of firms are simpler. Define:

$$H^t_0 = \emptyset$$

$$H^t_i = [\bigcup_{F \in \mathcal{F}} F \times \text{Supp } F \times R_+ \times \{H,L\} \times \{G,B\} \times R^+_j]$$

$$H^t = \bigcup_{i=0} H^t_i.$$  

In every period in which a firm operates, a firm observes the price distribution and the price it posts itself, the mass of consumers which accept to trade, the effort level exerted, the outcome obtained, and the mass of consumers staying. As a firm exits as soon as all consumers quit, histories in which the mass of consumers staying is zero are ignored. Elements of $H^t_i$ ($H^t(S)$ and $H^t_i(S)$) for consumer $i$ are denoted by $h^t_i(Q)$ and $h^t_i(S)$, respectively, and similarly elements of $H^t$ for firm $j$ are denoted by $h^t_j$.

**Strategies**

A strategy for consumer $i$ specifies for each $t \geq 0$, and each history $h^t_i \in H^t_i$, two decisions: whether to trade or not, and possibly with which firm, and at the end of the period, whether to stay or quit, in case she has not just chosen the outside option. That is, it consists of two mappings, $\xi_i(X), \rho_i(X), X \in \{S,Q\}$:

$$\xi_i(Q): H^t_i(Q) \times \mathcal{F} \to R \cup \{O\}$$

$$\xi_i(S): H^t_i(S) \times \mathcal{F} \to \{T,O\}$$

$$\rho_i(Q): H^t_i(Q) \times \{F \times \text{Supp } F\} \times \{G,B\} \to \{S,Q\}$$

$$\rho_i(S): H^t_i(S) \times \{T\} \times \{G,B\} \to \{S,Q\}.$$  

Thus, the mapping $\xi_i$ specifies whether consumer $i$ agrees to trade in period $t$, and possibly at which price, while $\rho_i$ specifies whether the consumer accepts to stay or quits.

A strategy for a good firm $j$ consists of two mappings, which for each $t \geq 0$ and each history $h_j^t \in H^t_j$ specify which price to post and whether to exert high effort. That is,  

$$p^t_j: H^t \to R_+ \cup \{O\}$$

$$\tau^t_j: \mathcal{F} \to \{H,L\}.$$  

A strategy for a bad firm $j$ is a function $p^t_j$, for each $t \geq 0$, mapping $H^t_j$ into reals.

**Markovian Strategies**

The belief $\theta'$ of consumer $i$ about the proportion of good firms and her belief $\theta$ about her current firm, if any, are the only variables which are directly payoff-relevant to her, along with the prices she may observe. The state of the game for consumer $i$ is defined to be her belief $\omega^i \in \Omega$, about the distribution of $(\theta', \theta) \in \{0,1\}$.  

Markovian strategies for consumer $i$ are defined as follows. If $\Omega_i$ is induced by a history $h^t_i \in H^t_i(S)$, for some $t$, then consumer $i$ decides whether and with which firm to trade according to the mapping $\xi^i(Q)$:

$$\xi^i(Q): \Omega_i \times \mathcal{F} \to R \cup \{O\}$$

$$\text{with } \xi^i(Q)(\omega, F) \in \text{Supp } F \cup \{O\}$$

while if $\Omega_i$ is induced by a history $h^t_i \in H^t_i(S)$, she decides whether to trade according to the mapping $\xi^i(S)$:

$$\xi^i(S): \Omega_i \times R \to \{T,O\}.$$  

Her decision to stay or switch is given by:

$$\rho^i: \Omega_i \to \{S,Q\}.$$  

The state of the game for firm $j$ is similarly defined as its belief $\omega^1 \in \Omega_j$ about the distribution of $(\theta', \theta) \in \{0,1\}$. Markovian strategies for (good) firm $j$ are given by mappings:

$$p^1_j: \mathcal{F} \to R_+ \cup \{O\}$$

$$\tau^1_j: \Omega_j \times \{H,L\}.$$
The pricing rule \( p^j \) specifies a price as a function of the mass of loyal consumers and of \( j \)'s beliefs. The effort decision \( \tau^j \) specifies the effort level as a function of the mass of consumers trading and of \( j \)'s beliefs. Similarly, a Markovian strategy for a bad firm \( j \) is a pricing rule \( p^j: \mathbb{R}_+^n \times \Omega_j \rightarrow \mathbb{R} \).

