1. Compute the following limits:
   a) \( \lim_{x \to \infty} (1 + x)^{1/x} \)
   b) \( \lim_{x \to 0^+} x \ln x \)
   c) \( \lim_{x \to \infty} x^{1/x} \)
   d) \( \lim_{x \to 1^+} \left( \frac{1}{\ln x} - \frac{2}{x - 1} \right) \)
   e) \( \lim_{x \to \infty} x \tan(1/x) \)
   f) \( \lim_{x \to 0} \frac{\arcsin x - \arctan x}{x} \)

2. Compute the asymptotes of the following functions:
   a) \( f(x) = \frac{2x^3 - 3x^2 - 8x + 4}{x^2 - 4} \)
   b) \( f(x) = \frac{x^3}{x^3 + x^2 + x + 1} \)
   c) \( f(x) = 2x + e^{-x} \)
   d) \( f(x) = \frac{\sin x}{x} \)
   e) \( f(x) = \sqrt{x^2 + 1} \)
   f) \( f(x) = \frac{3x^2 - x + 2 \sin x}{x - 7} \)
   g) \( f(x) = \frac{e^x}{x} \)
   h) \( f(x) = xe^{1/x} \)
   i) \( f(x) = \frac{x}{e^x - 1} \)

3. (*)Find the Taylor polynomial of order 2 in \( a \) and, using that polynomial, compute the approximate value of the function on \( x = a + 0.1 \):
   a) \( f(x) = e^x \) in \( a = 0 \)
   b) \( f(x) = \sin x \) in \( a = 0 \)
   c) \( f(x) = \frac{\ln x}{x} \) in \( a = 1 \)

4. (*)Given the Taylor polynomial of order 2 in \( a = 0 \) of \( f \), determine if the function has a local maximum or minimum at the point \( (0, f(0)) \).
   a) \( P(x) = 1 + 2x^2 \)
   b) \( P(x) = 1 + x + x^2 \)
   c) \( P(x) = 1 - 2x^2 \)

5. Compute the (absolute and local) maxima and minima of \( f \) in the given intervals:
   a) (*) \( f(x) = 3x^{2/3} - 2x \) in \([-1, 2]\).
   b) \( f(x) = xe^{-x} \) in \([1/2, \infty), \ [0, \infty) \) and \( \mathbb{R} \).

6. (*)Compute in which point the slope of the tangent line to the graph of the function \( f(x) = -x^3 + 2x^2 + x + 2 \) takes its maximum value.

7. The first (*) and second drawings show the graphs of the derivatives of different functions \( f \). Determine the increasing/decreasing, concavity/convexity intervals of \( f \), and its local extreme and inflection points.

8. The following drawing shows the graph of the second derivative of \( f \). Determine the convexity intervals of \( f \) and the inflection points. Determine where the function is increasing and decreasing and the relative extrema of \( f \) assuming that \( f(-3) = f(0) = 0 \).
9. Let \( f(x) = \begin{cases} x^\alpha & \text{if } 0 \leq x \leq 1 \\ x^\beta & \text{if } 1 \leq x \end{cases} \) Discuss, depending on the values of \( \alpha \) y \( \beta \), when \( f \) is concave or convex.

10. (*)Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) convex, and let \( x > 0 \). Check graphically the following inequalities:

\[
f(1) < \frac{1}{2} \left( f(1-x) + f(1+x) \right) < \frac{1}{2} \left( f(1-2x) + f(1+2x) \right)
\]

11. (*)Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) concave, and let \( x > 0 \). Check graphically the following inequalities:

\[
f(1) > \frac{1}{2} \left( f(1-x) + f(1+x) \right) > \frac{1}{2} \left( f(1-2x) + f(1+2x) \right)
\]

12. (*)Let \( f : [0, \infty) \rightarrow \mathbb{R} \), convex, such that \( f'(1) = 0 \).

   a) Find the local extrema of \( f \).
   b) What can be said of the global extrema of \( f \)?
   c) Consider now \( f : [0, n] \rightarrow \mathbb{R} \). What can be said of the global extrema of \( f \) ?

13. (*)Let \( f : [0, \infty) \rightarrow \mathbb{R} \), concave, such that \( f'(1) = 0 \).

   a) Find the local extrema of \( f \).
   b) What can be said of the global extrema of \( f \)?
   c) Consider now \( f : [0, n] \rightarrow \mathbb{R} \). What can be said of the global extrema of \( f \) ?

14. Study and graph the following functions:
   a) \( f(x) = x + \cos x \)
   b) \( f(x) = \frac{e^{2x}}{e^x - 1} \)
   c) \( f(x) = \frac{x}{\ln x} \)
   d) \( f(x) = \sqrt{|x - 4|} \)

15. (*)Given the cost function \( C(x) = 4000 + 10x + 0.02x^2 \) and the demand function \( p(x) = 100 - (x/100) \), find the price \( p \) per unit that gives the maximum benefit.

16. (*)Let \( p(x) = x^2 - x + 1/3 \) be the sale price of 1 kilo of plutonium when \( x \) units are sold. Taking into account that the firm sells in the market a maximum of 2 kilos, find the value of \( x \) that maximizes the profits of the firm. We can assume that the Government pays all the costs of the firm.

17. (*)Let \( p(x) = 100 - x^2/2 \) be the demand function of a product and \( C(x) = 48 + 4x + 3x^2 \) its cost function. What is the production \( x \) that minimizes the average cost? And if there exists a maximum production \( x^* \)?

18. A firm that has a cost function \( c(x) = x^2 + 1 \) faces a demand given by the function \( p(x) = \begin{cases} 10 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 < x \leq 10 \end{cases} \). Find the production that gives the maximum profit.

19. (*)A manufacturer sells 5000 units per month for 100 euros per unit and he believes that his sales would increase by 500 units for each 5 euros of decrease on the unitary price.

   a) Find the demand, revenues and marginal revenues functions.
   b) If the cost of production of \( x \) units is \( C(x) = 1000 + 0.12x \), find the marginal profit function.