1. Find the points where the following functions have horizontal tangent.
   a) \( f(x) = x^3 + 1 \)
   b) \( f(x) = 1/x^2 \)
   c) \( f(x) = x + \sin x \)
   d) \( f(x) = \sqrt{x - 1} \)
   e) \( f(x) = e^x - x \)
   f) \( f(x) = \sin x + \cos x \)
   a) \( x=0 \)
   b) never
   c) \( x = \pi + 2k\pi \)
   d) never
   e) \( x = 0 \)
   f) \( x = \frac{\pi}{4} + k\pi \).

2. (*)Prove that the tangent lines to the graphs of \( y = x \) and \( y = 1/x \) in their points of intersection are perpendicular to each other.

3. In what point is the tangent to the curve \( y^2 = 3x \) parallel to the line \( y = 2x \)?
   The point of the curve is \((\frac{3}{16}, \frac{3}{4})\).

4. (*)Calculate the intersection point with the x axis of the tangent line to the graph of \( y = x^2 \) in the point \((1,1)\).
   The intersection point is \( x = 1/2 \).

5. Calculate \( a \) so that the tangent to the graph of \( f(x) = a/x + 1 \) in the point \((1, f(1))\) intersects the horizontal axis in \( x = 3 \).
   Therefore, the intersection point will be \( x = 3 \) when \( a = 1 \).

6. (*)Find the tangent and normal lines to \( f(x) = \arctan \left( \frac{\sin x}{1 + \cos x} \right) \) in \( x = 0 \).
   Equation of the tangent line: \( y - 0 = \frac{1}{2}(x - 0) \); equation of the normal line: \( y - 0 = -2(x - 0) \).

7. Find the derivatives of the following functions.
   a) \( f(x) = (\sin x + \tan 3x) \sin 2x \)
   b) \( f(x) = \frac{x \sqrt{x^2 - 1}}{2x + 6} \)
   c) \( f(x) = 4x^{3/2} \cos 2x \)
   d) \( f(x) = 5x \ln(8x + \sin 2x) + e^{\tan 5x} \)

8. (*)Let \( f(x) = 2[\ln(1 + g^2(x))]^2 \). Using that \( g(1) = g'(1) = -1 \), calculate \( f'(1) \).
   \( f'(1) = 4 \ln(2) \).

9. (*)Using that \( e^b = e^{b \ln a} \), differentiate \( f(x) = x^{\sin x} \) and \( g(x) = (\sqrt{x})^x \).
   \( f'(x) = x^{\sin x} \cos x \ln x + x \sin x / x \).
   \( g'(x) = (\sqrt{x})^x (\ln x + 1) / 2 \).

10. (*)Let \( f(x) = \ln(1 + x^2) \) and \( g(x) = e^x + e^{3x} \). Calculate \( h(x) = f(g(x)) \), \( v(x) = g(f(x)) \), \( h'(0) \) and \( v'(0) \).
    \( h(x) = \ln(1 + e^x + e^{6x} + 2e^{5x}) \), \( h'(0) = 4 \)
    \( v(x) = (1 + x^2)^2 + (1 + x^2)^3 \), \( v'(0) = 0 \).

