Exercise List 1: Time and Uncertainty in a Competitive Economies

1. There is only one perishable commodity available for consumption today and tomorrow, and two consumers, Ann and Bob, whose preferences for alternative consumption streams today and tomorrow, $(x, y) \in \mathbb{R}^2_+$, are described by the utility functions $u^A(x, y) = \min\{x, y\}$, and $u^B(x, y) = xy$, respectively.

(a) Assume that the endowment streams are (0,30) for Ann and (20,0) for Bob. Ann and Bob participate in a competitive credit market in which they are the only participants. Calculate the competitive equilibrium rate of interest and consumption streams, and determine whether this allocation is Pareto optimal.

(b) Now assume that in addition to the credit market there is real investment process which yields three units of the good tomorrow for each unit of good today used as input. The investment process is operated by a competitive firm owned by Ann and Bob in proportions λ and $(1 - \lambda)$, where $\lambda \in (0, 1)$. Determine the competitive equilibrium interest rates, production plans, and consumption streams.

2. An exchange economy operates over an infinite number of consecutive dates. There is a single perishable consumption good. Every date t, N_t individuals are born. Individuals live only two dates. Thus, every date t only the consumers born at $t - 1$ (the elderly), and those born at t (the young) interact. The preferences of individuals for consumption when young, x, and old, y, are represented by the utility function $u(x, y) = xy$, and their endowments are $\bar{x} = 10$ and $\bar{y} = 4$. Consider three cases regarding the evolution of the population: (i) $N_t = 2N_{t-1}$, (ii) $N_t = N_{t-1}$, and (iii) $N_t = N_{t-1}/2$ in your discussion to the questions (a) and (b) below

(a) Discuss why even if there is a credit market there is no trade is the unique competitive equilibrium. (Note that only the young can borrow or lend.) Verify that the equilibrium allocation is not Pareto optimal.

(b) Suppose now that there is a stock of money held initially by the elderly at date 1. Specifically, at date 1 each elder owns €8. Money must be accepted as a mean of exchange, i.e., every date t there is a competitive market in which the good is exchanged for money at a price of p_t (euros per unit). Write the budget constraints of a consumer. (Use the notation $\rho_t = p_t/p_{t+1}$, and note that young consumers have no money.) Calculate the set of stationary equilibrium prices (i.e., those in which ρ_t is constant over time), and identify the price ρ^* supporting the *Golden Rule.*

3. Andy's income this year is €20. He is retiring next year, and his income depends on whether Social Security remains financially sound (S), or it goes bankrupt (N). Andy's income in state S will be €20, whereas it will be only €10 in state N (the amount paid by his private pension plan). Beth's income this year is also €20, and her income next year depends on the state of nature: in state S Beth will have to contribute to SS, leaving her with an income of only €10; in state N Beth's income is €20. Andy's preference is represented by the utility function $u^A(x, y_S, y_N) = x + 5\ln y_S + 6\ln y_N$, and Beth's by the function $u^B(x, y_S, y_N) = x + 10\ln y_S + 3\ln y_N$, where x denotes the individual's spending this year, y_S denotes spending next year in state S, and y_N denotes spending next year in state N. Income is perishable; i.e., one unit of income dated this year is worthless next year.

(a) Determine the competitive allocation and prices assuming that there markets for all goods.

(b) Suppose that only spot markets and a credit market exist. Is the equilibrium allocation Pareto efficient?

(c) Assume now that there are only spot markets, and the only securities are shares in the firm Gamma Technologies and shares in the firm Delta Insurance. Each share of Gamma will yield €2 in state S and €1 in N. Each share of Delta will yield €1 in state S and €2 in N. Determine the equilibrium security prices and Andy's and Beth's holdings of securities. What portfolio would one have to hold in order to guarantee oneself a return of €1 next year whether the state is S or N? What would be the cost of that portfolio? What would you say is the interest rate?

4. In an exchange economy that operates over two periods, today and tomorrow, there are two individuals A and B and a single perishable good, which we refer to as consumption. The state of the world tomorrow can be either sunny or cloudy. Both individuals are endowed with ten units of the good today and fifteen tomorrow, and their preferences are described by the utility function

$$u_i(x, y_s, y_c) = x + 5\pi_i \ln y_s + 5(1 - \pi_i)\ln y_c,$$

where x denotes consumption today, (y_s, y_c) denotes consumption when (sunny, cloudy), and π_i denotes individual i's subjective probability assessment that tomorrow will be sunny. Mr. A believes that the two states are equally likely, but Ms. B believes there is a $3/4$ chance the state will be sunny.

(a) If the only market available is a credit market (in which contracts are not state-contingent), what will the equilibrium interest rate (r) be, and how much will each individual lend (l)? (Normalize spot prices to one.)

(b) If there is a complete set of contingent claims markets, what would be the equilibrium prices and consumptions. (Set $p_x = 1$.)

(c) Now suppose that there is a credit market and a market for a real asset that pays one unit of the good tomorrow if cloudy and nothing if sunny. Determine the equilibrium interest r and asset price q. How much will each individual save or borrow, and how many units of the asset will each one buy or sell?

5. In an exchange economy that extends over two periods, today and tomorrow, there are two consumers, A and B, and a single perishable good. The state of nature tomorrow can either be H or L. Consumers' preferences over consumption today and tomorrow are represented by the utility functions $u_A(x, y_H, y_L) = xy_H$, and $u_B(x, y_H, y_L) = xy_L$, and both consumers are endowed with four units of the good each date regardless of the state of nature. In this economy there are no markets for contingent contracts, but the are the spot markets as well as a credit market at date 1. Normalize the spot market prices to $p_0 = p_H = p_L = 1$, and denote by r the interest rate in the credit market.

(a) What will be the competitive equilibrium interest rate r^*? How much will each person borrow $b_i(r^*)$? Is the resulting allocation Pareto optimal?

(b) Now suppose there is also a security which for a price q pays one unit of consumption tomorrow if event H occurs. Determine the competitive equilibrium interest rate and security price, (r^*, q^*). Is the resulting allocation Pareto optimal? Check whether equilibrium involve either consumer *selling short*, i.e., selling more units of the security than the consumer is endowed with at state H.