Chapter 3
Applications

October 23, 2019

1 Credibility and Commitment

Example 1.1.

- Two firms: Firm 1 and Firm 2;
- Two prices: L ($4) or H ($5);
- 3000 captive consumers per firm;
- 4000 floating consumers who go to the firm with the lowest price.

The payoffs are as follows

		Firm 2 L	Firm 2 H
Firm 1	L	$20, 20$	$28, 15$
	H	$15, 28$	$25, 25$

Non Commitment

The situation may be described as a prisoner's dilemma. Both firms prefer (H, H) but the unique NE is (L, L). Firms cannot credibly commit to play H. If one firm commits to H, the other firm should respond with L. How to resolve this?

One possibility is a **low price guarantee**. Each firm announces that it will match the lowest price in the market. Now the situation is as follows

		Firm 2 L	Firm 2 H
Firm 1	L	$20, 20$	$20, 20$
	H	$20, 20$	$25, 25$

Non Commitment

Now (H, H) is a NE with payoff $u_1 = u_2 = 25$.

Example 1.2. Suppose a market in which there is a monopolist M. The total number of customers in the market is $X = 2a + b = 180$ with $a = 50$ and $b = 80$. The monopolist sells a product at the price $p_h = 20$. The cost of production per unit is $c = 10$. Thus, the present profit of the monopolist is $\pi_M = (p_h - c)X = 1800$.

Suppose now that there is a potential entrant E in the market. The cost of entry is $c_E = 600$. If E enters the market, then both firms decide simultaneously between two prices $p_l = 15$ and $p_h = 20$. Once the firms have chosen the prices, the market divides as follows: Each firm captures a (unconditional) customers regardless of the prices. The remaining b customers buy the product from the firm that charges the lowest price. That is, they play a simultaneous game with the following payoffs.

E

		p_l	p_h
M	p_l	$(a+b/2)(p_l-c),(a+b/2)(p_l-c)-c_E$	$(a+b)(p_l-c),a(p_h-c)-c_E$
	p_h	$a(p_h-c),(a+b)(p_l-c)-c_E$	$(a+b/2)(p_h-c),(a+b/2)(p_h-c)-c_E$

$c_1 = c_2 = c$

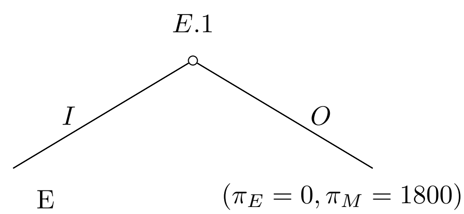
Plugging in the values $a = 50$, $b = 80$, $p_l = 15$, $p_h = 20$, $c = 10$ and $c_E = 600$, we obtain the following simultaneous game

E

		p_l	p_h
M	p_l	$450, -150$	$650, -100$
	p_h	$500, 50$	$900, 300$

$c_1 = c_2 = c = 10$

This game has a unique NE: (p_h, p_h) with profits $\pi_E = 300$, $\pi_M = 900$. Now we consider the situation in which E is considering wether to enter or not the market. This may be described by the following dynamic game.



E

		p_l	p_h
M	p_l	$450, -150$	$650, -100$
	p_h	$500, 50$	$900, 300$

$c_1 = c_2 = c = 10$

Player E at the node $E.1$ anticipates that if he chooses I they will play the NE (p_h, p_h) with profits $\pi_E = 300$, $\pi_M = 900$. Hence, in the SPNE player E chooses I at node $E.1$. The SPNE is $((I, p_h), p_h)$ with payoffs $\pi_E = 300$, $\pi_M = 900$.

Imagine now that M has the possibility of investing in a new technology that would reduce its unitary costs to $c_l = 3$. The cost of adopting this technology is $c_t = 1300$. If firm 1 adopts this technology and E enters, the game faced by M and E is the following.

E

		p_l	p_h
M	p_l	$(a+b/2)(p_l-c_l)-c_t,(a+b/2)(p_l-c)-c_E$	$(a+b)(p_l-c_l)-c_t a(p_h-c)-c_E$
	p_h	$a(p_h-c_l)-n-c_t,(a+b)(p_l-c)-c_E$	$(a+b/2)(p_h-c_l)-c_t,(a+b/2)(p_h-c)-c_E$

$c_1 = 3, c_2 = 10$

Plugging in the values $a = 50$, $b = 80$, $p_l = 15$, $p_h = 20$, $c = 10$ and $c_E = 600$, we obtain the following simultaneous game

