University Carlos III of Madrid **Departament of Economics**
Game Theory: Problem set 3.

Problem 1: Two Airlines, Fly and Connect, sell a round trip in economic class between two cities for \$400. Each one of the Airlines serves half the market and it thinks that if it lowers its fare by a 10% and the competitor does not change its fare, its demand would rise by a 40% (which means that its price elasticity is -4). Of this increase, 20% represents clients of the other company who change attracted by lower fares, and the remaining 20% represent new clients attracted by lower fares. If both companies lower their price by 10% both are going to increase by a 20 their respective demands due to new clients (price elasticity is -2). The unitary cost for carrying each passenger is \$200 for each of the Airlines. Assume that they have to simultaneously decide whether to lower the prices by 10% or not.

(a) Represent the normal form of the game and find the Nash equilibrium.

problem-set-3-1a

(b) Assume now that the two Airlines agree to have their total revenues accrue into a common pool which is then shared equally, while each airline pays its own costs. As before, the Airlines have to decide whether or not lowering their fares by 10%. Represent the game in its normal form and find the Nash equilibrium.

problem-set-3-1b

(c) Do you believe that companies are going to act as in (b). Why?

problem-set-3-1c

Problem 2: In December 1994, when Transco Energy's shares traded at \$12.625, Williams Companies announced a two-tier bid for Transco Energy Co. In the first tier, Williams offered \$17.50 for each share conditional on securing 51% of shares (this is to say that the offer was effective only if 51% of shares were tendered in the first tier). In the second tier, the remaining shares were going to be converted into Williams's shares so as to secure a value of \$15.00 for each share of Transco (once obtained at least 51% of Transco's shares in the first tier, Williams was going to control Transco and was going to be able to make this conversion without having to rely on remaining shareholders' votes). Imagine that you hold one share of Transco and that you have to decide what to do. Because the final outcome does not depend on your own decision (you only hold one share!) you can assume that the probability that Williams succeeds in its takeover attempt, p, is independent of your decision.

(a) Draw the decision tree that represent your decision problem assuming that the value of a Transco's share if Williams's attempt fails is v (Williams's offer may reveal the market information that Williams has on Transco, and Transco's value after Williams's offer, v, may therefore be different from its initial \$12.625).

problem-set-3-2a

(b) What is your optimal decision? Does it depend on v? Why or why not?

(c) Suppose now that all of Transco's shareholders are small (they each hold few shares of Transco) and that, therefore, it is reasonable to assume that no individual shareholder's decision has an impact on the final outcome. Do you think that Williams's attempt will be successful or not? Why?

(d) Imagine now that Williams's offer is not conditional, but that Williams offers \$17.50 to all shareholders regardless of whether it will gain the majority of 51% or not. Draw the decision tree that represents your decision problem.

(e) What is your optimal decision? Does it depend on v? ¿Why or why not?

(f) What do you think the final outcome will be? Does it depend on the value of v?

(g) Why did Williams choose to make the two-tier conditional offer described above?

Problem 3: Two theaters are located on the same street. They advertise and, by so doing, attract customers to the area so that each of the two theaters' advertising is beneficial to both. Let x_1 be the advertising level of theater 1 and x_2 be the advertising level of theater 2 (for instance, x_1 could denote the number of potential customers reached by theater 1's advertising). The profit functions for theaters 1 and 2 are respectively:

$$\begin{aligned} \pi_1(x_1, x_2) &= (30 + x_2)x_1 - 2x_1^2 \\ \pi_2(x_1, x_2) &= (30 + x_1)x_2 - 2x_2^2 \end{aligned}$$

(a) What are the Nash equilibrium advertising levels of the two theaters?

(b) What are their equilibrium profits?

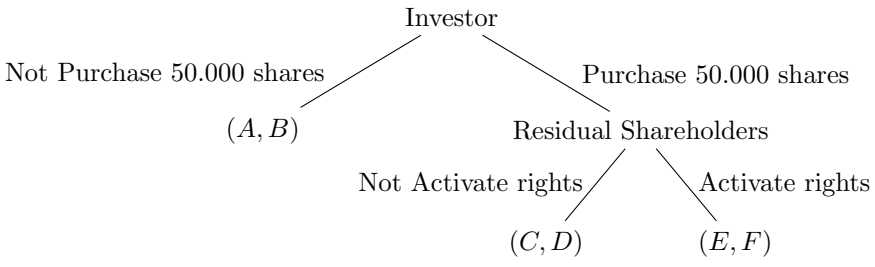
(c) Could the theaters increase total profits if they could commit to given advertising levels? What advertising levels would they choose?

