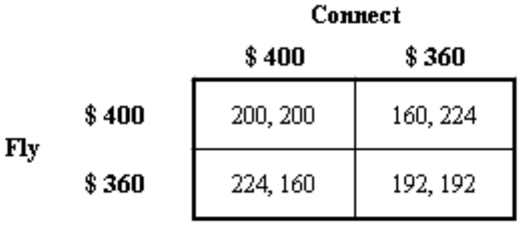
Universidad Carlos III
Master in Industrial Economics and Markets
Game Theory: Problem set 3

Problem 1

(a)



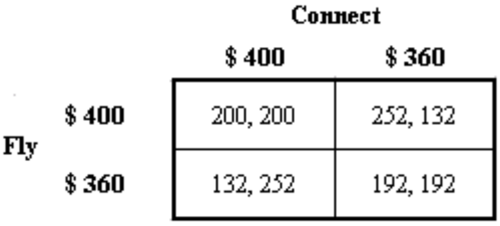
		Connect	
		$ 400	$ 360
Fly	$ 400	200, 200	160, 224
	$ 360	224, 160	192, 192

NE: ($360, $ 300)

Observe: Fly fix $ 400
Connect fix $ 400

Both have unitary margin of $200 with ($360, $ 400). Fly margin is $160 while Connect one is $200 but Fly sales are multiplied by 1.4 and Connect sales are multiplied by 0.8. The other payoffs are obtained in a similar way.

(b)



		Connect	
		$ 400	$ 360
Fly	$ 400	200, 200	252, 132
	$ 360	132, 252	192, 192

Observe

I. With ($400, $400) and ($360, $360) sales and cost are the same for the two companies it is irrelevant that they equally share the income.
II. With ($360, $400) Fly sales are multiplied by 1.4 and Connect ones by 0.8 so income, costs, and profits are the following:

	Fly	**Connect**
Income	360(1.4) = 504	400(0.8) = 320
Costs	200(1.4) = 280	200(0.8) = 160
Profit	$\frac{504 + 320}{2} – 280= 132$	$\frac{504 + 320}{2} –160 = 252$

The NE (in dominant strategies) is ($400, $400).

(c) Yes: The common pool reduce the incentives to lower prices and lead to high equilibrium prices.

Problem 2

(a)

Let :

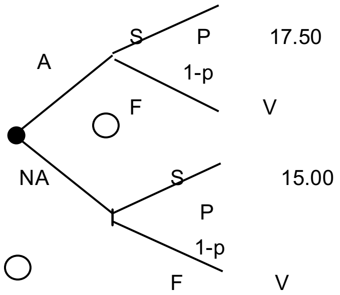
A: Accept Williams´ offer

NA: Do not accept Williams´ offer

S: At least 51% are offered by Transco shareholders in the 1st tier.

F: Less than 51% are offered by Transco shareholders in the 1st tier.

Decision tree



(b) Expected profit from:

A	$17.50P + (1-P)\,V$
NA	$15.00P + (1-P)\,V$

Observe that

$$17.50P + (1-P)V \geq 15P + (1-P)V \quad \forall\ P \in [0,1]$$

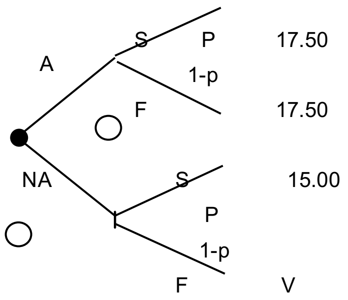
$$17.50P + (1-P)V > 15P + (1-P)V \quad \forall\ P \in (0,1]$$

A is weakly dominant so it is optimal

Being the offer conditional on securing 51% of shares, if Williams attempt fails, the payment V does not depend from shareholders´decisions.

(c) If all shareholders think this way, all accept the offer

(d)



(e) Expected profit from:

A: 17.50
NA: 15.00P + (1-P) V
If V ≤ 17.50, then 17.50 ≥ (15.00)P + (1-P)V so A is weakly dominant.

If V > 17.50:
17.50 ≥ (15.00)P + (1-P)V whenever the probability of success is high enough.

$$\left(P \geq \frac{V - 17.5}{V - 15.00} \right)$$

Yes because for V > 17.50 A is no longer dominant.

