University Carlos III of Madrid **Departament of Economics**

Game Theory: Problem set 1.

Problem 1: For each one of the following normal form games find

(a) The pure-strategy Nash equilibria and their payoffs;

(b) The mixed-strategy Nash equilibria and their payoffs;

(c) Are there any equilibria in dominant strategies? If yes, which ones?

(a)

	C	D
A	$-50, -50$	$100, 0$
B	$0, 100$	$0, 0$

(b)

	C	D
A	$-1, 2$	$1, -2$
B	$1, -2$	$-1, 2$

(c)

	C	D
A	$-10, -10$	$0, -20$
B	$-20, 0$	$10, 10$

(d)

	C	D
A	$3, 3$	$0, 0$
B	$0, 0$	$1, 1$

(e)

	C	D
A	$4, -8$	$-2, -7$
B	$1, -5$	$-1, -6$

(f)

	C	D
A	$2, 5$	$8, -1$
B	$4, -8$	$-2, -3$

(g)

	C	D
A	$10, 10$	$-2, -7$
B	$1, -5$	$1, 1$

Problem 2: For the normal form games below determine

(a) the strategies that survive the iterated elimination of strictly dominated strategies.

(b) the pure strategy Nash equilibria of the game.

(c) the mixed strategy Nash equilibria of the game.

(a)

	X	Y	Z
A	$20, 10$	$10, 20$	$1, 1$
B	$10, 20$	$20, 10$	$1, 1$
C	$1, 1$	$1, 1$	$0, 0$

(b)

	D	E	F
A	$2, 0$	$1, 1$	$4, 2$
B	$3, 4$	$1, 2$	$2, 3$
C	$1, 3$	$0, 2$	$3, 0$

(c)

	D	E	F
A	$2, 2$	$1, 1$	$4, 3$
B	$1, 4$	$0, 5$	$3, 3$
C	$3, 3$	$-2, 2$	$3, 0$

(d)

	D	E	F
A	$4,-1$	$3,4$	$2,0$
B	$1,4$	$-3,5$	$-3,-3$
C	$3,3$	$2,-2$	$8,-3$

(e)

	D	E	F
A	$2,-1$	$2,-1$	$4,-2$
B	$2,-1$	$-2,-1$	$3,-2$
C	$1,3$	$-2,2$	$3,4$

(f)

	A	B	C	D
X	$1,4$	$2,2$	$4,1$	$2,0$
Y	$0,0$	$1,3$	$2,5$	$0,0$
Z	$0,1$	$5,0$	$8,1$	$3,3$

Problem 3: For the normal form game

	X	Y	Z
A	$10,30$	$0,20$	$20,30$
B	$15,35$	$10,40$	$10,40$
C	$25,25$	$5,25$	$5,25$

determine all the Nash equilibria of the game.

Problem 4: Consider a game in which there is a prize worth \$30. There are three contestants, A, B, and C. Each can buy a ticket worth \$15 or \$30, or not buy a ticket at all. They make these choices simultaneously and independently. Once the ticket purchase decisions have been made, the prize is awarded. If no one bought the ticket, the prize is not awarded. Otherwise, the ticket is awarded to the buyer of the higher cost ticket if there is only one such player, or split equally between two or three players if there are ties among the highest cost ticket buyers.

1. Write the normal form of the game. (Hint: Draw a box for each of player C?s possible strategies.)
2. Identify the Nash equilibria in pure strategies of this game.

Problem 5: Two firms, 1 and 2, produce heterogeneous products. If the two firms set prices p_1 and p_2, respectively, the quantities demanded of the products of the two firms will be,

$$\begin{aligned} x_1(p_1,p_2) &= \max\left\{0, 180 - p_1 - \left(p_1 - \frac{p_1+p_2}{2}\right)\right\} \\ x_2(p_1,p_2) &= \max\left\{0, 180 - p_2 - \left(p_2 - \frac{p_1+p_2}{2}\right)\right\} \end{aligned}$$

The above demand functions describe a situation in which the products are not perfectly homogeneous. For instance, the demand for good 1, even if decreasing in the price set by firm 2, is positive also when $p_1 > p_2$ (which is not the case when the good are homogeneous), or, in a different way, the demand for good 1 will be lower than $180 - p_1$ when the price of good 1 is higher than the mean of the prices (and therefore higher than p_2) and higher than that when the price of good 1 is lower than the mean of the prices (and therefore lower than p_2). Assume that the firms have constant marginal costs $\mathrm{MC}_1 = 20$ and $\mathrm{MC}_2 = 20$. Assume also that the characteristics of the market are such that the firms have to set simultaneously the prices.

