5. In an exchange economy that extends over two periods, today and tomorrow, there are two consumers, A and B, and a single perishable good. The state of nature tomorrow can either be H or L. Consumers' preferences over consumption today and tomorrow are represented by the utility functions $u_A(x, y_H, y_L) = xy_H$, and $u_B(x, y_H, y_L) = xy_L$, and both consumers are endowed with four units of the good each date regardless of the state of nature. In this economy there are no markets for contingent contracts, but the are the spot markets as well as a credit market at date 1. Normalize the spot market prices to $p_0 = p_H = p_L = 1$, and denote by r the interest rate in the credit market.

(a) What will be the competitive equilibrium interest rate r^*? How much will each person borrow $b_i(r^*)$? Is the resulting allocation Pareto optimal?

CONSUMERS' BUDGET CONSTRAINTS:

$$x = 4 + b$$
$$y_H = 4 - (1+r)b$$
$$y_L = 4 - (1+r)b$$

CONSUMER $i \in \{A, B\}$ PROBLEM

$$\max_{b \in \mathbb{R}} (4+b)[4-(1+r)b]$$

HENCE $b^A(r) = b^B(r) := b(r)$.

MARKET CLEARING: $0 = b^A(r) + b^B(r) = 2b(r) \iff b(r) = 0$

HENCE CE ALLOCATION IS THE INITIAL ENDOWMENTS: $[(4,4,4),(4,4,4)]$

THE ALLOCATION $[(4,8,0),(4,0,8)]$ IS PARETO SUPERIOR.

(b) Now suppose there is also a security which for a price q pays one unit of consumption tomorrow if event H occurs. Determine the competitive equilibrium interest rate and security price, (r^*, q^*). Is the resulting allocation Pareto optimal? Check whether equilibrium involve either consumer *selling short*, i.e., selling more units of the security than the consumer is endowed with at state H.

PRICE NORMALIZATION : $P_x = P_H = P_L = 1$.

CONSUMERS' BUDGET CONSTRAINTS:

$$x = 4 + b - qz$$

$$y_H = 4 - (1+r)b + z$$

$$y_L = 4 - (1+r)b$$

CONSUMER A's PROBLEM:

$P_L > 0 \Rightarrow y_L^A = 0 \Rightarrow b^A(r,q) = \dfrac{4}{1+r}$,

AND z^A SOLVES :

$$\max_{z \in \mathbb{R}} \left(4 + \frac{4}{1+r} - qz\right)(\cancel{4-4}+z)$$

$$z^A(r,q) = \frac{2(2+r)}{(1+r)q}$$

CONSUMER B's PROBLEM:

$P_H > 0 \Rightarrow y_H^B = 0 \Rightarrow b^B = \dfrac{4+z}{1+r}$

AND z^B SOLVES :

$$\max_{z \in \mathbb{R}} \left(4 + \frac{4+z}{1+r} - qz\right)[4-(4+z)]$$

$$z^B(r,q) = \frac{2(2+r)}{(1+r)q-1}$$

MARKET CLEARING :

CREDIT : $\frac{4}{1+r} + \frac{4 - \frac{2(2+r)}{(1+r)q-1}}{1+r} = 0$

SECURITY : $\frac{2(2+r)}{(1+r)q} + \frac{2(2+r)}{(1+r)q-1} = 0$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad \begin{aligned} r^* &= 0 \\ q^* &= \frac{1}{2} \end{aligned}$$

CE ALLOCATION : $\left[(4,8,0), (4,0,8) \right]$.