(f) V ≤ 17.50: all shareholders accept
V > 17.50: there are 2 equilibria:

I. No one accepts the offer (P = 0 and it´s optimal not to accept it).
II. All accept the offer (P = 1 and it is optimal to accept it).

(g) Because it is sure to buy also when V > 17.5.

Problem 3

(a)

$$\underset{x_1}{Max} \quad \Pi_1(x_1, x_2) = \underset{x_1 \geq 0}{Max} \quad (30 + x_2)x_1 - 2x_1^2$$

FOC

$30 + x_2 - 4\,x_1 = 0$ **(1)**

Everything is symmetric: Set $x_1 = x_2$ in (1)

$30 + x - 4x = 0$
$x_1^* = x_2^* = x = 10$

(b) Substituting $x_1^* = x_2^* = 10$ into Π_1

$\Pi_1^* = \Pi_1(10, 10) = 200$
$\Pi_2^* = 200$, by symmetry

(c) Max $\Pi_1 (x_1, x_2) + \Pi_2 (x_1, x_2) = (30+x_2)x_1 - 2x_1^2 + (30 + x_1)x_2 - 2x_2^{\,2}$
x_1, x_2

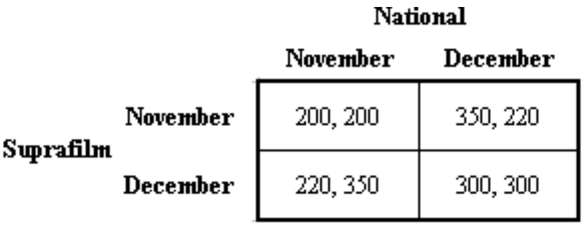
FOC

$$\begin{cases} 30 + x_2 - 4x_1 + x_2 = 0 \\ 30 + x_1 - 4x_2 + x_1 = 0 \end{cases} \Longrightarrow x_1^{**} = x_2^{**} = 15$$

$\Pi_1 (15, 15) + \Pi_2 (15, 15) = 450 > 200 + 200$
$= \Pi_1 (10, 10) + \Pi_1 (10, 10)$

Problem 4

(a)



		National	
		November	December
Suprafilm	November	200, 200	350, 220
	December	220, 350	300, 300

(b)
(December, November), (November, December)

Question 5.9

(a) Max $\Pi_A (q_A, q_{B,} q_C) = [10 – 2(q_A + q_B + q_C)]q_A - 2 q_A$
$q_{A\geq 0}$

$$\frac{\partial \Pi_A}{\partial q_A} = 10 - 2q_B - 2q_C - 4q_A - 2 = 0$$

By symmetry

$8 – 8q = 0 \qquad q^* = q_A^* = q_B^* = q_C^* = 1$

(b) $\Pi_A (1, 1, 1) = (10 –2(3))1 – 2(1) =$
$= 4 –2$
$= 2$
$= \Pi_B (1, 1, 1)$
$= \Pi_C (1, 1, 1)$

(c) Max $\Pi_A (q_A, q_B) = [10 – 2 (q_A + q_B)] q_A - 2q_A$
q_A

$$\frac{\partial \Pi_A}{\partial q_A} = 10 - 2q_B - 4q_A - 2 = 0$$

By symmetry $8 – 6q = 0 \qquad q^* = \underline{8} = \underline{4} = q_A^* = q_B^*$

6 3

$$\Pi_A\left(\frac{4}{3},\frac{4}{3}\right)=\left(10-2\frac{8}{3}\right)\frac{4}{3}-\frac{8}{3}=\frac{14}{3}\frac{4}{3}-\frac{8}{3}$$

The maximal amount that firm A would be willing to pay is $\frac{32}{9}-2=\frac{32-18}{9}=\frac{14}{9}$

(d) Yes, because $\Pi_B\left(\frac{4}{3},\frac{4}{3}\right)=\frac{32}{9}>2=\Pi_B(1,1,1)$ Notice, however, that the minimum price firm C would require to sell is $\Pi_C(1,1,1)=2>\frac{14}{9}$ so that A will not buy from C.