(a) Write down firms' profits, $\pi_1(p_1,p_2)$ and $\pi_2(p_1,p_2)$, as functions of the prices they set.

(b) Find firm 1's best response to a price set by firm 2, $\mathrm{BR}_1(p_2)$.

(c) Find firm 2's best response to a price set by firm 1, $\mathrm{BR}_2(p_1)$.

(d) Find the Nash equilibrium (p_1^*, p_2^*).

(e) Determine equilibrium profits, $\pi_1(p_1,^* p_2^*)$ and $\pi_2(p_1^*, p_2^*)$.

(f) Are equilibrium profits zero? Why?

(g) Draw the best response function and the Nash equilibrium in the $p_2 - p_1$ plane (p_2 on the horizontal axis and p_1 on the vertical axis).

Problem 6: Two firms, 1 and 2, produce heterogeneous products. If the two firms set prices p_1 and p_2, respectively, the quantities demanded of the products of the two firms will be:

$$\begin{aligned} x_1(p_1, p_2) &= 140 - p_1 - \left(p_1 - \frac{p_1 + p_2}{2}\right) \\ x_2(p_1, p_2) &= 140 - p_2 - \left(p_2 - \frac{p_1 + p_2}{2}\right) \end{aligned}$$

Suppose the two firms have constant marginal costs MC1 = 40 and MC2 = 40 and that they have to set their prices simultaneously.

(a) What is the Nash equilibrium ? What are the Nash equilibrium profits for firms 1 and 2?

(b) Are the profits in equilibrium equal to 0? Why or why not?

Problem 7: Assume that two firms, 1 and 2, produce heterogeneous products and the quantities demanded by the market, when these firms fix prices p_1 and p_2, are, respectively:

$$\begin{aligned} xi_1(p_1, p_2) &= 200 - p_1 + p_2/3 \\ x_2(p_1, p_2) &= 200 - p_2 + p_1/3 \end{aligned}$$

These demand functions describe a situation in which products are not perfectly homogenous. Suppose that both firms have constant marginal costs MC= 50 and MC2 = 50 and that the features of the market are such that both firms have to set prices simultaneously.

(a) Find the Nash equilibrium (p_1^*, p_2^*).

(b) Determine equilibrium profits, $\pi_1(p_1,^* p_2^*)$ and $\pi_2(p_1^*, p_2^*)$. Are equilibrium profits zero? Why?

(c) Suppose now that firm 1 and 2 could make an agreement on how to make their products different so that the demand functions become,

$$\begin{aligned} xi_1(p_1, p_2) &= 200 - p_1 + p_2/2 \\ x_2(p_1, p_2) &= 200 - p_2 + p_1/2 \end{aligned}$$

Do you think that it pays for the firms to agree on these new differentiated products?

Problem 8: Firms 1 and 2 compete in a market with a homogeneous product. Firms have to decide how much to produce and put up for sale. Let q_1 and q_2 denote the quantities produced and put up for sale by firm 1 and 2, respectively. The clearing market price of the homogeneous product on sale depends on quantities produced according to the following inverse demand function: $P(Q) = \max\{0, 10 - Q\}$, where $Q = q_1 + q_2$ is the total number of units of the product in the market. Total production costs are $c_1(q_1) = q_1^2$ and $c_2(q_2) = q_2^2$. This is the Cournot model of oligopoly competition that you will study in the course of Industrial Organization.

(a) Describe the situation as a normal form game.

(b) Find the Nash equilibrium.