In the equilibria defined below, however, consumers strategies only depend on prices and on their beliefs about their firm and about the proportion of good firms (as these strategies are symmetric, it is straightforward to extend their domain to the state space \( \Omega_j \)). The notation used hereafter is accordingly simpler.

**APPENDIX B**

**PROOF OF PROPOSITION 2:**

Recall equation (4):

\[
V_t = p_t - c + \alpha \delta \frac{n_{t+1}}{n_t} V_{t+1}
\]

\[\geq p_t + \beta \delta \frac{n_{t+1}}{n_t} V_{t+1}, \quad \forall t \geq 0 \text{, as well as equation (3):}
\]

\[n_t = \alpha \varphi_{t-1} + \beta (1 - \varphi_{t-1}) \quad t \geq 1, \quad n_0 = 1.
\]

Using equation (4), one gets that

\[
V_{t+1} = \frac{n_t}{n_{t+1}} \left( \frac{c}{\delta (\alpha - \beta)} \right).
\]

Substituting back into equation (4), both for \( V_t \) and \( V_{t+1} \), one obtains:

\[
\text{(B1) } p_t^* = \frac{c}{\delta (\alpha - \beta)} \left( \frac{n_{t-1}}{n_t} \varphi_{t-1} - \beta \delta \right)
\]

and upon using equation (3),

\[
\text{(B2) } p_t^* = \frac{c}{\delta (\alpha - \beta)} \left( \frac{\alpha \varphi_{t-1}}{\varphi_t} - \beta \delta \right)
\]

It is clear from the last expression of equation (B2) that \( \{ p_t^* \}_{t=1}^\infty \) is an increasing sequence, and converges to:

\[
p_t^* = \frac{c}{\delta (\alpha - \beta)} \left[ (\alpha - \beta) + \beta (1 - \delta) \right]
\]

\[= \left( 1 + \frac{(1 - \delta)/\delta}{(\alpha - \beta)/\alpha} \right) c.
\]

As for \( p_0^* = -[\beta/(\alpha - \beta)]c \), it follows from \( V_0 = 0 \), given that

\[V_1 = \frac{n_1}{T} \left( \frac{c}{\delta (\alpha - \beta)} \right).
\]

Direct inspection yields \( p_0 < p_1 \).

**PROOF OF THEOREM 1:**

As by construction, good firms are willing to exert high effort and consumers are willing to trade on the equilibrium path under the assumption of the theorem, it remains to show that firms do not want to deviate from equilibrium pricing and to complete the description of strategies and beliefs.

Define consumer \( i \)'s beliefs as follows. At the end of period \( t - 1 \), consumer \( i \)'s belief about the proportion of good firms remaining in period \( t \), \( \theta^i \), equals \( \varphi_t \) [defined by equation (2)]. Define then, for \( F \in \mathcal{F} \), and \( \theta^i \in [0,1] \), \( \Phi_{\theta^i}[F] \): Supp \( F \rightarrow [0,1] \) by \( \Phi_{\theta^i}[F](p) = \varphi_t \), if \( p = p_t \) and \( \theta^i = \varphi_t \) and \( \Phi_{\theta^i}[F](p) = 0 \) otherwise. This mapping is used by consumer \( i \), after a history \( h^i_t \in H^i_t(Q) \), to update her beliefs, given her prior beliefs and the observed price distribution, and also determines her belief after her own choice (to trade and with which firm to trade). After a history \( h^i_t \in H^i_t(S) \), consumer \( i \) uses the rule \( \Phi_{\theta^i}: \mathbb{R} \rightarrow [0,1] \) defined by \( \Phi_{\theta^i}(p) = \varphi_t \), if \( p = p_t \) and \( \theta^i = \varphi_t \), and \( \Phi_{\theta^i}(p) = 0 \) otherwise, to update her beliefs given the price she observes. After updating her belief either way, she revises them again, in the event she agrees to trade, using Bayes’ rule \( \theta^i: [0,1] \times (G,B) \rightarrow [0,1] \), given that good firms exert high effort and that either a good outcome \( G \) or a bad one \( B \) is observed.
After a history $h_i' \in H_i^e(Q)$, consumer $i$ decides whether to trade and with which firm to trade according to the mapping $\xi_i^e(Q)$ defined as follows. Given $\Phi_p[F]$, define $\xi_i^e(Q) : F \to \text{Supp } F \cup \{0\}$ by $\xi_i^e(Q)(F) = p$ where $p$ maximizes $\alpha \Phi_p[F](p) + \beta(1 - \Phi_p[F](p)) - p$ over $p \in \text{Supp } F$ if this maximum exceeds $\gamma$, and let $\xi_i^e(Q)(F) = 0$ otherwise. Hence, $\xi_i^e(Q)(F) = p = \min_{p \in \text{Supp } F} \alpha \Phi_p[F](p)$, or $\xi_i^e(Q)(F) = p_*$, or $\xi_i^e(Q)(F) = 0$. Notice in particular that $\xi_i^e(Q)(F) = p$ only if $p < 0$. After a history $h_i' \in H_i^e(S)$, consumer $i$ decides whether to trade according to the mapping $\xi_i^e(S) : \Re \times [0,1]$ defined by $\xi_i^e(S)(p, \varphi_i') = T$ if $\alpha \varphi_i' + \beta(1 - \varphi_i') - p \geq \gamma$ and by $\xi_i^e(S)(p, \varphi_i') = O$ otherwise, where $\varphi_i'$ is her belief after observing price $p$ (in equilibrium, of course, $\varphi_i' = \Phi_p[p]$).

