Chapter 5
Static Games with asymmetric information

December 18, 2019

1 The Bayesian equilibrium

Definition 1.1. A Bayesian game G consists of the following,

$$G = (N, A, T, \{p_i\}_{i=1}^n, \{u_i\}_{i=1}^n)$$

where we have

- The set of *players* $N = \{1, 2, \dots, n\}$.
- The space of *actions* $A_1, A_2, \dots, A_n$ of each player. $A = A_1 \times A_2 \times \cdots \times A_n$, $a = (a_1, a_2, \dots, a_n)$
- The set of possible *types* $T_1, T_2, \dots, T_n$ for each player. The type $t_i \in T_i$ is known only by player $i = 1, 2, \dots, n$.
- The set of all the types is $T = T_1 \times T_2 \times \cdots \times T_n$. We use the notation $t = (t_1, t_2, \dots, t_n)$, $t_{-i} = (t_1, t_2, \dots, t_{i-1}, t_{i+1}, \dots t_n)$.
- Each player i has a *conjecure* $p_i : T_i \to \Delta(T_{-i})$ about the types of the other players. It may depend on its own type
$$p_i = p_i(t_{-i}|t_i)$$
and it is interpreted as the information that agent i has on the types of the other agents (t_{-i}), given that his type is t_i.
- The conjectures of the agents have to be consistent and compatible with Bayes rule: There is a distribution of probabilities $q \in \Delta(T)$ such that
$$p_i(t_{-i}|t_i) = \frac{q(t_1 \dots, t_n)}{\sum_{s_{-i} \in T_{-i}} q(t_i, s_{-i})}$$
We interpret $q(t_1, \dots, t_n)$ as the probability that agent 1 is of type t_1, agent 2 is of type t_2, ... and agent n is of type t_n. The denominator $\sum_{s_{-i} \in T_{-i}} q(t_i, s_{-i})$ is the probability that the agent of type i be t_i. That is, $p_i(t_{-i}|t_i)$ is, according to Bayes rule, the probability that the other agents have the type t_{-i}, given that the type of agent i is t_i.
- A **pure strategy** of player i,
$$s_i : T_i \to A_i$$
specifies the action of each of player for each of his possible types i.
- A **mixed strategy** of player i,
$$\sigma_i : T_i \to \Delta(A_i)$$
is a vector $\sigma_i(t_i) = (\sigma_i(a_1, t_i), \dots, \sigma_i(a_n, t_i))$. Here, $\sigma_i(a_k, t_i)$ is the probability that agent i of type t_i plays the strategy a_k.

- *The funcions of utility over outcomes:* of the agents $u_1, u_2, \ldots, u_n$ are of the form

$$u_i : A \times T \to \mathbb{R}$$

$u_i(a;t)$ depends on the types $t = (t_1, \ldots, t_n)$ and on the actions of all the agents $a = (a_1, \ldots, a_n)$.

- *Utility functions over strategies:* The expected utility of agent i when he plays strategy a_i, given that the other agents are playing the pure strategies s_k is

$$U_i(s_1, \ldots, s_{t_i-1}, a_i, s_{t_i+1}, \ldots s_n) =$$

$$\sum_{t_{-i} \in T_{-i}} u_i\big(s_1(t_1), \ldots, s_{t_{i-1}}(t_{i-1}), a_i, s_{t_{i+1}}(t_{i+1}), \ldots s_n(t_n); t\big) p_i(t_{-i}|t_i)$$

- The notion of equilibrium is the NE with the above expected utility functions.

2 Cournot competition with asymmetric information

Cournot Duopoly

- Suppose there are two companies which compete in quantities (Cournot's competion).
- The inverse demand is

$$P(q) = a - q$$

where

$$q = q_1 + q_2$$

and q_1, q_2 are the amounts produced by the companies.

- The cost function of Firm 1 is

$$c_1(q_1) = cq_1$$

- The cost function of Firm 2 is

$$c_2(q_2) = \begin{cases} c_a q_2 & \text{with probability } \theta \\ c_b q_2 & \text{with probability } 1 - \theta \end{cases}$$

with $c_b < c_a$.