Problem 5

(a) A = 0

(b) $B=\underbrace{50000}_{\#\ of\ shares}\cdot\underbrace{20}_{\substack{value\ of\ each\\ share}}=1000000$

(c) $C=\underbrace{50000}_{\#\ of\ shares}\cdot(\underbrace{20\cdot 1.2}_{\substack{value\ of\ each\\ share\ under\ new\\ management}}-\underbrace{21}_{\substack{price\ paid\\ for\ each\ share}})=50000\cdot 3=150000$

(d) $D=\underbrace{50000}_{\#\ of\ shares}\cdot\underbrace{20\cdot 1.2)}_{\substack{value\ of\ each\\ share\ under\ new\\ management}}=1200000$

(e)

$$E=\frac{\overbrace{50000}^{shares\ bought\ by\ I}}{\underbrace{50000+50000+50000}_{Total\ number\ of\ shares}}(\underbrace{2400000}_{\substack{Value\ of\ company\\ under\ new\ management}}+\underbrace{500000}_{\substack{increase\ in\ value\ (assets)\\ of\ company\ deriving\ from\\ sale\ of\ new\ shares\ under\\ the\ shareholder\ rights\ plan}})-\underbrace{50000}_{\substack{\#\ of\ shares\\ bought\ by\ I}}\cdot\underbrace{21}_{\substack{price\ paid\\ for\ each\\ share}}$$

$$=\frac{1}{3}(2900000)-1050000=966666.67-1050000$$

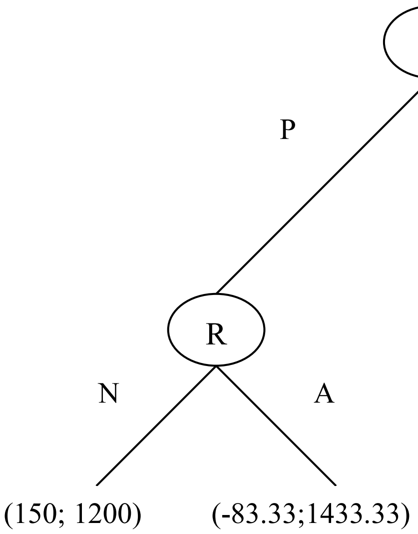
$$=-83333.33$$

(f)

$$F = \frac{\overbrace{50000}^{\substack{\#\ \text{of shares}\\ \text{already owned}\\ \text{by residual}\\ \text{shareholders}}} + \overbrace{50000}^{\substack{\#\ \text{of shares already}\\ \text{purchased by residual}\\ \text{shareholders under plan}}}}{\underbrace{50000+50000+50000}_{\text{Total}\ \#\ \text{of shares}}}(\underbrace{2400000}_{\substack{\text{value of company}\\ \text{under new}\\ \text{management}}} + \underbrace{500000}_{\substack{\text{increase in value}\\ \text{(assets) of company}\\ \text{deriving from sales of}\\ \text{new shares under plan.}}}) - \underbrace{50000}_{\substack{\#\ \text{of shares purchased}\\ \text{by residual shareholders}}} \cdot \underbrace{10}_{\substack{\text{price paid}\\ \text{for each}\\ \text{share}\\ \text{under plan}}}$$

$$= \frac{2}{3}(2900000) - 500000 = 1933333.33 - 500000$$
$$= 1433333.33$$

(g)



Notice: payoffs are in thousands of €.

By backward induction SPE is (NP, A)

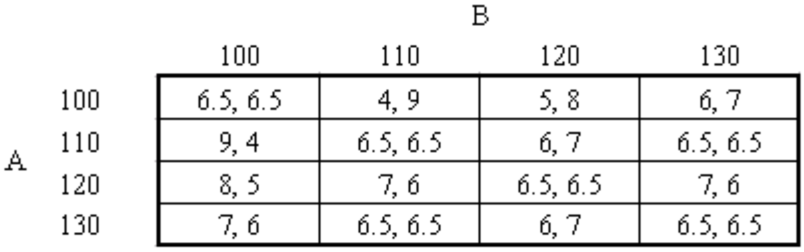
“Remark: Another legitimate answer to parts (e) and (f) is

$$E = \frac{1}{3}(2000000 + 500000)\,1.2 - 50000(21)$$
$$= \frac{1}{3}\,3000000 - 1050000 = -50000$$
$$F = \frac{2}{3}(2000000 + 500000)1.2 - 50000(10)$$
$$= \frac{2}{3}\,3000000 - 500000$$
$$= 1.500.000$$