Problem 4: For most new films, the demand for movie theaters is highest in the first few days after opening, then taper off. Accordingly, studios must time new releases very carefully. Two key factors affecting potential demand are the season and the timing of other releases. Suppose that both Suprafilm and National are producing major action movies. The two studios simultaneously must choose between release in November or December. If both films open on November, each will sell 200,000 tickets. If one opens on November and the other in December, then the early release will sell 350,000 tickets, while the later release will sell 220,000. If both open in December, each will sell 300,000 tickets.

(a) Construct a game in normal form to illustrate the situation.

(b) Identify the Nash equilibrium or equilibria.

Problem 5: An investor has received an offer to purchase 50,000 shares of company T currently held by the majority group. The price the majority group requests is \$21, despite of the fact that company T is currently valued at \$2 million and that it has 100,000 shares (a share of company T is currently trading at \$20 on the stock exchange). It is known, however, that if the investor gets the majority of company T, and therefore its control, he will bring in new, more competent management that will cause the value of company T to go up by 20%. Company T has currently in place a shareholder rights plan that has created one right for each share. Under the plan, if any party acquires 10,000 or more shares, the residual shareholder can activate the rights. Each right allows the purchase of one newly issued share at half the current market price (in current conditions this would be \$10). Any rights owned by the party whose acquisition activated the rights, however, would be canceled. This situation is described by the following game tree where A, C and E denote the payoffs to the investor (depending on players' strategies) and where B, D, and F denote the payoff to the residual investors (depending on players' strategies).



Before turning to the questions, notice the following:

(i) If the rights are activated, the value of the company is increased by the amount paid by residual shareholders, but the number of shares of the company increases.

(ii) The payoffs to the players have to keep into account the value of the shares they hold (which depends on the value of the company and the number of shares) as well as what they had to pay for (some or all of) the shares they hold.

Answer the following questions:

(a) Find A, the payoff to the investor if he decides not to purchase the shares.

(b) Find B, the payoff to the residual shareholders if the investor decides not to purchase the shares.

(c) Find C, the payoff to the investor if he decides to purchase the shares and if the residual shareholders decide not to activate the rights.

(d) Find D, the payoff to the residual shareholder if the investor decides to purchase the shares and if the residual shareholders decide not to activate the rights.

(e) Find E, the payoff to the investor if he decides to purchase the shares and if the residual shareholders decide to activate the rights.

(f) Find F, the payoff to the residual shareholder if the investor decides to purchase the shares and if the residual shareholders decide to activate the rights.

(g) Find the subgame perfect Nash equilibrium.

Problem 6: Two executives, A and B, have to submit a proposal to a firm's board about the price of the firm's product. There are four possible prices: 100, 110, 120 or 130 euros. The board is made up of 13 members and conversations prior to the meeting have led four members to prefer the price of 100, two members to prefer the price of 110, one member prefers 120, and six members prefer the price of 130. Both executives have to submit their proposals simultaneously. Once the proposals are received, each member of the board will vote for one of the executives as the next CEO. Each member will vote for the executive whose price proposal is closer to his own ideal price. If both executives propose the same price, members vote with probability 1/2 for each. If one member has to decide between two equidistant prices (from his ideal price), he will vote for each executive with probability 1/2. The executive with more votes will be the next CEO.

(a) Design a table with the (expected) votes each executive would get depending on both proposals for prices. (Suggestion: Place executive A's proposed prices in rows and B's prices in columns) Suppose that both executives only care about being chosen as CEO and that they do not have any other preference regarding a particular price.

(b) Find the normal form of this game.

(c) Find the pure strategy Nash equilibria.

(d) Comment on the result.

Problem 7: Firm B is in a position to sell 10000 units of an electronic device at a unit price of \$200. In order to produce this device a component is necessary whose patent belongs to another firm A. Firm A can only produce the component if it builds a factory that costs \$1 million. The factory can only be used for the production of such a component and there are no other potential buyer for the component apart from firm B. Once the factory is built, the marginal cost of production of each component is \$25. The unitary production cost of the device is \$50 plus the price that A receives for the component. The temporal sequence is as follows:

- January 1st, 2003: Firm A decides whether to build the factory or not.
- November 30th, 2003: If firm A has built the factory, then firm B offers a price P for each of the 10000 components.

- December 1st, 2003: A decides whether to accept or reject the offer.
- December 2nd, 2003: If A accepts the offer, the production components and devices starts.
- January 1st, 2004: Devices are sold and B receives the money. Firm B pays $10.000P$ to firm A.

In what follows, assume that discount rate is 0.

(a) Draw the extensive form game.

(b) Find the subgame perfect Nash equilibrium.