- There is asymmetric information:
  - Firm 2 knows its cost cost function (c_a or c_b) and knows the cost of Firm 1.
  - But, Firm 1 only knows its cost function. It also knows that the marginal cost of Firm 2 is c_a with probability θ and c_b with probability $1 - \theta$.
  - Firm 2 has more information than Firm 1.

- First we represent this situation as a **Bayesian Game**.
- **The players** are $N = \{1, 2\}$.
- **The types** are the cost functions of the companies $T_1 = \{c\} \quad T_2 = \{c_a, c_b\}$.
- **The utility functions** are the profits,

$$\begin{aligned} \pi_1(q_1, q_2, c) &= (a - q_1 - q_2)q_1 - cq_1 = (a - q_1 - q_2 - c)q_1 \\ \pi_2(q_1, q_a, c_a) &= (a - q_1 - q_a)q_a - c_a q_a = (a - q_1 - q_2 - c_a)q_a \\ \pi_2(q_1, q_b, c_b) &= (a - q_1 - q_b)q_b - c_b q_b = (a - q_1 - q_2 - c_b)q_b \end{aligned}$$

- The **sets of actions** are $A_c = A_a = A_b = [0, \infty)$.
- **The conjectures of the firms** are

$$p_2(t_1 = c | c_a) = p_2(t_1 = c | c_b) = 1$$

and

$$p_1(t_2 = c_a | c) = \theta, \quad p_1(t_2 = c_b | c) = 1 - \theta$$

Best response of Firm 2

- If the cost function of Firm 2 is c_a then it solves the following problem

$$\max_{q_a} (a - q_1 - q_a) q_a - c_a q_a = \max_{q_2} (a - q_1 - q_a - c_a) q_a$$

- The FOC's are $a - q_1 - 2q_a - c_a = 0$ so the best response for Firm 2 provided its cost is c_a and that Firm 1 produces q_1 is

$$q_a = q_2(c_a) = \frac{a - q_1 - c_a}{2}$$

- If the cost function of Firm 2 is c_b then it solves the following problem

$$q_b = \max_{q_2} (a - q_1 - q_b) q_b - c_b q_b = \max_{q_2} (a - q_1 - q_b - c_b) q_b$$

- The FOC's for the firm are $a - q_1 - 2q_b - c_b = 0$ so the best response for Firm 2 if its cost is c_b and firm 1 produces q_1 is

$$q_b = q_2(c_b) = \frac{a - q_1 - c_b}{2}$$

Best response of Firm 1

- Firm 1 does not know the cost function of Firm 2. It maximizes **expected profit (EP)**

$$\max_{q_1} \quad \theta \underbrace{(a - q_1 - q_2(c_a) - c) q_1}_{\text{EP if firm 2's cost is } c_a} + (1 - \theta) \underbrace{(a - q_1 - q_2(c_b) - c) q_1}_{\text{EP if firm 2's cost is } c_b}$$

- The FOC is

$$\theta (a - 2q_1 - q_2(c_a) - c) + (1 - \theta) (a - 2q_1 - q_2(c_b) - c) = 0$$

- We obtain the reaction function of Firm 1

$$q_1 = \frac{\theta (a - q_2(c_a) - c) + (1 - \theta) (a - q_2(c_b) - c)}{2}$$

- The NE satisfies the equations

$$q_1 = \frac{\theta (a - q_2(c_a) - c) + (1 - \theta) (a - q_2(c_b) - c)}{2}$$
$$q_2(c_a) = \frac{a - q_1 - c_a}{2}$$
$$q_2(c_b) = \frac{a - q_1 - c_b}{2}$$

Solving for $q_2(c_a)$ and $q_2(c_b)$ in the first equation, we obtain

$$2q_1 = \theta \left(a - \frac{a - q_1 - c_a}{2} - c \right) + (1 - \theta) \left(a - \frac{a - q_1 - c_b}{2} - c \right)$$