In this case the answer to (g) would be the same”

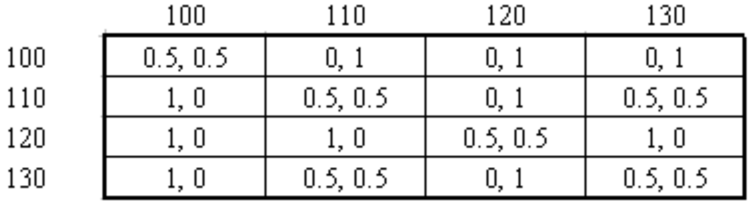
Problem 6

(a)



		B			
		100	110	120	130
	100	6.5, 6.5	4, 9	5, 8	6, 7
A	110	9, 4	6.5, 6.5	6, 7	6.5, 6.5
	120	8, 5	7, 6	6.5, 6.5	7, 6
	130	7, 6	6.5, 6.5	6, 7	6.5, 6.5

(b)



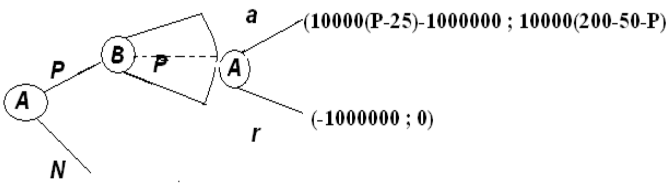
	100	110	120	130
100	0.5, 0.5	0, 1	0, 1	0, 1
110	1, 0	0.5, 0.5	0, 1	0.5, 0.5
120	1, 0	1, 0	0.5, 0.5	1, 0
130	1, 0	0.5, 0.5	0, 1	0.5, 0.5

(c) (120, 120)

(d) Each one of the two executives proposes the median of the distribution of preferred prices.

Problem 7

(a)



P: build plant
N: do not build plant
A: accept offer P
R: reject offer P

(b) Suppose A received offer P. It should accept it if

$$10000(P-25) - 1000000 \geq -1000000 \quad \text{i.e if } P \geq 25$$

Given A will accept any offer $P \geq 25$ at second node (the cost of the plant is SUNK) the best offer for B is $P = 25$. Given this at first node, A knows that if it builds the plant its final payoff will be 10000(25-25) – 1000000 = -1000000 and therefore it prefers not to build the plant.

The subgame perfect Nash equilibrium is

A: (N, accept any $P \geq 25$) B: (offer $P = 25$)

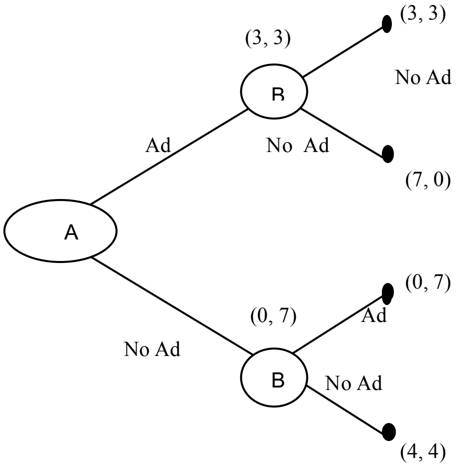
Problem 8

(a)

		B	
		Add	**No Add**
A	**Add**	3,	7, 0
	No Add	0, 7	4, 4

Both firms have a dominant strategy: to advertise. Unique NE is {Advertise, Advertise}.

(b)



SPNE A : Ad

B : {Advertise, Advertise}

Outcome: (3, 3)

		B			
		AdAd	Ad-NAd	NAd-Ad	NAd-NAd
A	Ad	3, 3	3, 3	7, 0	7, 0
	NAd	0, 7	4, 4	0, 7	4, 4

NE

A: AD; B : {Ad, Ad}

i. Without commitment power of B, both timing are equivalent as they yield inefficient eq.