11. Let \( f : [-2,2] \rightarrow [-2,2] \) be continuous and bijective.
   a) Suppose that \( f(0) = 0 \) and \( f'(0) = \alpha \), \( \alpha \neq 0 \). Find \( (f^{-1})'(0) \).
   b) Now suppose that \( f(0) = 1 \) and \( f'(0) = \alpha \), \( \alpha \neq 0 \). Find \( (f^{-1})'(1) \).
   c) Now suppose that \( f(1) = 0 \) and \( f'(1) = \alpha \), \( \alpha \neq 0 \). Find \( (f^{-1})'(0) \).
   a) \( (f^{-1})'(0) = \frac{1}{\alpha} \).
   b) \( (f^{-1})'(1) = \frac{1}{\alpha} \).
   c) \( (f^{-1})'(0) = \frac{1}{\alpha} \).
12. (*)Supposing that the following equations define $y$ as a differentiable function of $x$, calculate $y'$ in the given points:
   a) $x^3 + y^3 = 2xy$ in $(1, 1)$.
   b) $\sin x = x(1 + \tan y)$ in $(\pi, 3\pi/4)$.
   c) $x^2 + y^2 = 25$ in $(3, 4), (0, 5)$ and $(5, 0)$.
   a) $y' = -1$. b) $y' = \frac{1}{2\pi x}$. c) $y' = \frac{3}{2}$ in $(3, 4)$. $y' = 0$ in $(0, 5)$. It doesn’t exist derivative in $(5, 0)$.

13. Calculate the derivative of the following functions showing where they are not differentiable.
   a) $(*) f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0 \\ 0 & \text{if } x > 0 \end{cases}$
   b) $(*) g(x) = \begin{cases} 1/|x| & \text{if } x \leq -2 \\ (x + 2)^2 & \text{if } -2 < x \leq 0 \\ 3 + \sin(x + \frac{\pi}{2}) & \text{if } x > 0 \end{cases}$
   c) $h(x) = \begin{cases} \arctan^2 x & \text{if } x \leq 0 \\ \sin^2 x & \text{if } 0 < x \leq 2\pi \\ \sin x & \text{if } 2\pi < x \end{cases}$
   a) $f'(x) = \begin{cases} 2x & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases}$ Obviously, $f$ is not differentiable in $0$.
   b) $g'(x) = \begin{cases} 1/x^2 & \text{if } x < -2 \\ 2(x + 2) & \text{if } -2 < x < 0 \\ \cos(x + \frac{\pi}{2}) & \text{if } x > 0 \end{cases}$ Obviously, $f$ is not differentiable in $-2$ nor in $0$.
   c) $h'(x) = \begin{cases} 2\arctan x & \text{if } x \leq 0 \\ 3\sin^2 x \cos x & \text{if } 0 \leq x < 2\pi \\ \cos x & \text{if } 2\pi < x \end{cases}$ Obviously, $f$ is not differentiable in $2\pi$.

14. (*)Find $a$ and $b$ so that the function $f(x) = \begin{cases} 3x + 2 & \text{if } x \geq 1 \\ ax^2 + bx - 1 & \text{if } x < 1 \end{cases}$ is differentiable.
   $f$ differentiable in $1$ when $a = -3, b = 9$.

15. Apply the mean value theorem to $f$ in the given interval and find the $c$ values of the thesis of the theorem.
   a) $f(x) = x^2$ in $[-2, 1]$ 
   b) $f(x) = -2\sin x$ in $[-\pi, \pi]$ 
   c) $f(x) = x^{2/3}$ in $[0, 1]$ 
   d) $f(x) = 2\sin x + \sin 2x$ in $[0, \pi]$ 

16. Let $f : [a, b] \rightarrow [a, b]$ be a continuous function in $[a, b]$ and differentiable in $(a, b)$. Prove that,
   if $f'(x) \neq 1$ in $(a, b)$, then $f$ has a unique fixed point in $[a, b]$.

17. Prove that the function $f$ has a unique fixed point.
   a) $f(x) = 2x + 1/2\sin x$ 
   b) $f(x) = 2x + 1/2\cos x$ 

18. (*)Let $f(x) = x^3 - 3x + 3$, $f : [-3, 2] \rightarrow \mathbb{R}$. Determine the global extrema.
   The minimum is reached in $-3$ and the maximum is reached in $-1$ and in $2$.

19. Let $f : [-5, 5] \rightarrow \mathbb{R}$ such that $f$ reaches the maximum in $x = 2$ and the minimum in $x = -3$. Let $g(x) = -f(-x)$. What can be said about the maximum and the minimum of $g$?