- and from here we see that

$$q_1 = \frac{a - 2c + \theta c_a + (1-\theta)c_b}{3}$$

- Substituting this value we obtain that

$$\begin{aligned} q_1^* &= \frac{a - 2c + \theta c_a + (1-\theta)c_b}{3} \\ q_2^*(c_a) &= \frac{a - 2c_a + c}{3} + \frac{1-\theta}{6}(c_a - c_b) \\ q_2^*(c_b) &= \frac{a - 2c_b + c}{3} - \frac{\theta}{6}(c_a - c_b) \end{aligned}$$

Comparison with the Cournot equilibrium with complete information

- With complete information and cost functions c_1, c_2 Cournot's equilibrium is

$$\bar{q}_1 = \frac{a - 2c_1 + c_2}{3} \quad \bar{q}_2 = \frac{a - 2c_2 + c_1}{3}$$

- We observe that the case with complete information can be obtained from the incomplete information case by setting $c_2 = c_a = c_b$.

- Let us call

$$\bar{q}_1(c_a) = \frac{a - 2c_1 + c_a}{3}$$

  the production of Firm 1 if it knows that the cost function of Firm 2 is c_a.

- and

$$\bar{q}_1(c_b) = \frac{a - 2c_1 + c_b}{3}$$

  the production of Firm 1 if it knows that the cost function of Firm 2 es c_b.

- Then,

$$\bar{q}_1(c_a) \geq q_1^* \geq \bar{q}_1(c_b)$$

- Note that

$$\begin{aligned} q_2^*(c_a) &> \frac{a - 2c_a + c_1}{3} \\ q_2^*(c_b) &< \frac{a - 2c_b + c_1}{3} \end{aligned}$$

- This happens because Firm 2 adjusts its production not only to its own cost, but it also takes into account that Firm 1 doesn't know the cost function of Firm 2 and adopts a production in between those that would adopt if it knew that the cost of Firm 2 is either a or b.

3 Sealed bid auctions

- Two agents, $i = 1, 2$, participate in an auction. The agents bid simultaneously. The highest bidder wins: gets the object and pays his bid. In case of a tie, the winner is determined by a fair lottery.

- Private value auctions: Each bidder i knows only his valuation of the object, $v_i \in [0, 1]$, but the valuation v_j of the other bidder. He only knows the distribution function of v_j. Let us assume that the valuations of the bidders are independent and uniformly distributed on the interval $[0, 1]$. That is,

$$\text{prob}(v_j \leq x) = x$$

- Agents are risk neutral: If his valuation is v, wins the object and pays p, his payoff is $v - p$.

- We write this mechanism as a Baysian Game and find the equilibria.

  1. The types of the agents are $T_1 = T_2 = [0, 1]$.
  2. The probability of each type is described by $\text{prob}(v_j \le x) = x$. Theses are the conjectures of the agents.
  3. The sets of actions are $A_1 = A_2 = [0, 1]$. We denote de action of agent $i = 1, 2$ by $b_i \in A_i$.

Utility functions

- The utility function of agent $i = 1, 2$, given that

  – his valuation is v_i,
  – he bids b_i, and
  – the other agent bids b_j

  is

$$u_i(b_i, b_j|v_i) = \begin{cases} v_i - b_i, & \text{if } b_i > b_j; \\ 0, & \text{if } b_i < b_j; \\ (v_i - b_i)/2 & \text{if } b_i = b_j. \end{cases}$$

- A strategy for agent i is a function $b_i(v_i)$ from T_i into A_i.

- Given that $j = 1, 2$ uses the strategy $b_j(v_j)$, the best reply of agent i is the solution of the following maximization problem

$$\max_{b_i} \quad (v_i - b_i)\text{prob}(b_i > b_j(v_j)) + \frac{1}{2}(v_i - b_i)\text{prob}(b_i = b_j(v_j))$$

  Since, $\text{prob}(b_i = b_j(v_j)) = 0$ the above problem is equivalent to the following one

$$\max_{b_i}(v_i - b_i)\text{prob}(b_i > b_j(v_j))$$

- Ww show now that there is an equilibrium that is *symmetric and linear*. That is,

$$b_i(v) = b_j(v) = Av$$

- and we have to determine the value of A.
- The maximization problem becomes

$$\begin{aligned} \max_{b_i}(v_i - b_i)\text{prob}(b_i > b_j(v_j)) &= \max_{b_i}(v_i - b_i)\text{prob}(b_i > Av_j) = \\ = \max_{b_i}(v_i - b_i)\text{prob}\left(v_j < \frac{b_i}{A}\right) &= \max_{b_i}(v_i - b_i)\frac{b_i}{A} \end{aligned}$$

- The FOC is

$$v_i - 2b_i = 0$$

- And we see that

$$b_i = \frac{v_i}{2}$$

  is a symmetric BE.