Problem 9

a) Consider a contract that pays the agency $\overline{W}$ in case of success and $\underline{W}$ in case of failure. For the agency to accept the contract and exert the high effort it is necessary that

Expected Payoff with high effort $\geq$ Expected Payoff with low effort

$$0.75\overline{W} + 0.25\underline{W} - 400000 \geq 0.5\overline{W} + 0.5\underline{W} - 200000 \qquad \textbf{(1)}$$

Expected Payoff with high effort $\geq$ Reservation level

$$0.75\overline{W} + 0.25\underline{W} - 400000 \geq 0 \qquad \textbf{(2)}$$

To minimize cost require (2) to hold with equality

$$\frac{1}{4}\underline{W} = 400000 - \frac{3}{4}\overline{W} \Longleftrightarrow \underline{W} = 1600000 - 3\overline{W} \qquad \textbf{(3)}$$

From (1)

$$\frac{1}{4}\overline{W} - \frac{1}{4}\underline{W} \geq 200000 \Longleftrightarrow \overline{W} - \underline{W} \geq 800000 \qquad \textbf{(4)}$$

Substitute (3) in (4)

$$\overline{W} - 1600000 + 3\overline{W} \geq 800000 \Longleftrightarrow 4\overline{W} \geq 2400000$$

$$\Longleftrightarrow \overline{W} \geq \frac{2400000}{4} = 600000 \qquad \textbf{(5)}$$

Set $\overline{W} = 60000$ and substitute it into (3)

$\underline{W} = -200000.$

Notice: if payment be negative no contract can induce agency to accept it and exert high effort.

b) Consider alternative contract with fixed payment W = 200000. With a fixed payment the agency exerts low effort (has no incentives to exert high) and therefore accepts contract.
Expected payoff to your company: (0.5*600.000 + 0.5*0) – 200000 = 100000 with contract of previous part.

Expected payoff to your company 0.75(600000–600000) + 0.25(0–200000) = 50000

Since payoff in first case is higher than in second, firm has no interest in inducing agency to exert high effort.

Problem 10

a) $\max_{x_H} \quad \Pi_H(x_H, x_S) = (30 - x_S)x_H - \frac{1}{2} x_H^2$

FOC

$30 - x_S - x_H = 0$

By symmetry

$30 = 2x \qquad x_S^* = x_H^* = 15$

b) $\Pi_H^* = \Pi_H(x_H^*, x_S^*) = 15*15 + 40*15 - 0.5*15^2 = 712.5 = \Pi_S^*$

c) 712.5

d) $\max_{x_H, x_S} \quad \Pi_H(x_H, x_S) + \Pi_S(x_H, x_S) = (30 - x_S)\, x_H + 40\, x_S - 0.5\, x_H^2 + (30 - x_H)\, x_S + 40\, x_H - 0.5\, x_S^2$

FOC

with respect to x_H : $30 - x_S - x_H - x_S + 40 =$

By symmetry

$30 - 3x + 40 = 0 \qquad \tilde{x} = 70/3 = 23.333 = \tilde{x}_S = \tilde{x}_H$

$$\tilde{\Pi} = \Pi_H(\tilde{x}_H, \tilde{x}_S) + \Pi_S(\tilde{x}_H, \tilde{x}_S) = 2\left[\left(\frac{90}{3} - \frac{70}{3}\right)\frac{70}{3} + 40\,\frac{70}{3} - \frac{1}{2}\,\frac{4900}{9}\right]$$

$$= \frac{2800 + 16800 - 4900}{9} = \frac{14700}{9}$$

$$= 1633.33$$

e) $1633.33 - 712.5 = 920.83 > 712.5$

Problem 11

a) $30 + 250q + 30(1-q) - 100 = 30 + 250q + 30 - 30q - 100$

$= 220q - 40$

b) $(250 - 100)q \qquad = 150q$

c) $220q - 40 > 150q \qquad 70q > 40 \qquad q > \frac{4}{7}$

d) Expected profit to firm with information

$(30 + 250 - 100)q + (1-q)*0 = 180q$

Notice: $180q \geq 220q - 40$ for $q \in [0, 1]$

$180q > 150q$ for $q \in [0, 1]$

Increase in profit with information

If $q > \frac{4}{7}$ $180q - 220q + 40 = 40(1-q)$

If $q < \frac{4}{7}$ $180q - 150q = 30q$

Consultant should ask for increase in profit with information

e) No, because $0.5 * 30 - 120 = -105 < 0$

f) Yes, because $0.5 * 250 - 120 = 5 > 0$

g) $30 + (0.5*250)q + 30(1-q) - 100 = 95q - 40$

h) $(0.5*250 - 100)q = 25q$

i) Invest now is better than wait and possibly invest if

$$95q - 40 > 25q \Longleftrightarrow q > \frac{4}{7}$$

Increase in profit with information

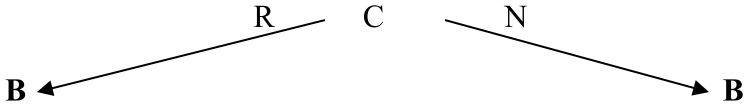
If $q > \frac{4}{7}$ $(30 + 0.5*250 - 100)q - 95q + 40 = 40(1-q)$

