

Consider an economy in which there are two goods x and y , one firm whose production set is

$$Y = \{(-x, y) \in \mathbb{R}_- \times \mathbb{R}_+ \mid y \leq 2\sqrt{x}\},$$

and two consumers with identical preferences, represented by $u(x, y) = xy$, and endowments, $(\bar{x}_i, \bar{y}_i) = (24, 0)$, and with fractions $\theta \in [0, 1]$ and $1 - \theta$ of the firm's property.

Calculate the set of Pareto optimal and competitive equilibrium allocations for $\theta = 1$ and $\theta = 1/2$.

THE CE OF THE ECONOMY

w : price of good x (leisure) } $w = \frac{w}{p}$: real wage
 p : " " " " (consumption) }

Firm

$$\max_{x \geq 0} p(2\sqrt{x}) - wx \Leftrightarrow \max_{x \geq 0} 2\sqrt{x} - wx$$

Solution: $\frac{1}{\sqrt{x}} = w \Rightarrow x_f^d(w) = \frac{1}{w^2}$ (Firm's Labor Demand)

Hence: $y^s(w) = 2\sqrt{x^d(w)} = \frac{2}{w}$. $\pi(w) = \frac{2}{w} - w\left(\frac{1}{w^2}\right) = \frac{1}{w}$

Consumer

$$\max_{(x,y) \in \mathbb{R}_+^2} xy$$

$$\text{s.t. } wx + y \leq 24w + \pi(w)\theta_i$$

$$\begin{cases} \text{MRS} = \frac{y}{x} = w \\ wx + y = 24w + \pi(w)\theta_i \end{cases}$$

Here

$$x_i(w) = 12 + \frac{\pi(w)\theta_i}{2w} = 12 + \frac{\theta_i}{2w^2} \quad (\text{WORKER'S LABOR SUPPLY})$$

$$y_i(w) = 12w + \frac{\theta_i}{2w}$$

MARKET CLEARING :

$$(24 - x_1(w)) + (24 - x_2(w)) = x^d(w)$$

$$24 - \frac{\theta_1 + \theta_2}{2w^2} = \frac{1}{w^2} \Leftrightarrow 24 = \frac{3}{2} \left(\frac{1}{w^2} \right) \Leftrightarrow w^* = \frac{1}{4}$$

