Name (Print): ______________________________

University Carlos III de Madrid

Game Theory
Master in Economic and Master in Industrial Economics and Markets

Quiz 2 – 12/02/2022.

Time Limit: 40 Minutes.

Question:	1	2	3	Total
Points:	10	50	100	160
Score:				

Instructions : Please, check that this exam contains 2 pages and 3 problems.

Consider the following normal form game.

		Player 2		
		A	B	C
Player 1	A	$5,5$	$1,12$	$0,4$
	B	$8,1$	$7,6$	$0,5$
	C	$3,0$	$0,0$	$1,1$

The stage game G

1. (10 points) Find the Nash equilibria in pure strategies.

   Solution: There are two NE in pure strategies: (B, B) with payofss $u_1 = 7$, $u_2 = 6$ and (C, C) with payofss $u_1 = u_2 = 1$.

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2. (50 points) Assume that the above stage game is played two times. After the first round, players observed the moves done by the other player. The total payoffs of the repeated game are the sum of the payoffs obtained in each round. Is there a subgame perfect Nash equilibrium in pure strategies in which (A, A) is played in the first round?

   Solution: Let us consider the following strategy profile S: player $i = 1, 2$ at

   - $t = 1$ play (A, A);
   - $t = 2$ play (B, B) if (A, A) was played at $t = 1$ and play (C, C) otherwise.

   Is S a SPNE of the repeated game? At $t = 2$ the strategy profile S suggests the players to play a NE. Hence, no player has an incentive to deviate at $t = 2$. Anticipating the payoffs at $t = 2$ the players anticipate the following payoffs at $t = 1$.

		Player 2		
		A	B	C
Player 1	A	$12,11$	$2,13$	$1,5$
	B	$9,2$	$8,7$	$1,6$
	C	$4,1$	$1,1$	$2,2$

   and we see that playing A is not a best reply for player 2, when player 2 chooses 1. Hence playing (A, A) at $t = 1$ is not part of a SPNE of the repeated game.

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3. (100 points) Assume that the above stage game is played three times. After the first and second round, players observed the moves done by the other player. The total payoffs of the repeated game are the sum of the payoffs obtained in each round. Is there a subgame perfect Nash equilibrium in pure strategies in which (A, A) is played in the first round?

Solution: Let us consider the following strategy profile S: player $i = 1, 2$ at

- $t = 1$ play (A, A);
- $t = 2$ play (B, B) if (A, A) was played at $t = 1$. Otherwise, play (C, C);
- $t = 3$, play (B, B) if (A, A) was played at $t = 1$ and (B, B) was played at $t = 2$. Otherwise, play (C, C).

Let us check that the above strategy profile S is a SPNE. In period $t = 3$, it dictates players to play a NE of the stage game. Thus S induces a NE at all subgames that start at $t = 3$.

Let us consider now a subgame that starts at $t = 2$ after (A, A) was played in period $t = 1$. Ignoring the payoffs at $t = 1$ and anticipating the strategy S at $t = 3$, yields the following future payoffs at $t = 2$ plus the payoffs at $t = 3$.

Player 2

Player 1	A	B	C
A	$6, 6$	$2, 13$	$1, 5$
B	$9, 2$	$14, 12$	$1, 6$
C	$4, 1$	$1, 1$	$2, 2$

and (B, B) is a NE of this subgame.

Let us consider now a subgame that starts at $t = 2$ after (A, A) was not played in period $t = 1$. Ignoring the payoffs at $t = 1$ and anticipating the strategy S at $t = 3$, yields the following future payoffs at $t = 2$ plus the payoffs at $t = 3$.

Player 2

Player 1	A	B	C
A	$6, 6$	$2, 13$	$1, 5$
B	$9, 2$	$8, 7$	$1, 6$
C	$4, 1$	$1, 1$	$2, 2$

and (C, C) is a NE of this subgame.

Finally, let us check that it is also a NE of the subgame that starts at $t = 1$. Anticipating the strategy S at $t = 2, 3$ yields the following payoffs at $t = 1$.

Player 2

Player 1	A	B	C
A	$19, 17$	$3, 14$	$2, 6$
B	$10, 3$	$9, 8$	$2, 7$
C	$5, 2$	$2, 2$	$3, 3$

And (A, A) is a NE of this subgame. Hence, no player has incentives to deviate and S is a SPNE.

Remark: The following two strategies are also possible.

- **Strategy S_1:**
  - $t = 1$ play (A, A);
  - $t = 2$ play (B, B) if (A, A) was played at $t = 1$. Otherwise, play (C, C);
  - $t = 3$, play (B, B) if (A, A) was played at $t = 1$. Otherwise, play (C, C).

- **Strategy S_2:**
  - $t = 1$ play (A, A);
  - $t = 2$ play (B, B) if (A, A) was played at $t = 1$. Otherwise, play (C, C);
  - $t = 3$, play (B, B) if (B, B) was played at $t = 2$. Otherwise, play (C, C).