In the event in which consumer $i$ traded in period $t$, her switching decision is then taken according to $\rho_0 : [0,1] \to \{S,Q\}$, defined by $\rho_0(\varphi_i') = S$ if and only if $\varphi_i' \geq \phi(\theta_i, G)$: she stays if and only if her belief updated using Bayes’ rule, $\varphi_i'$, exceeds or equals her belief revision about the averaging operating firm, $\phi(\theta_i, G)$, given that only firms having experienced successes still operate in period $t + 1$.

Observe at this point that if firm $j$ does not set price $p_i$ in period $t$, its consumers leave at the end of the period, and will have traded in period $t$ only if the price posted, $p_i^*$, was sufficiently negative. Hence, firm $j$ cannot realize positive profits by deviating from the equilibrium price.

Firm $j$’s belief about the distribution of beliefs of consumers about its own type and about the fraction of good firms remaining in the economy, $f_i' \in F_i'$ and $g_i' \in G_i'$, respectively, is defined in the obvious way: firm $j$ believes that all consumers have common belief $\theta_i = \varphi_i$ at the beginning of period $t$. If it has any loyal consumer, it also assigns probability one that they all believe that firm $j$ is good with probability $\varphi_i$. These beliefs are then updated in accordance with the belief functions of consumers and their description is omitted. Define then, both for good and bad firms, the pricing rule $p_i' : F_i' \times G_i' \times \Re \to \Re$ as mapping its beliefs and the measure of its consumers into the price that maximizes payoffs given consumers’ strategies. In equilibrium, $f_i'$ and $g_i'$ are degenerate and assign probability one to the distribution assigning probability one to the belief $\varphi_i$ in period $t$. Similarly, for good firms, define $\varphi_i' : F_i' \times G_i' \times \Re_+ \to \{H,L\}$ as mapping firm $j$’s beliefs and the measure of its consumers into the effort level which maximizes payoff. Obviously, high effort is specified by this mapping on the equilibrium path.

**PROOF OF PROPOSITION 3:** Notice that the binding incentive constraints imply that:

$$p_i = \frac{c}{\delta(\alpha - \beta)} \left( \frac{n_{t-1} - \beta \delta}{n_t} \right)$$

and $p_0 = - \frac{\beta}{\alpha - \beta} c$. However, consumers must be indifferent among offerings, or for that matter, indifferent between what a firm of age $i > 0$ offers and what an entrant offers. That is,

$$\alpha \varphi_i + \beta(1 - \varphi_i) - p_i = \alpha \varphi_0 + \beta(1 - \varphi_0) - p_0$$

from which it immediately follows that

$$p_i = (\alpha - \beta)(\varphi_i - \varphi_0) = \frac{\beta}{\alpha - \beta} c.$$

Combining equations (B3) and (B4), one gets

$$n_i = \frac{c}{\delta(\alpha - \beta)^2(\varphi_i - \varphi_0)} n_{t-1} \forall i > 0.$$

As before, since all firms which produce a failure exit, it must be that

$$\lambda_i = (\alpha \varphi_{i-1} + \beta(1 - \varphi_{i-1})) \lambda_{i-1} \quad i \geq 1$$

and $\lambda_0 = 1$ by normalization. By normalization also, $\sum_{i=0}^{\gamma} \eta_i \lambda_i = 1$, provided that there exists $n_0 > 0$ such that the series defined by the left-hand side coverage, which is the case whenever an equilibrium exists, as shown below.