4 Public Goods

- Two neighbors live in an island with no highways. The government plans to build one at the cost of 20 million. The **monetary value** of the road for a user is the following.
  - 30 million if the user has a car
  - 0, if the user does not have a car.
- the probability that a neighbor owns a car is

$$p(\text{car}) = p$$

- This information is public and known by the agents.
- To take a decision wether to build the road or not to build the road the government considers the two following possible procedures.

4.1 Mechanism A

Mechanism A

- Each neighbor fills up a questionnaire stating wether he has a car or not.
- It is understood that a neighbor that claims to have a car is willing to contribute up to 30 million towards the construction of the road.
- If the number of neighbors that claim to have a car is enough to cover the cost of the road, then the road tis build and the cost it is divided evenly among all those neighbors.
- If the number of neighbors that claim to have a car is NOT enough to cover the cost of the road, then the road is build.
- Let us describe this mechanism as a Bayesian game and compute the equilibria.
- The types of agents are $T_1 = T_2 = \{\text{car}, \text{no car}\}$. But it will be more convenient to use the value that the each type of user assigns to building the road $T_1 = T_2 = \{0, 30\}$.
- The probability of each type is $p(0) = 1 - p$, $p(30) = p$. These are the conjectures of the agents

$$p_i(t_j = 30|t_i) = p$$

- The set of actions of the agents is $A_1 = A_2 = \{\text{yes}, \text{no}\}$.
- Let us denote the action of agent $i = 1, 2$ by a_i. The strategy of agent $i = 1, 2$ is a function $T \to A_i$.

Utility functions

- The utility function of agent $i = 1, 2$ is (it is understood that $i \neq j$)

$$u_i(a_1, a_2|t_i) = \begin{cases} t_i - 20, & \text{if } a_i = \text{yes y } a_j = \text{no;} \\ t_i, & \text{if } a_i = \text{no y } a_j = \text{yes;} \\ t_i - 10, & \text{if } a_i = a_j = \text{yes;} \\ 0, & \text{if } a_i = a_j = \text{no.} \end{cases}$$

- Note that

$$u_i(a_1, a_2|t_i = 0) = \begin{cases} -20, & \text{if } a_i = \text{yes y } a_j = \text{no;} \\ 0, & \text{if } a_i = \text{no y } a_j = \text{yes;} \\ -10, & \text{if } a_i = a_j = \text{yes;} \\ 0, & \text{if } a_i = a_j = \text{no.} \end{cases}$$

That is, the action

$$a_i(0) = \text{no}$$

is a dominant strategy.

Complete information

- Suppose that $\boxed{t_1 = 30 \text{ y } t_2 = 0}$. The game is

	yes	no
yes	20, -10	10, 0
no	30, -20	0,0

The unique NE is (yes, no).

- Suppose that $\boxed{t_1 = t_2 = 30}$. The game is

	yes	no
yes	20, 20	10, 30
no	30, 10	0,0

- There are three NE
  - (yes, no), (no, yes) and
  - a NE in mixed strategies

$$\left(\frac{1}{2}\text{yes} + \frac{1}{2}\text{no}, \frac{1}{2}\text{yes} + \frac{1}{2}\text{no}\right)$$

Incomplete information

- Let us search for a **symmetric equilibrium in pure strategies**.
- **Symmetric equilibrium** means that $a_1(t) = a_2(t)$ for every $t \in T$.
- In equilibrium, $a_1(0) = a_2(0) = \text{no}$.
- $\boxed{\text{Is } a_1(30) = a_2(30) = \text{yes}}$ an equilibrium?
- Suppose that $t_i = 30$. (the agent i has a car)
- The expected payoff of this agent with the above strategy is

$$\begin{aligned} pu_i(\text{yes}, a_j(30)|30) &+ (1-p)u_i(\text{yes}, a_j(0)|30) = \\ pu_i(\text{yes}, \text{yes}|30) &+ (1-p)u_i(\text{yes}, \text{no}|30) = \\ p(30-10) &+ (1-p)(30-20) = 10p + 10 \end{aligned}$$