E

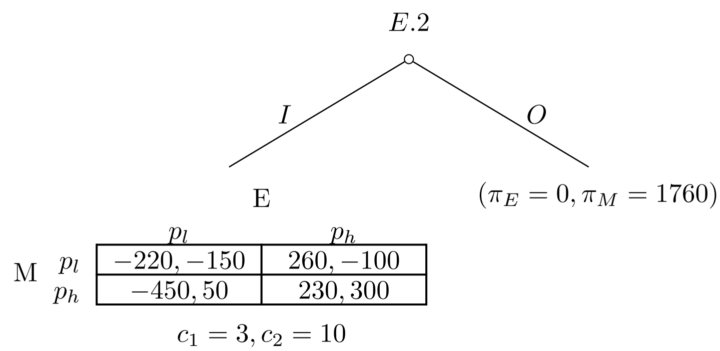
		p_l	p_h
M	p_l	$-220, -150$	$260, -100$
	p_h	$-450, 50$	$230, 300$

$c_1 = 3, c_2 = 10$

Now, the unique NE is (p_l, p_h) with payoffs $\pi_E = -100$, $\pi_M = 260$.

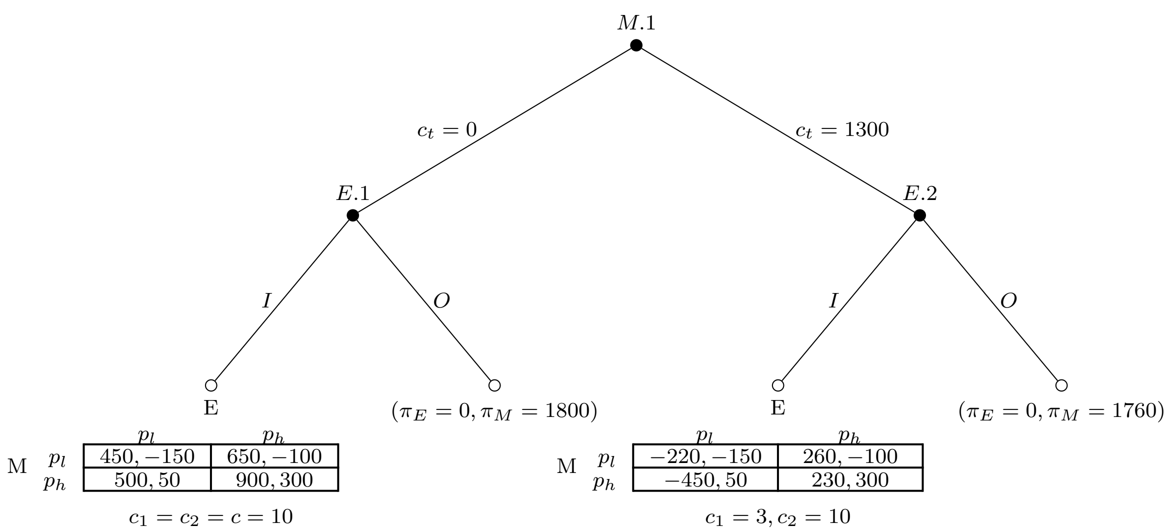
Note that if M adopts the new technology and E does not enter, then M operates alone in the market, charges price $p_h = 20$ and its profits are $\pi_M = (2a+b)(p_h - c_l) - c_t = 1760$. So, if it were alone M would not adopt the new technology because its profit before adopting the technology was $\pi_M = (2a+b)(p_h - c) = 1800$.

The game tree that describes the new situation after M has adopted the new technology is now the following.



In this game, player E at the node $E.2$ anticipates that if he chooses I they will play the NE (p_l, p_h) with profits $\pi_E = -100$, $\pi_M = 260$. Hence, in the SPNE player E chooses 0 at node $E.2$. The SPNE is $((0, p_h), p_l)$ with payoffs $\pi_E = 0$, $\pi_M = 1760$.

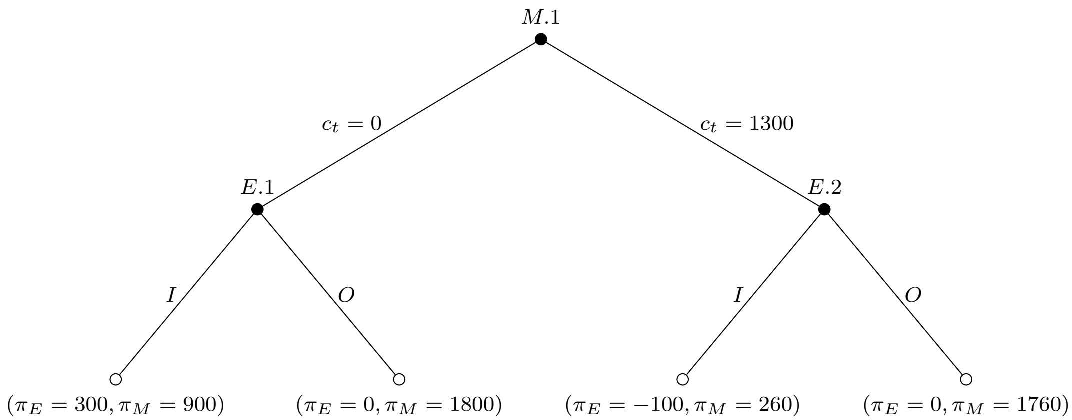
Now, let us consider the situation of M when it is deciding wether to adopt the new technology or not. This may be described by the following extensive form game.