**PROOF OF THEOREM 2:**

First, consider the partial sums $\sum_{i=0}^{\gamma} n_i \lambda_i$, defined for $n_0 > 0$, $\lambda_0 = 1$, and where $n_i, \lambda_i$, $i > 0$ are defined by induction in equation (B5) and (B6) for $n > 0$. Notice that the (positive) ratio $n_{i+1} \lambda_{i+1} / n_i \lambda_i$ converges to
\[\alpha c/\delta (\alpha - \beta)^2(1 - \varphi_0)\alpha \text{ as } i \to \infty. \text{ Hence, the series converge for } c < \delta (\alpha - \beta)^2(1 - \varphi_0)/\alpha \text{ and diverge for } c > \delta (\alpha - \beta)^2(1 - \varphi_0)/\alpha. \text{ Suppose then that } c = \delta (\alpha - \beta)^2(1 - \varphi_0)/\alpha. \text{ In that case, observe that}
\]
\[
i \left(\frac{n_{i+1} \lambda_{i+1}}{n_i \lambda_i} - 1\right) \to \frac{i \left(\frac{\beta}{\alpha} - \alpha \varphi_0 + \beta (1 - \varphi_0)\right)}{\alpha \varphi_0} \to 0
\]
and the series diverge by Raabe’s test. Hence the series converge if and only if
\[
c < \frac{\delta (\alpha - \beta)^2(1 - \varphi_0)}{\alpha}
\]
and, as these (positive) series are homogeneous of degree 1 in \(n_0\), one may always pick \(n_0\) so that \(\sum_{i=0}^{n_0} \lambda_i = 1\), which was the desired normalization.

Second, observe that consumers are willing to trade on the equilibrium path as long as the utility from trade exceeds \(\gamma\), but as the utility from trade does not depend on the age of the firm by construction, it is sufficient that trade with an entrant is preferred to the outside option. That is,
\[
\gamma \leq \alpha \varphi_0 + \beta (1 - \varphi_0) + \frac{\beta}{\alpha - \beta} c
\]
which is precisely condition (B2).

It remains, as for Theorem 2, to construct an equilibrium by defining strategies, belief functions and by verifying that strategies are best responses, given beliefs. As the model has no initial node, histories are complicated objects whose definition is therefore omitted. To avoid confusion, consumers are indexed by the letter \(k\), firms by \(j\), and \(i\) refers to the age of a firm. As before, it is useful to distinguish sets of histories \(H^f_i(Q)\) and \(H^f_i(S)\), corresponding to sets of histories up to period \(i\) along which consumer \(k\) has quit her firm or used her outside option at the end of period \(i - 1\) and to sets of histories along which she stayed with her firm, respectively. Let \(n_{-1} = 0\) and define \(i^* \in \mathbb{N} \cup \{\infty\}\) such that \(n_i \equiv n_{i+1} \forall i \leq i^* - 1\), and \(n_i \equiv n_{i+1} \forall i \geq i^*\), where the sequence \(\{n_i\}_{i=0}^{\infty}\) is given by Proposition 3 (as \(n_{i+1}/n_i\) decreases in \(i\), \(i^*\) is uniquely determined). Also, let \(\mathcal{F}\) now be the set of (joint) distributions over \(\mathbb{R} \times \{Y, N\}\), with generic element \(F\), where \(Y\) (\(N\)) refers to the presence (absence) of a customer base, and let \(\sigma_i\) be the first coordinate mapping. For histories \(h^k_i \in H^f_i(Q)\), and for any \(F \in \mathcal{F}\), define then the updating rule \(\Phi[F]\): \(\text{Supp } F \to [0,1]\) as follows:
\[
\Phi[F](p,Y) = \varphi_i, \text{ if } p = p_i, \\
\Phi[F](p,N) = \varphi_{i*} \text{ if } p = p_{i*}, \\
\Phi[F](z) = 0 \text{ otherwise.}
\]