Symmetric equilibrium in pure strategies

- On the other hand, if he deviates and follows the strategy $b_i = \text{no}$, his expected payoff is

$$\begin{aligned} pu_i(\text{no}, a_i(30)|30) &+ (1-p)u_i(\text{no}, a_i(0)|30) = \\ pu_i(\text{no}, \text{yes}|30) &+ (1-p)u_i(\text{no}, \text{no}|30) = \\ p(30-0) &+ (1-p)\times 0 = 30p \end{aligned}$$

- Therefore, the strategy
$$a_1(30) = a_2(30) = \text{yes}$$
is a BE if $10 + 10p \geq 30p$, that is if $p \leq 1/2$.

- Summary: if $p \leq 1/2$ there is a symmetric BE in pure strategies
$$\begin{aligned} a_1(0) &= a_2(0) = \text{no} \\ a_1(30) &= a_2(30) = \text{yes} \end{aligned}$$

- This equilibrium is **efficient**: The road is build whenever $t_1 + t_2 > 20$.

Is there any other symmetric equilibrium in pure strategies?

- Is $a_i(30) = a_i(0) = \text{no}$ $i = 1, 2$ a BE?
- The expected payoff with this strategy is 0.
- Whereas if an agent deviates, his expected payoff is 10.
- Therefore, $a_i(30) = a_i(0) = \text{no}$ $i = 1, 2$ is not a BE.

Incomplete information. Symmetric equilibrium in mixed strategies

- Now we ask if there exists a symmetric BE in **mixed strategies** when $p > 1/2$.
- The symmetric equilibrium in mixed strategies must be of the form
$$a_1(30) = a_2(30) = w\,\text{yes} + (1-w)\,\text{no} =$$

- Suppose that $\boxed{\text{the agent } i \text{ is of type } t_i = 30}$. Recall that a necessary condition for the above strategy to be NE in mixed strategies is that the agent be indifferent among $a_i(30) = \text{yes}$ and $a_i(30) = \text{no}$. Let us express this condition.

- When the agent declares $a_i(30) = \text{yes}$ and the other agent $j \neq i$ follows the mixed strategy $a_j(30) = (w, 1-w)$ then the expected payoff for agent i is

$$\begin{aligned} p\Big[w\,u_i(\text{yes}, \text{yes}|30) &+ (1-w)\,u_i(\text{yes}, \text{no}|30)\Big] \\ &+ (1-p)u_i(\text{yes}, \text{no}|30) = \\ p\Big[w(30-10) &+ (1-w)(30-20)\Big] + (1-p)(30-20) = \\ &= 10 + 10pw \end{aligned}$$

- If the agent declares $a_i(30) = \text{no}$ and the other agent $j \neq i$ follows the mixed strategy $a_j(30) = (w, 1-w)$ then the expected payoff for agent i is

$$\begin{aligned} p\Big[w\, u_i(\text{no},\text{yes}|30) \;&+\; (1-w)\, u_i(\text{no},\text{no}|30)\Big] \\ &+\; (1-p)u_i(\text{no},\text{no}|30) = \\ p\Big[w \cdot 30 \;&+\; (1-w)\cdot 0\Big] + (1-p)\cdot 0 = \\ &= 30pw \end{aligned}$$

- Therefore, the strategy $a_1(30) = a_2(30) = w\,\text{yes} + (1-w)\,\text{no}$ is a BE if

$$10 + 10pw = 30pw$$

that is, if

$$w = \frac{1}{2p} \qquad \frac{1}{2} \leq p \leq 1$$

- Hence, we have found that

1. If $p \leq 1/2$ then

$$a_i(0) = \text{no}, \quad a_i(30) = \text{yes}$$

is a BE.