CE : FIRM'S LABOR : $x_f(w^*) = \left(\frac{1}{w^*} \right) 2 = 16$

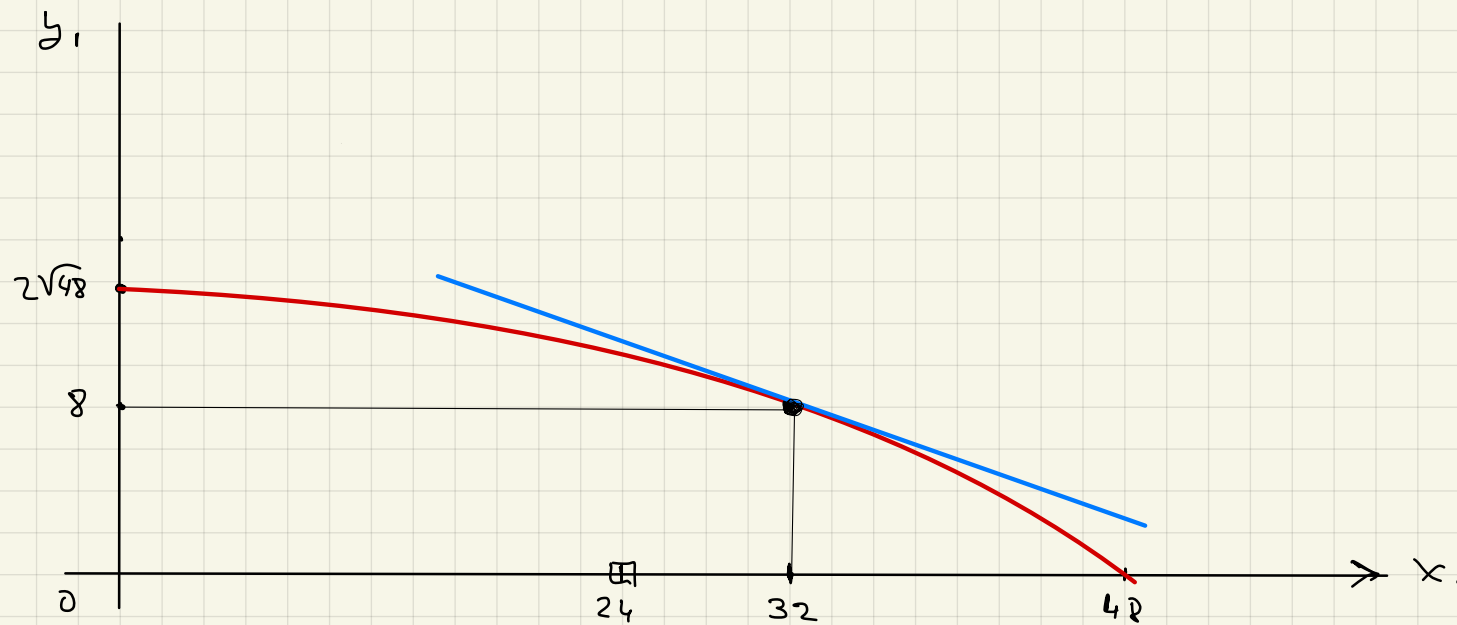
" OUTPUT : $2\sqrt{16} = 8$

" PROFIT : $\frac{1}{w^*} = 4$

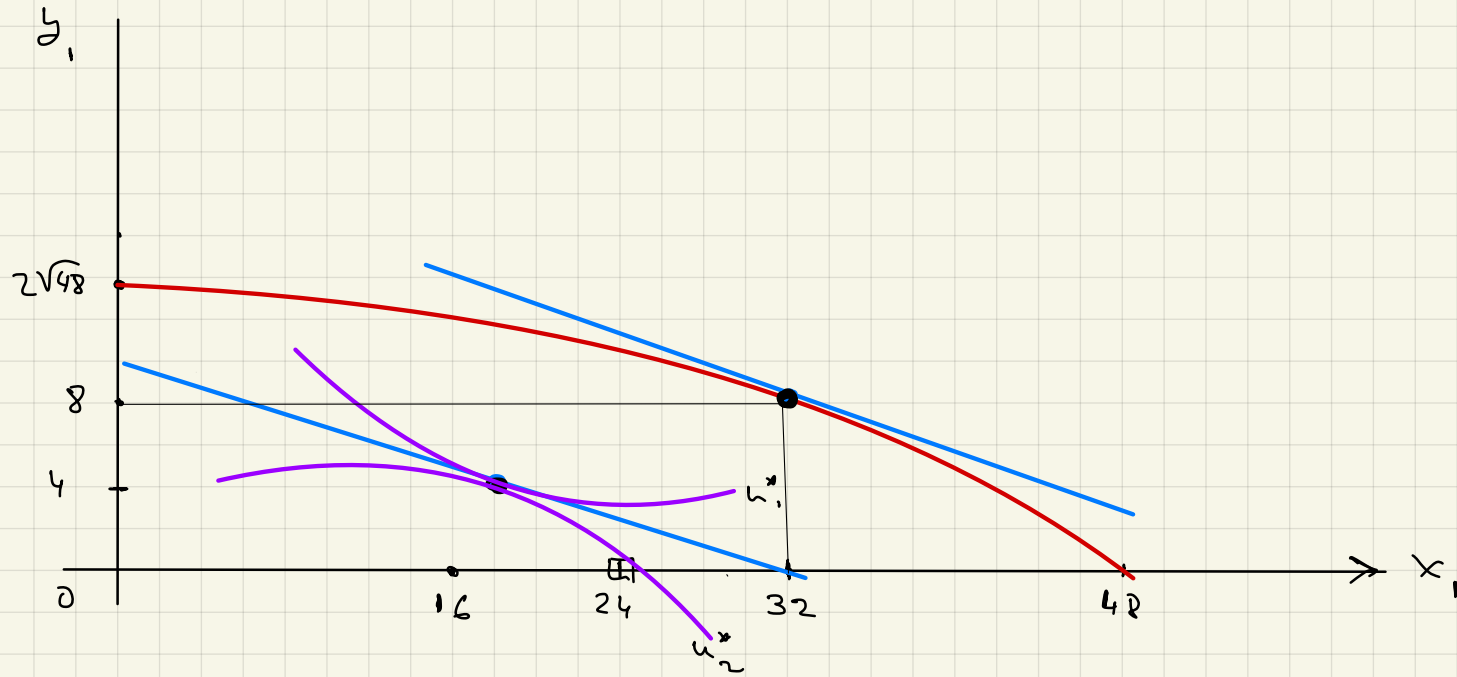
CE ALLOCATION :