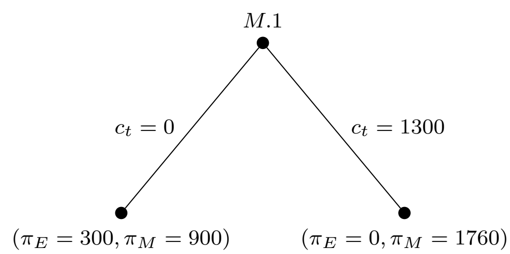
Let us compute the SPNE. We have already seen that

- in the subgame that starts after $E.1$ has chosen I there is a unique NE: $((I, p_h), p_h)$ with payoffs $\pi_E = 300$, $\pi_M = 900$.
- in the subgame that starts after $E.2$ has chosen I there is a unique NE: $((I, p_l), p_h)$ with payoffs $\pi_E = -100$, $\pi_M = 260$.

We substitute these subgames by their payoffs in the original game and we obtain the game



Hence, at node $E.1$, player E chooses I and at node $E.2$, player E chooses O. We substitute the payoffs of these actions into the original game and we obtain.



And M chooses $c_t = 1300$ at node $M.1$. The SPNE is

$$((c_t = 1300, p_h, p_l), (I, p_h; O, p_h))$$

with payoffs $(\pi_E = 0, \pi_M = 1760)$. By adopting the new technology M deters E from entering the market.

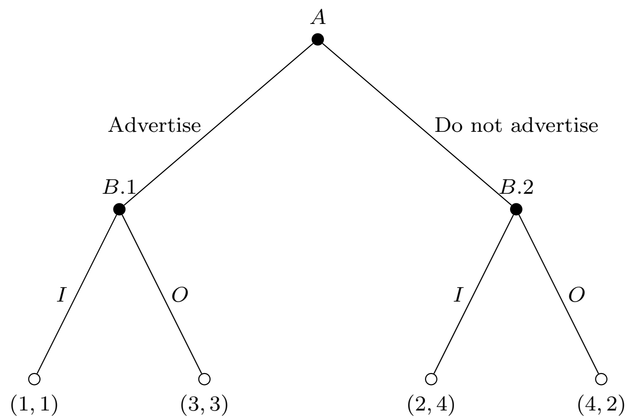
2 First mover advantage

Example 2.1. Consider a Senate race game. Suppose a particular Senate seat is currently occupied by A (the incumbent) It is known that the only likely challenger for A's seat is Congresswoman B. Politician A has to decide whether to launch an advertising campaign for her seat. Politician B has to decide whether to enter the race or not.

The utilities of the players are

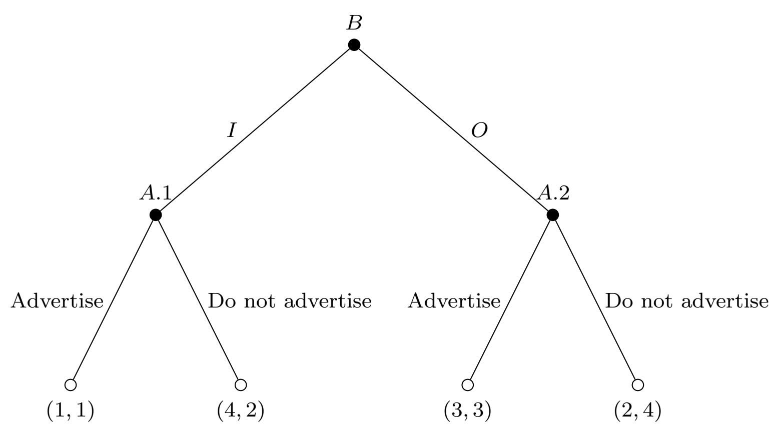
- A advertises and B enters: $u_A = 1$, $u_B = 1$.
- A advertises and B stays out: $u_A = 3$, $u_B = 3$.
- A does not advertise and B enters: $u_A = 2$, $u_B = 4$.
- A does not advertise and B stays out: $u_A = 4$, $u_B = 2$.

Suppose first that politician A decides first whether to advertise and then politician B decides whether to enter the race or not. The following game describes the situation.