2. If $p > 1/2$ then

$$a_i(0) = \text{no}, \quad a_i(30) = \frac{1}{2p}\,\text{yes} + (1 - \frac{1}{2p})\,\text{no}$$

is a BE.

Efficiency

- What is the probability that it is efficient to build the road?
- There are four possible cases,

	$(30,30)$	$(30,0)$	$(0,30)$	$(0,0)$
Efficient	yes	yes	yes	no
prob	p^2	$p(1-p)$	$p(1-p)$	$(1-p)^2$

- The probability that building the road is efficient is

$$p^2 + 2p(1-p) = 2p - p^2$$

- What is the probability that the road is built if mechanism A is adopted?
- if $p \leq 1/2$,

	$(30,30)$	$(30,0)$	$(0,30)$	$(0,0)$
prob	p^2	$p(1-p)$	$p(1-p)$	$(1-p)^2$
prob. of building	1	1	1	0

- The probability of building the road is

$$p^2 + 2p(1-p) = 2p - p^2 \leq \frac{1}{2}$$

- The probability of building the road coincides with the probability that it is efficient to build it.

- But, if $p > 1/2$,

	$(30,30)$	$(30,0)$	$(0,30)$	$(0,0)$
prob	p^2	$p(1-p)$	$p(1-p)$	$(1-p)^2$
prob. of building	$1-(1-w)^2 = 2w - w^2$	w	w	0

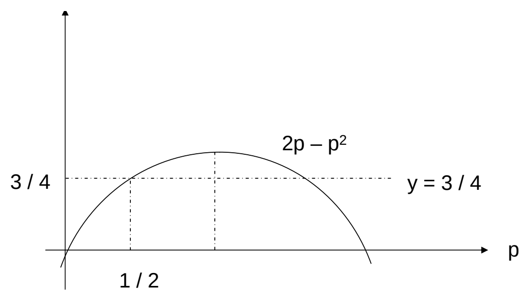
- The probability of building the road is

$$p^2(2w - w^2) + 2p(1-p)w = \frac{3}{4}$$

- The mechanism is inefficient with probability

$$p^2(1-w)^2 + 2p(1-p)(1-w) = 2p - p^2 - \frac{3}{4}$$

- The graph of the function $2p - p^2$ is



- We see that if $1/2 < p \leq 1$, then

$$2p - p^2 > \frac{3}{4}$$

- The probability of building the road is lower than the probability that it is efficient to build it.

4.2 Mechanism B

Mechanism B

- We order the agents $i = 1, 2$. agent 1 indicates how much he is willing to pay in order to build the road: $\xi_1 \in [0, 20]$.
- Agent 2 knowing ξ_1 informs of his decision
  1. **'yes'**: then the road is build and agents pay the amounts ξ_1 and $\xi_2 = 20 - \xi_1$.
  2. **'no'**: then the road is not build and agents pay the amounts $\xi_1 = 0$ and $\xi_2 = 0$.
- Let us write this mechanism as Bayesian game and compute the equilibria.
- the types of the agents are $T = T_1 = T_2 = \{\text{car}, \text{no car}\}$. As a type we continue to use the value that each assigns to building the road $T = T_1 = T_2 = \{0, 30\}$.
- the probability of each type is $p(0) = 1 - p$, $p(30) = p$. These are the conjectures of the agents

$$p_i(t_j = 30) = p$$

- the sets of *actions of the agents are* $A_1 = [0, 20]$, $A_2 = \{yes, no\}$.
- We denote the action of agent $i = 1, 2$ by a_i. The strategy of agent 1 is a function $a_1 : T_1 \to A_1$. The strategy of agent 2 is a function $a_2 : T_2 \times A_1 \to A_2$.

