**WORKSHEET 5: Integration**

1. (*) Calculate the following integrals:

   a) \( \int \frac{x^2 + x + 1}{x \sqrt{x}} \, dx \)  
   b) \( \int x e^{-2x} \, dx \)  
   c) \( \int \sin^{14} x \cos x \, dx \)

   d) \( \int (x + 1)(2 - x)^{1/3} \, dx \)  
   e) \( \int \frac{x^4}{1 + x^5} \, dx \)  
   f) \( \int (\frac{1}{x})^{3/2} \, dx \)

   g) \( \int \sin^3 x \, dx \)  
   h) \( \int x e^{ax^2} \, dx \)  
   i) \( \int \frac{1}{3 + x^2} \, dx \)

   j) \( \int \frac{\sqrt{x} - 1}{1 + \sqrt{x} - 1} \, dx \)  
   k) \( \int \frac{x}{\sqrt{16 - x^2}} \, dx \)  
   l) \( \int x^4 \ln x \, dx \)

   m) \( \int \frac{4x + 6}{\sqrt{x^3 - \sqrt{x}}} \, dx \)  
   n) \( \int (\ln x)^2 \, dx \)  
   o) \( \int \frac{2x + 6}{(x - 2)^2} \, dx \)  
   p) \( \int \frac{1}{x^2 - 2x + 4} \, dx \)

   q) \( \int \frac{1}{x^2 - 2x + 4} \, dx \)  
   r) \( \int \frac{2x + 1}{x^3 + 6x} \, dx \)  
   s) \( \int \frac{1}{x^2 - 2x + 4} \, dx \)

2. How many different intersection points can two different primitives of the same function have?

3. (*) Let \( f : [0, 2] \rightarrow \mathbb{R} \) be continuous, increasing in \((0, 1)\), decreasing in \((1, 2)\) and, also, satisfying that: \( f(0) = 3 \), \( f(1) = 5 \) and \( f(2) = 4 \). Between which values can we guarantee that \( \int_0^2 f(x) \, dx \) is located?

4. (*) Certain company has determined that its marginal cost is \( \frac{dC}{dx} = 4(1 + 12x)^{-1/3} \). Find the cost function if \( C = 100 \) when \( x = 13 \).

5. (*) Given that the marginal cost of producing \( x \) units is \( x + 5 \) and the average cost has a minimum in \( x = 4 \), find the fixed costs of the firm.

6. (*) Calculate \( F''(x) \) in the following cases:

   a) \( \int_x^{x^3} t \cos t \, dt \)  
   b) \( \int_1^{x^2} \sqrt{t^4 + 2t} \, dt \)  
   c) \( \int_1^{x^2} (t^2 - 2t + 5) \, dt \)

7. Calculate \( F''(x) \) in the following cases:

   (a) \( \int_{-\pi}^x \tan^2 t \, dt \), supposing that \( x^2 < \frac{\pi}{2} \).

   (b) \( \int_{x^2}^{2x} f^2(2t) \, dt \), supposing that \( f \) is continuous.

8. (*) What are the values of \( x \) where \( F(x) = \int_{-3}^x \frac{t^2 - 4}{3t^2 + 1} \, dt \) has a local maximum or minimum?

9. Let \( F(x) = \int_{x^2}^{2x} f(t^2) \, dt \) be such that \( f(1) = 1 \), \( f(2) = f(4) = 4 \) and \( f \) is continuous. Calculate \( F'(1) \).
10. (*) Calculate observing the symmetry of the functions:
   \[ a) \int_{-\pi/2}^{\pi/2} sin^27x \cos^28 x \, dx \quad b) \int_{-\pi/4}^{\pi/4} (\sqrt{x^5} \cos 3x + \cos \frac{x}{3} + \tan^3 x) \, dx \]

11. Let \( f \) be a function with period \( T \), such that \( \int_0^T f = b \). Find \( \int_{a+nT}^a f \).

12. (*) Find the area located between the following curves:
   \[ a) f(x) = x^2 - 4x + 3, \ g(x) = -x^2 + 2x + 3 \]
   \[ b) f(x) = (x - 1)^3, \ g(x) = x - 1 \]
   \[ c) f(x) = x^4 - 2x^2 + 1, \ g(x) = 1 - x^2 \]

13. (*) Graph the functions \( y = 2e^{2x} \) and \( y = 2e^{-2x} \). Calculate the area located between those graphs and the lines \( x = -1 \) and \( x = 1 \).

14. Let \( f : [1, 3] \rightarrow [2, 4] \) be increasing, continuous and bijective such that \( \int_1^3 f \, dx = 5 \). Calculate \( \int_2^4 f^{-1}(x) \, dx \).

15. (a) Given \( f : [0, 4] \rightarrow \mathbb{R} \), convex and increasing with values \( f(0) = 0, \ f(2) = \alpha, \ f'(2) = \beta, \ f(4) = 16 \). Estimate as a function of \( \alpha \) and \( \beta \), the value of \( \int_0^2 f(x) \, dx \).
   
   (b) Given \( f : [0, 4] \rightarrow \mathbb{R} \), concave and increasing with values \( f(0) = 0, \ f(2) = \alpha, \ f'(2) = \beta, \ f(4) = 2 \). Estimate as a function of \( \alpha \) and \( \beta \), the value of \( \int_0^2 f(x) \, dx \).

16. The sales of a product are given by the formula \( S(t) = 10 + 5 \sin\left(\frac{\pi t}{6}\right) \) where \( S \) is measured in thousands of units and time \( t \) in months. Calculate the average sales during the year \( (0 \leq t \leq 12) \).

17. Calculate:
   \[ a) \int_0^1 \frac{1}{\sqrt{x}} \, dx \quad b) \int_0^3 \frac{1}{x^3} \, dx \quad c) \int_1^\infty \frac{1}{x^2} \, dx \]
   
   \[ d) \int_1^\infty e^{-x} \, dx \quad e) \int_{-\infty}^\infty \frac{dx}{1+x^2} \quad f) \int_{-2}^4 \frac{dx}{x^2} \]

18. Calculate \( \int_0^\infty \frac{dx}{\sqrt{x}(1 + x)} \)