(c) Comment on the result. In particular, do you believe that the outcome is satisfactory for both firms? Do you believe that there may exist arrangements both firms would agree on that improve the outcome? Which?

Problem 8: Two firms, A and B, compete in a market. The market size is \$8 billion. Each firm can choose whether to advertise. Advertising costs \$1 billion for each firm that chooses to do so. If one firm advertises and the other doesn't, then the former captures the whole market. If both firms advertise, they split the market equally and pay for the advertising. If neither advertises, they split the market equally but without the expenses of advertising.

(a) Consider the case in which both firms move simultaneously. Write the normal form of the game and find the Nash equilibrium or equilibria of this game.

(b) Consider now the case in which A moves first and B moves after having observed A's move. Write the normal form of the game and find the Nash equilibrium or equilibria of this game. Also find subgame perfect Nash equilibria for this game.

(c) Is any of the two games preferable from the point of view of the firms? Why or why not?

Problem 9: Your firm has developed a new product. If the commercial launching is successful, the new product will yield profits of \$600.000. If the commercial launching is not successful, profits will be 0. You are considering the possibility of contracting an advertising agency. If you contract the agency, this could choose an effort level high or low, without you being able to observe it. If the agency chooses high effort, it will pay a cost of \$400.000, while a low effort only costs \$200.000. High effort generates a commercial success probability of 0.75, while low effort generates a success probability 0.50. As it is not possible to observe the effort level of the advertising agency, the only way to give the agency incentives for exerting high effort is offering a contingent contract that specifies different payments, one for commercial success and the other for failure

(a) Which contract would you offer to the agency in order for it to choose high effort?

(b) Is it really in your interest that the firm chooses high effort?

Problem 10: Two firms, H and S, produce complementary hardware and software products. This means that if one firm dedicates more resources to investment, it increases not only the demand of its own product but also the demand of the product of the other firm. Letting x_H and x_S denote the investment expenditures of H and S, respectively, their profits are

$$\begin{aligned} \pi_H(x_H, x_S) &= (30 - x_S)x_H + 40x_S - \frac{x_H^2}{2} \\ \pi_S(x_H, x_S) &= (30 - x_H)x_S + 40x_H - \frac{x_S^2}{2} \end{aligned}$$

(a) If firms H and S simultaneously decide their investment expenditures, what are the equilibrium investment expenditures?

(b) What are the equilibrium profits?

(c) Suppose now that firm H can make a take-it-or-leave-it offer to buy firm S from its owners. What is the lowest amount that the owners of firm S would be willing to accept?

(d) Suppose that the owners of firm S accept the offer, and that the new firm will decide both x_H and x_S. What are the optimal investment expenditures for the new firm? What profit will it obtain?

(e) Suppose that firm H makes the lowest offer that would be accepted by the owners of firm S, that they accept such offer and that the new firm will then set the optimal investment expenditure levels that you have found in part (d). Would the profits of firm H (net of the price paid to buy firm S) increase? If so, by how much?

Problem 11: If firm A invests \$100 million it can enter a new market. Profit in period 1 will be \$30 million, but it is known that there is a probability of q (greater than 0 but less then 1) that the market grows in period 2 and that in this case profit will be \$250 million; with probability $1 - q$ the market would stay the same and the profit in period 2 would again be \$30 million. Firm A can invest in period 1 (without knowing whether the market will grow in period 2 or not) or it can wait until the beginning of period 2 to find out whether the market has grown or not before deciding whether to enter or not (in this case the profit in period 1 would be lost).

(a) What is the expected profit to firm A if it invests in period 1?

(b) What is the expected profit to firm A if it delays the entry decision until the beginning of period 2?

(c) For what values of q is firm A better off by investing immediately?

(d) Imagine that in period 1 you are able to foresee with no possibility for error whether the market in period 2 will grow or not. How much would you ask of firm 2 for such information?

(e) Suppose now that firm A has a potential competitor, firm B, which is in a position of entering the market in period 2 after having learned whether the market has grown or not. Suppose that the entry cost for B is \$120 million and that if firm B enters, period 2's profits are equally shared by the two firms. Will firm B enter if it finds out that the market has not grown?

(f) Will firm B enter if it finds out that the market has grown?

(g) Using the answers to the previous two parts, determine the expected profits to firm A if it invests immediately.

(h) Determine the expected profits to firm A if it delays the entry decision until the beginning of period 2.

(i) Imagine again that in period 1 you are able to foresee with no possibility for error whether the market in period 2 will grow or not. If you could sell this information to firm A, would the price you ask for it change because of the existence of a potential competitor? Explain your answer.