For any \(F \in \mathcal{F}\), given \(\Phi[F]\), define the function \(\xi^d(Q)\): \(\text{Supp } F \cup \{O\} \to [0,1]\) which maps choices (firm or outside option) into probabilities. Specifically, if \((p_i,Y) \in \text{Supp } F \forall i \geq 1\), \((p_{i*},N) \in \text{Supp } F\) and if they all achieve max\(\gamma \in \text{Supp } F\) \(\alpha \Phi[F](z) + \beta (1 - \Phi[F](z)) - \sigma_i(z)\), larger than \(\gamma\), then let:
\[
\xi^d(Q)(p_{i+1}, Y) = \frac{\lambda_{i+1} (n_{i+1} - n_i)}{\sum_{j=1}^{n_{i+1}} \lambda_{i+1} (n_{i+1} - n_i)}
\]
\[
\xi^d(Q)(p_{i*}, N) = \frac{\lambda_{i*} (n_{i*} - n_i)}{\sum_{j=1}^{n_{i*}} \lambda_{i*} (n_{i*} - n_i)}
\]
and \(\xi^d(Q)(z) = 0\) for any other \(z \in \text{Supp } F \cup \{O\}\).

Otherwise, if the maximum defined above is larger than \(\gamma\), let \(\xi^d(Q)\) assign equal weight to all the maximizers and no weight to other elements of \(\text{Supp } F \cup \{O\}\). Finally, if the maximum is below \(\gamma\), let \(\xi^d(Q)(O) = 1\).

For histories, \(h^k_i \in H^f_i(S)\), define \(\Phi: [0,1] \times \mathbb{R} \to [0,1]\) by \(\Phi(\varphi_i, p) = \varphi_i\) if \(p = p_i\) (for some \(p_i\) of the sequence defined in Proposition 3) and consumer \(k\)'s prior belief \(\varphi^d = \varphi_i\) and by \(\Phi(\varphi, p) = 0\) otherwise. Define then \(\xi^d(S): [0,1] \times \mathbb{R} \to \{T, O\}\) by \(\xi^d(S)(\varphi, p) = T\) if \(\alpha \varphi + \beta (1 - \varphi) - p \equiv \gamma\), and by \(\xi^d(S)(\varphi, p) = O\) otherwise.
Consumer $k$’s belief is thus updated by applying either $\Phi$ or $\Phi^{-1}$. This revised belief is then again updated according to Bayes’ rule, by applying the mapping $\Phi$ $[0,1] \times (G,B) \rightarrow [0,1]$, given that good firms exert high effort and the outcome $X \in \{G,B\}$ is observed. As a function of this belief, consumer $k$ then decides whether to stay or quit. That is, her decision is a mapping $\rho^k : [0,1] \rightarrow [0,1]$, specifying the probability with which consumer $k$ quits given her belief. As pointed out in footnote 4, the attention is restricted to parameters $\alpha$ and $\beta$ such that the belief generated by a string of good outcomes is different from the belief generated by a string of good outcomes followed by a bad outcome, irrespective of the lengths of these strings (it is straightforward to verify that this condition determines a generic subset of $[0,1]^2$). Let then $\rho^k(\varphi_{i+1}) = 0$ for $i \leq i^* - 1$, $\rho^k(\varphi_{i+1}) = 1 - n_{i+1}/n_i$ for $i \geq i^*$ [where $\varphi_i$ is given by equation (1)] and $\rho^k(\varphi) = 1$ otherwise.

It remains to specify the firms’ strategies. Those are exactly defined as in the basic model, except that $p^i : \mathbb{R}_+ \times F^i \times G^j \rightarrow \mathbb{R} \cup \{E\}$, where $E$ stands for exit, which is chosen if and only if the payoff from any other strategy is nonpositive, unless $j$ is an entrant, in which case $p^i$ takes the value $p_{0,j}$. As in the basic model, a deviation in price from the specified sequence is punished by beliefs which are so pessimistic that only a negative price may prevent loyal consumers from taking their outside option. As all consumers leave for sure at the end of the period, such a deviation cannot therefore be worthwhile. High effort is optimal for good firms by construction, and exit is voluntary as firms without consumers cannot do better than zero profits. It is straightforward to verify that consumers’ behavior is optimal too. This concludes the argument.

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