The SPNE is (Advertise, OI) with payoffs $u_A = 3$, $u_B = 3$.

Let us now consider the situation in which B decides first whether to enter the race and then A decides whether to advertise. The game that describes this is the following.



The SPNE is (I, (Do not advertise, Do not advertise)) with payoffs $u_A = 2$, $u_B = 4$. The player who moves first is better off.

3 Contracts

Read Section 9.4.2 from Harrington's book.

4 Deterring a potential competitor

A firm A can spend \$200 million on an investment to enter a new market. The value of the market today (period $t = 1$) is \$50 million. The future (period $t = 2$) value of the market is uncertain. It may

- grow to \$500 million with probability q.
- remain the same \$50 million with probability $1 - q$.

The firm has two options.

1. Invest today under uncertainty.
2. Wait until tomorrow and then invest only if market has grown.

What should it do? if it invests today, the expected profit is

$$\mathrm{E}_A(I) = -200 + 50 + 500q + 50(1-q) = 450q - 100$$

If it waits until it knows wether the market has grown, the expected profit is

$$\mathrm{E}_A(W) = 0 + (500 - 200)q = 300q$$

The **option value** of waiting is

$$\mathrm{E}_A(W) - \mathrm{E}_A\, v(I) = 300q - (450q - 100) = 100 - 150q$$

Thus, A should wait iff $q \leq \frac{2}{3}$.

Suppose now that there is a potential competitor B in the market. Firm B is behind firm A in developing the product. In particular, firm B is not yet ready to enter the market and may enter only in the second period (after A knows if the market has grown or not). But then, B will benefit from some of A's innovations, so the cost of entry for firm B is \$100 million.

Now there is a countervailing force. By waiting, the firm risks facing competition by competitors and a worsening off its position in the market. Does this change the value of waiting?

4.1 Option Value with an informed competitor.

Suppose that in the second period B knows if the market has grown or not. Therefore, B will enter only if the value of the market has grown. In that case the expected profits for A are the following

$$\begin{aligned} \mathrm{E}_A(I) &= -200 + 50 + 250q + 50(1-q) = 200q - 100 \\ \mathrm{E}_A(W) &= 0 + (250 - 200)q = 50q \end{aligned}$$

Thus, the **option value** of waiting is

$$\mathrm{E}_A(W) - \mathrm{E}_A(I) = 50q - (200q - 100) = 100 - 150q$$

the same as before. That is A should wait iff $q \leq \frac{2}{3}$.

4.2 Option Value with an uninformed competitor.

Suppose that in the second period B does not know if the market has grown or not. Therefore, when B has to decide wether to enter or not:

- it knows if A entered in period $t = 1$.
- it only knows that in period $t = 2$ the value is \$500 million with probability q and \$50 million with probability $1 - q$.
- It is common knowledge that A knows if the market has grown.

Let us consider B's decision wether to enter in period $t = 2$ or not. Suppose first that A has entered at $t = 0$. In this case, the expected profits of B are

$$\begin{aligned} \mathrm{E}_B(E) &= -100 + 250q + 25(1-q) = 225q - 75 \\ \mathrm{E}_B(NE) &= 0 \end{aligned}$$

Thus B should enter iff $q \geq \frac{1}{3}$, if it knows that A entered at $t = 0$.

Suppose now that A has not entered at $t = 0$. In this case, the expected profits of B are

$$\begin{aligned} \mathrm{E}_B(E) &= -100 + 250q + 50(1-q) = 200q - 50 \\ \mathrm{E}_B(NE) &= 0 \end{aligned}$$

Thus, if B knows that A did not enter at $t = 0$, it should enter iff $q \geq \frac{1}{4}$.

Let us assume now that

$$\frac{1}{4} < q < \frac{1}{3}$$

For this value of q, B's decision is

- Do not enter if A entered at $t = 0$.
- Enter if A did not enter at $t = 0$.

Firm A knows the above. Hence, the expected profits of A are now the following.

$$\begin{aligned} \mathrm{E}_A(I) &= -200 + 50 + 500q + 50(1-q) = 450q - 100 \\ \mathrm{E}_A(W) &= 0 + (250 - 200)q = 50q \end{aligned}$$

And the **option value** of waiting is

$$\mathrm{E}_A(W) - \mathrm{E}_A(I) = 50q - (450q - 100) = 100 - 400q$$

which es negative because $q > \frac{1}{4}$. Hence, we conclude that A enters now the market to deter B from entering in period $t = 2$.

The bottom line is that if $\frac{1}{4} < q < \frac{1}{3}$ and

- with no potential competitor, A would enter the market in period $t = 2$ only if the market has grown.
- in the presence of a potential competitor, A would enter the market in period $t = 1$ to deter the competitor from entering in $t = 2$