If $q < \frac{4}{7}$ $(30 + 0.5*250 - 100)q - 25q = 30q$

Consultant should request same fee as without competitor.

Problem 12

a)



		R	N
A	R	1, 2, 2	1, $\underline{3}$, $\underline{2}$
	N	$\underline{3}$, $\underline{2}$, $\underline{2}$	$\underline{2}$, 1, 0

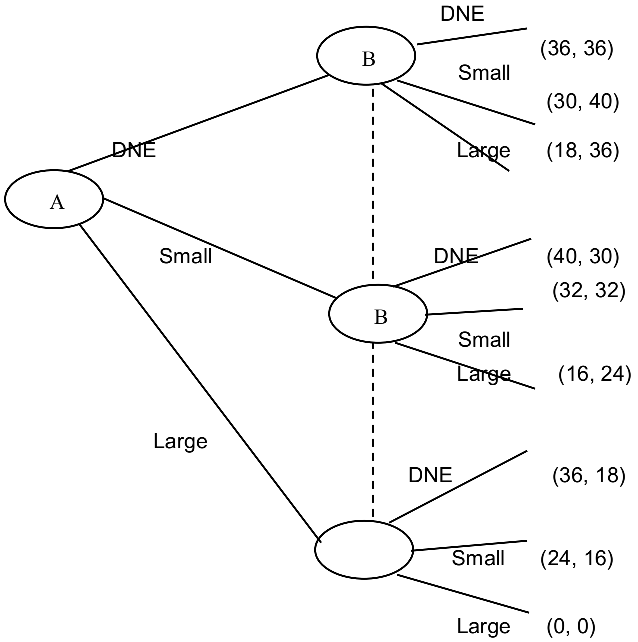
		R	N
A	R	1, $\underline{2}$, $\underline{3}$	0, 1, 1
	N	$\underline{2}$, 0, 1	$\underline{2}$, $\underline{1}$, $\underline{1}$

b) (N, R, R) (N, N, N)

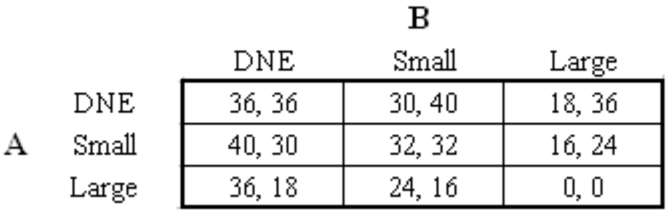
c) N is a dominat strategy for A. Given this each of B and C will want to play R if the other also play R and N if the other also plays N.

Problem 13

a) Extensive form



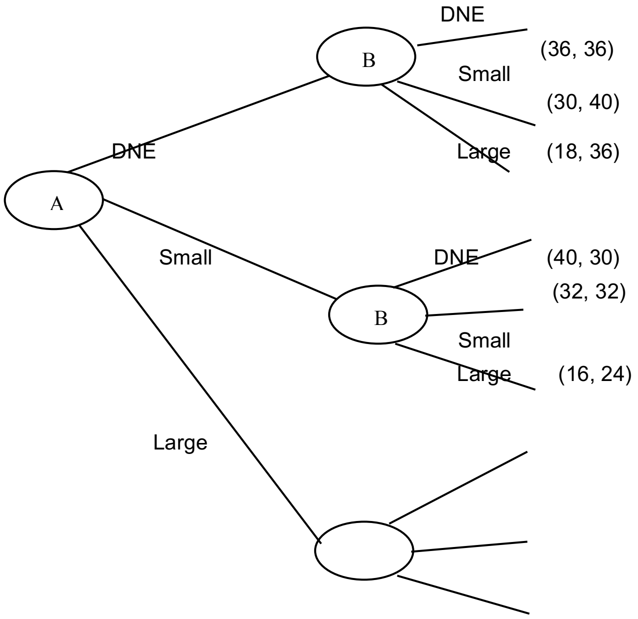
Normal Form



		B DNE	Small	Large
	DNE	36, 36	30, 40	18, 36
A	Small	40, 30	32, 32	16, 24
	Large	36, 18	24, 16	0, 0

b) (Small, Small)

c)



