University Carlos III
Department of Economics
Mathematics II. Final Exam. January 9th 2019

Last Name:		**Name:**
ID number:	**Degree:**	**Group:**

IMPORTANT

- **DURATION OF THE EXAM: 2h**
- Calculators are **NOT** allowed.
- **Scrap paper:** You may use the last two pages of this exam and the space behind this page.
- **Do NOT UNSTAPLE** the exam.
- You must show a valid ID to the professor.

Problem	Points
1	
2	
3	
4	
5	
Total	

(1) Consider the set $A = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, 2y + x \leq 2\}$ and the function $f(x, y) = x - y^2$.

(a) (**0.5 points**) Draw the set A, its interior and boundary. Justify if the set A is open, closed, bounded, compact or convex.

(b) (**0.5 points**) State Weierstrass' Theorem. Determine if it is possible to apply Weierstrass' Theorem to the function f defined on A.

(c) (**1 point**) Using the level curves of f above, determine if this function attains a maximum and/or a minimum on the set A. If so, compute the points where the extreme values are attained and the maximum and/or minimum values of f on the set A.

(2) Consider the function $f(x, y, z) = ax^2 + ay^2 + cz^2 + 2abxy$ en $\mathbb{R}^2$, with $a < 0$, $c \neq 0$.

(a) (**1 point**) Discuss, according to the valores of the parameters a, b and c, if the function f is strictly concave or strictly convex on $\mathbb{R}^2$.

(b) (**1 point**) Using the above results, determine if the set $A = \{(x, y, z) \in \mathbb{R}^3 : 4 - x^2 - xy - y^2 - z^2 \geq 0\}$ is convex.

(3) Consider the equation

$$ye^{xz} + y^2 z = 3$$

(a) (**0.5 points**) Using the implicit function theorem prove that the above equation defines a function z near the point $x = 0$, $y = 1$, $z = 2$.

(b) (**1 point**) Compute

$$\frac{\partial z}{\partial x}(0, 1), \quad \frac{\partial z}{\partial y}(0, 1),$$

(c) (**0.5 points**) Write the equation of the tangent plane to the graph of the function $z(x, y)$, computed in part (a), at the point $q = (0, 1)$.

(4) Consider a function $f(x, y, z) : \mathbb{R}^3 \to \mathbb{R}$ and three functions $x(u, v), y(u, v), z(u, v) : \mathbb{R}^2 \to \mathbb{R}^3$. Consider the composite function $h : \mathbb{R}^2 \to \mathbb{R}$ defined by $h(u, v) = f(x(u, v), y(u, v), z(u, v))$.

(a) (**1 point**) State the chain rule for

$$\frac{\partial h}{\partial u}, \quad \frac{\partial h}{\partial v}$$

(b) (**1 point**) Use part (a) to compute

$$\frac{\partial h}{\partial u}, \quad \frac{\partial h}{\partial v}$$

for the functions

$$f(x, y, z) = x^2 - y^2 + xz$$

and

$$x(u, v) = 2u + v, \quad y(u, v) = u - 2v, \quad z(u, v) = uv$$

(5) Let $f(x, y, z) = x \ln(yz) - x^2 - y^2 - z^2$, $v = (a, b, c) \in \mathbb{R}^3$.

(a) (**1 point**) Compute the gradient of f at the point $p = (0, 1, 1)$. Determine for what values of a, b, c we have that $D_v f(p) = 0$.

(b) (**1 point**) Write the de Taylor polynomial of order 2 of the function $f(x, y, z)$ near the point $p = (0, 1, 1)$.