

**CHAPTER 1: Introduction to the Topology of Euclidean Space  $\mathbb{R}^n$ .**

- 2-1. Sketch the following subsets of  $\mathbb{R}^2$ . Sketch their boundary and the interior. Study whether the following are closed, open, bounded and/or convex.
- (a)  $A = \{(x, y) \in \mathbb{R}^2 : 0 < \|(x, y) - (1, 3)\| < 2\}$ .
  - (b)  $B = \{(x, y) \in \mathbb{R}^2 : y \leq x^3\}$ .
  - (c)  $C = \{(x, y) \in \mathbb{R}^2 : |x| < 1, |y| \leq 2\}$ .
  - (d)  $D = \{(x, y) \in \mathbb{R}^2 : |x| + |y| < 1\}$ .
  - (e)  $E = \{(x, y) \in \mathbb{R}^2 : y < x^2, y < 1/x, x > 0\}$ .
  - (f)  $F = \{(x, y) \in \mathbb{R}^2 : xy \leq y + 1\}$ .
  - (g)  $G = \{(x, y) \in \mathbb{R}^2 : (x - 1)^2 + y^2 \leq 1, x \leq 1\}$ .
- 2-2. Let  $A$  be a subset of  $\mathbb{R}^2$ . Discuss which of the following assertions are true.
- (a)  $\text{Int}(A) = A - \partial(A)$ .
  - (b)  $\partial(A) = \partial(\mathbb{R}^2 - A) = \partial(A^C)$ .
  - (c)  $\partial(A)$  is bounded.
  - (d)  $A$  is closed if and only if  $A^C$  is open.
  - (e)  $A$  is bounded if and only if  $A^C$  is not bounded.
  - (f)  $A$  is closed if and only if  $\partial(A) \subset A$ .
  - (g)  $A$  is open if and only if  $(\partial A) \cap A = \emptyset$ .