University Carlos III of Madrid **Departament of Economics**
Game Theory: Problem set 4: Repeated Games.

Problem 1: Consider the following normal form game.

Player 2

		A	B	C
Player 1	A	$4,4$	$1,5$	$0,3$
	B	$7,1$	$3,4$	$0,1$
	C	$3,0$	$2,0$	$1,1$

The stage game G

1. Find the Nash equilibria in pure strategies.
2. Assume that the above stage game is played two times. After the first round, players observed the moves done by the other player. The total payoffs of the repeated game are the sum of the payoffs obtained in each round. Is there a subgame perfect Nash equilibrium in pure strategies in which (A, A) is played in the first round?
3. Assume that the above stage game is played three times. After the first and second round, players observed the moves done by the other player. The total payoffs of the repeated game are the sum of the payoffs obtained in each round. Is there a subgame perfect Nash equilibrium in pure strategies in which (A, A) is played in the first round?

Problem 2: Consider the following normal form game.

Player 2

		A	B
Player 1	A	$2,2$	$-2,6$
	B	$6,-2$	$0,0$

The stage game G

1. Find the all the Nash equilibria of the game G.
2. Assume that the above stage game is played infinitely many times. After each round, players observe the moves done by the other player. The total payoffs of the repeated game are the discounted (with discount factor δ) sums of the payoffs obtained in each round. For what values of the discount factor δ is there a subgame perfect Nash equilibrium in pure strategies in which (A, A) is played in every round?

Problem 3: Consider the following normal form game.

Player 2

		C	D
Player 1	C	$6,6$	$0,8$
	D	$0,2$	$4,0$

The stage game G

1. Find the all the Nash equilibria of the game G.
2. Assume that the above stage game is played 27 times. After each round, players observe the moves done by the other player. The total payoffs of the repeated game are the sum of the payoffs obtained in each round. Find all the subgame perfect Nash equilibrium of the repeated game.

3. Assume that the above stage game is played infinitely many times. After each round, players observed the moves done by the other player. The total payoffs of the repeated game are the discounted (with discount factor $\delta = 0.9$) sums of the payoffs obtained in each round. Is there a subgame perfect Nash equilibrium in pure strategies in which (C, C) is played in every round?

Problem 4: Two firms compete in a Cournot market with a homogeneous good. The market demand is $p = 33 - q_1 - q_2$ where q_1 and q_2 are the quantities produced buy the firms. Both firms have constant marginal cost $c = 3$.

(a) Suppose that the market opens only in one period. If there were only one firm in the market, what would be the monopoly profit? what would be the amount produced?

(b) Suppose that the market opens only in one period and the firms know it. Find the Cournot equilibrium and the profits in equilibrium.

(c) Assuming that the market opens only in one period and the firms know it, show that the profits if each firm produces half the monopoly quantity found in part (a) are bigger than the profits in the Cournot equilibrium computed in (b).

(d) Argue why if the market opens only in one period and firms know it, each firm producing half the monopoly quantity found in part (a) is not a reasonable outcome.

(e) Assume now that the market opens periodically in the following way: In every period, the probability that the market will continue tomorrow is $\frac{3}{4}$. With probability $\frac{1}{4}$ the market finishes at that period and never reopens. The discount factor is $\delta = 4/5$. Show that the trigger strategy sustains the monopoly outcome found in part (a) with each firm producing half of the monopoly quantity.

(f) In the context of the previous part show that the trigger strategy of part (e) is a subgame perfect Nash equilibrium.

Problem 5: Consider the following normal form game.

		Player 2	
		C	D
Player 1	C	$4, 4$	$0, 6$
	D	$6, 0$	$1, 1$

The stage game G

(a) Find the all the Nash equilibria of the game G.

(b) Assume that the above stage game is played infinitely many times. After each round, players observe the moves done by the other player. The total payoffs of the repeated game are the discounted (with discount factor $\delta = \frac{1}{2}$) sums of the payoffs obtained in each round. Consider the following strategy profile:

   i. both players play (C, C) until nobody deviates.

   ii. If somebody deviates, then, in the following period both players play (D, D) for n periods.

   iii. After this n periods of punishment, both players go back to the strategy in (i).

   For what values of n does the above strategy profile constitute a subgame perfect Nash equilibrium.

Problem 6: Consider the following stage game

		Player 2	
		A	B
Player 1	A	$0, 5$	$1, 1$
	B	$1, 1$	$5, 0$

The stage game G

Assume that G is played infinitely many times. For what values of the discount factor δ is there a subgame perfect Nash equilibrium in which in the equilibrium path players play (A, A) in odd periods and play (B, B) in even periods.