Utility functions

- the utility function of agent 1 is

$$u_1(a_1, a_2, t_1) = \begin{cases} t_1 - a_1, & \text{if } a_2 = \text{yes;} \\ 0, & \text{if } a_2 = \text{no.} \end{cases}$$

- the utility function of agent 2 is

$$u_2(a_1, a_2|t_1) = \begin{cases} t_2 - (20 - a_1), & \text{if } a_2 = \text{yes;} \\ 0, & \text{if } a_2 = \text{no.} \end{cases}$$

Second stage

- Let us compute the BE. Since, it is a sequential game, we solve it by backwards induction. First, we analyze the actions of *player 2*.
- if player 1 chooses the action a_1, the payoffs of player 2 are
  1. $u_2 = t_2 - (20 - a_1) = t_2 - 20 + a_1$, when $a_2 = \text{yes}$.
  2. $u_2 = 0$, when $a_2 = \text{no}$.
- the best reply of player 1 is

$$a_2(30, a_1) = \text{yes}, \qquad a_2(0, a_1) = \begin{cases} \text{no} & \text{if } a_1 < 20; \\ \text{yes}, & \text{if } a_1 = 20. \end{cases}$$

First stage

- player 1 anticipates the reaction of player 2.
- if the type of player 1 is $\boxed{t_1 = 0}$, his best strategy is to choose $a_1(0) = 0$.
- Suppose now that the type of player 2 is $\boxed{t_1 = 30}$ and let us find his optimal strategy.
- if his message is $a_1 = 20$, then his expected utility is 10.
- if his message is $a_1 < 20$, then his expected utility is

$$p(30 - a_1) + (1 - p) \cdot 0 = 30p - pa_1$$

- that is, among all the messages $a_1 < 20$ his optimal strategy is to choose $a_1 = 0$. And his expected utility is $30p$.

- Player 1 decides wether his message is $a_1 = 20$ or $a_1 = 0$.
- he chooses $a_1 = 0$ if $30p > 10$. or if

$$p > \frac{1}{3}$$

- Hence,

$$a_1(0) = 0 \qquad a_1(30) = \begin{cases} 0, & \text{if } p > 1/3; \\ 20, & \text{if } p < 1/3; \\ [0, 20], & \text{if } p = 1/3. \end{cases}$$

Equilibrium

- In brief, the BE is

$$a_1(0) = 0 \qquad a_1(30) = \begin{cases} 0, & \text{if } p > 1/3; \\ 20, & \text{if } p < 1/3; \\ [0,20], & \text{if } p = 1/3. \end{cases}$$

$$a_2(30, a_1) = \text{yes}, \qquad a_2(0, a_1) = \begin{cases} \text{no} & \text{if } a_1 < 20; \\ \text{yes}, & \text{if } a_1 = 20. \end{cases}$$

Efficiency

- What is the probability that the road is build if mechanism B is the one adopted?

	$(30,30)$	$(30,0)$	$(0,30)$	$(0,0)$
Efficient	yes	yes	yes	no
prob	p^2	$p(1-p)$	$p(1-p)$	$(1-p)^2$
road is build	yes	yes when $p < 1/3$ no when $p > 1/3$	yes	no

- The probability that building the road is efficient is $p^2 + 2p(1-p) = 2p - p^2$.
- The probability of building the road is

$$\begin{aligned} p^2 + 2p(1-p) = 2p - p^2 & \quad \text{if } p < 1/3 \\ p^2 + p(1-p) = p & \quad \text{if } p > 1/3 \end{aligned}$$

- If $p > 1/3$, the mechanism is inefficient with probability $p(1-p) = p - p^2$.
- Which of the two mechanisms is more efficient?
- Mechanism A is more inefficient than B if

$$2p - p^2 - \frac{3}{4} > p - p^2$$

that is, if

$$p > \frac{3}{4}$$

5 More information could be worse

- Consider the following game with incomplete information,

	L	M	R
T	**1,1/2**	**1,0**	**1,3/4**
B	**2,2**	**0,0**	**0,3**

1 **2** w_1 1/2

	L	M	R
T	**1,1/2**	**1,3/4**	**1,0**
B	**2,2**	**0,3**	**0,0**

w_2 1/2

- The agents believe that with probability 1/2 they play the game w_1 and with probability 1/2 they play the game w_2.
- The unique NE is (B, L). Each player gets an expected payoff of 2.
- Suppse now that player 2 knows if the true game played is w_1 or w_2. In this case,the unique NE is (T, R) ó (T, M). The payoff of player 2 is 3/4. He would have preferred not to be informed.
- But, can player 2 credibly commit to ignore the information he has received?