$$x_i^* = 12 + \frac{\theta_i}{2 \left(\frac{1}{4}\right)^2} = 12 + 8\theta_i$$

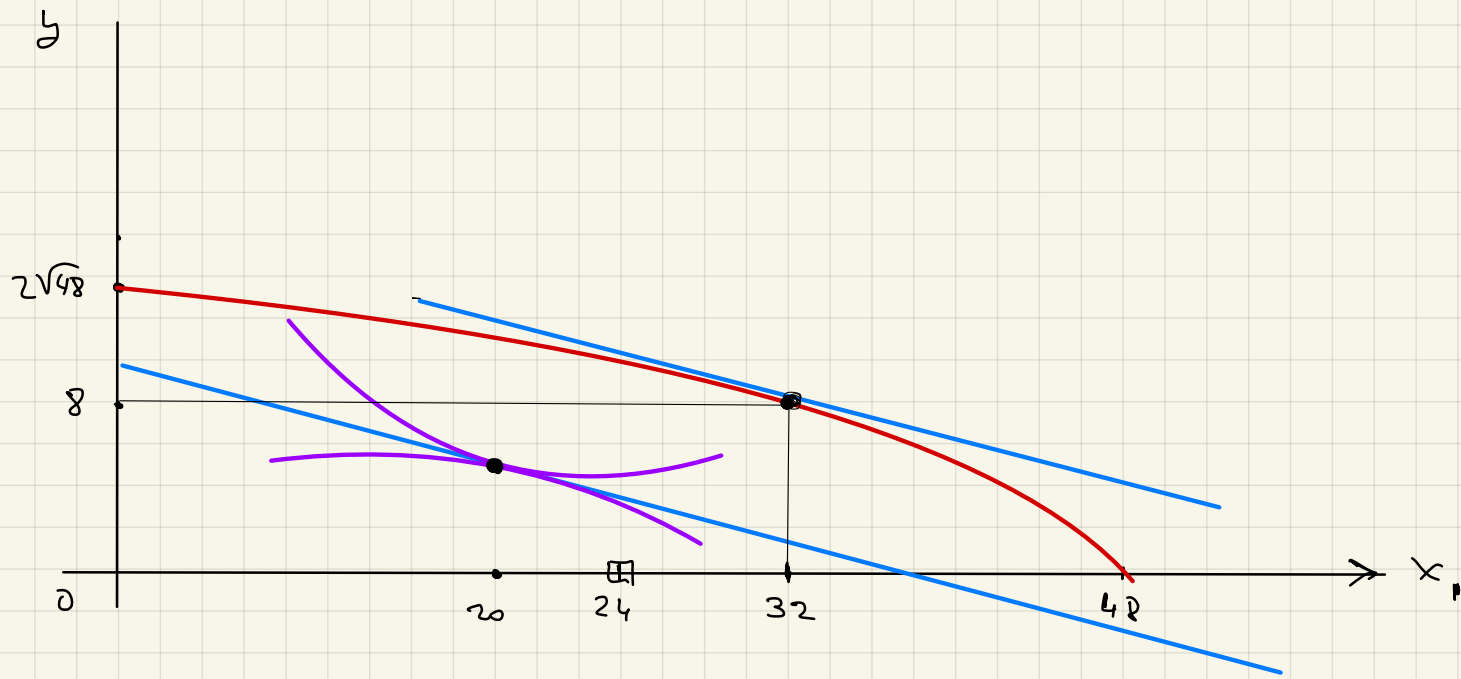
$$y_i^* = 12 \cdot \left(\frac{1}{4}\right) + \frac{\theta_i}{2 \left(\frac{1}{5}\right)} = 3 + 2\theta_i$$



$$\theta_1 = \theta_2 = \frac{1}{2} \Rightarrow (x_i^*, y_i^*) = (16, 4), \quad i \in \{1, 2\}$$



$$\theta_1 = 1, \theta_2 = 0 \Rightarrow (x_1, y_1) = (20, 5), (x_2, y_2) = (12, 3)$$



Two firms (rather than 1), same technology, $\theta_1 = (1, 0)$, $\theta_2 = (0, 1)$

MARKET CLEARING:

$$(24 - x_1(w)) + (24 - x_2(w)) = 2x^d(w)$$

$$2\left(12 - \frac{1}{2w^2}\right) = \frac{2}{w^2} \Leftrightarrow 24 - \frac{1}{w^2} = \frac{2}{w^2} \Leftrightarrow 8 = \frac{1}{w^2}$$

EMPLOYMENT: $\frac{2}{w^2} = 16$

OUTPUT $2(2\sqrt{8}) \approx 11.3$

CE ALLOCATION: $((16, 5.65), (16, 5.65))$