DNE	(36, 18)
Small	(24, 16)
Large	(0, 0)

Using backward induction we obtain:
A: Large
B: Response to A´s choice
DNE → Small
Small → Small
Large → DNE

d) Identical to (a) and to (b)

e) When A moves first but its move is not observed by B before B itself moves, the situation is identical to one in wich A and B simultanously.

Problem 14

a) In round 1 A would receive 4 votes
B would receive 2 votes
C would receive 3 votes

And B would be excluded

In round 2 the archenemies would vote for A
the enthusiasts would vote for C
the moderate would vote for C

and C would be the winner.

b) What are the possible outcomes in the first vote?

A is dropped; in round 2 B vs C
B is dropped; in round 2 A vs C
C is dropped; in round 2 A vs B

I. Round 2

If B vs C → 6 votes for B, 3 votes for C → B wins
If A vs C → 4 votes for A, 5 votes for C → C wins
If A vs B → 4 votes for A, 5 votes for B → B wins

If the archenemies vote for A in the first round, B will defeated (because the best response of the enthusiasts would be to vote for C) and C will result in the second round. But if the archenemies vote for B, A will be defeated in the first round and B will result in the second round, This is clarified by the normal form representation of first period play in the following page (that assumes equilibrium behavior in second round and the order of the payoffs in the tables is A,E,M).

A M C

B

A E	A	B	C
A	3, 1, 1	2, 2, 3	1, 3, 2
B	2, 2, 3	2, 2, 3	2,2, 3
C	1, 3, 2	2,2, 3	1, 3,2

A E	A	B	C
A	1, 3, 2	2, 2, 3	1, 3, 2
B	2, 2, 3	2, 2, 3	2, 2, 3
C	1, 3, 2	2,2,3	1,3,2

A E	A	B	C
A	2, 2, 3	2, 2, 3	1, 3, 2
B	2, 2, 3	2, 2, 3	2, 2, 3
C	1, 3, 2	2,2,3	2, 2, 3

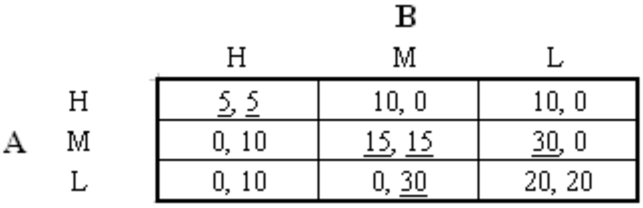
A : archenemies, row player. First payoff
E: enthusiasts, column player, second payoff
M: moderate, matriz player, third payoff

Payoff form top option: 3
Payoff form second option: 2
Payoff from worst option: 1

Payoff vectors when final outcome is
A: (3, 1, 1)
B: (2, 2, 3)
C: (1, 3, 2)

Problem 15

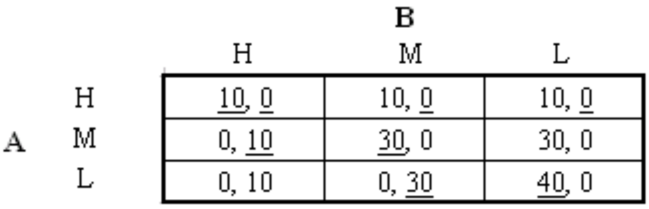
a)



		B		
		H	M	L
	H	5, 5	10, 0	10, 0
A	M	0, 10	15, 15	30, 0
	L	0, 10	0, 30	20, 20

b) (H, H), (M, M)

c)



		B		
		H	M	L
	H	10, 0	10, 0	10, 0
A	M	0, 10	30, 0	30, 0
	L	0, 10	0, 30	40, 0

d) (H, H)

e) The second because in the unique NE both bidders a high price.