Problem 12: The three members of the board of directors of a corporation, A, B, and C, have to vote on whether to raise their salaries. Assume that each director prefers a raise over no raise; however, each also knows that voting for a pay raise may reduce the probability of being reappointed at the end of his/her term. Thus, the best possible outcome for an individual director is to vote against a pay raise that still wins majority approval, and the worst possible outcome is to vote for a pay rise that fails. Not all directors agree on how to rank the other two outcomes. For directors B and C, the second-best outcome is to have a raise proposal pass with their vote being in favor. For director A, however, the second-best outcome is for a raise proposal to fail, with his vote being against. The payoff ranking below (where large numbers represent more favorable outcomes) summarizes this situation. (You should assume that each director knows the rankings of all three directors.) Assume that the directors vote simultaneously.

(a) Write the normal form of this game (Suggestion: Let A be the row player, B be the column player and C be the matrix player.)

Outcomes	**Director A's rankings**	**Director B's and C's rankings**
Best, 3 points	Raise passes, vote against	Raise passes, vote against
Second best,	Raise fails, vote against points	Raise passes, vote for
Third best, 1 point	Raise passes, vote for	Raise fails, vote against
Worst, 0 points 1 point	Raise fails, vote for	Raise fails, vote for

(b) Find the Nash equilibria of this game.

(c) Explain (in words) the results of part (b).

Problem 13: Consider two firms, A and B. Each of them is considering whether to expand its capacity to make cars or not. Assume, for simplicity's sake, that each faces three possibilities: Do Not Expand production capacity (DNE), a Small expansion, or a Large expansion. The profits to A and B depend on their expansion capacity and are summarized in the following table.

		B		
		DNE	Small	Large
A	DNE	36, 36	30, 40	18, 36
	Small	40, 30	32, 32	16, 24
	Large	36, 18	24, 16	0, 0

(a) Because of the long lead times, assume that the two players' actions are simultaneous. Draw the extensive form of the game (the game tree) and the normal form of the game (the payoff matrix).

(b) What is the (Nash) equilibrium?

(c) Suppose now that A moves first and that B observes A's decision before making its own decision. Draw the extensive form of the game (the game tree). What is the Subgame Perfect Nash equilibrium? [Hint: Can you use backward induction?]

(d) Suppose now that A moves first, but that B does not observe A's decision before making its own decision. Draw the extensive form of the game (the game tree) and the normal form of the game (the payoff matrix). What is the Subgame Perfect Nash equilibrium? [Hint: Can you use backward induction?]

(e) Explain the results of parts b, c and d.

Problem 14: The board of directors of a company is composed of 9 directors. The board of directors has to make a decision about the CEO and there are three possible choices

A: Fire the CEO .

B: Keep the CEO but not give him a pay raise.

C: Keep the CEO and give him a pay raise.

Directors have different opinions about the appropriate course of actions. Some would want to fire the CEO and get a new CEO on board. Some believe that the CEO should not be fired, but that there is no need to give him a pay raise. Some finally believe that the CEO should be kept, but that only a pay raise will make sure that he will not be snatched away by the competition.

The ordering of preferences is the following:

- Four directors (the 'archenemies') prefer A to B to C
- Three directors (the 'enthusiasts') prefer C to B to A

- Two directors (the 'moderate') prefer B to C to A

The board of directors has agreed to the following voting procedure. Each director may vote for one of the three choices. Then, the choice with the least number of votes is dropped, and the directors vote on the remaining two. Whichever choice receives the most votes in round two is the winner.

(a) What would happen if each director voted according to his true preferences?

(b) Consider now the case in which the directors are aware of the strategic implications of their actions. How would you vote if you were an archenemy? [Hint 1: Do not try to draw the extensive form because it is very complicated and it will not help you much. Hint 2: Ask yourself: 1) What are the possible outcomes of the first vote? 2) For each of these possible outcomes, how would directors vote in the second vote? 3) Given the answers to the previous questions, how do you think each type of director will vote in the first vote?]

Problem 15: Two bidders, A and B, participate in an auction to buy a plot of land. Each of the two bidders can only bid one of three possible prices, low, medium and high. The plot will be sold to the highest bidder and in case of a tie a coin will be tossed to determine the winner.

The value of obtaining the plot net of the price to be paid is the following:

Price	**Net value**
High	10
Medium	30
Low nt	40

(a) Write down the normal form of the game.

(b) Find the pure strategy Nash equilibria.

Consider now the possibility of modifying the rules of the auction so that in case of a tie the winner is A with probability 1.

(c) Write down the normal form of the game.

(c) Find the pure strategy Nash equilibria.

(c) If you wanted to sell the plot and therefore wanted to maximize the expected sale price, which of the two auction formats would you prefer? Explain your answer.