UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

TEST 2–December 14th, 2018

NAME:

Two graduate students share an apartment. They have to spend some time cleaning the apartment daily. If each of them spends x_1, x_2 hours cleaning, their utilities are

$$\begin{aligned} u_1(x_1, x_2) &= (18 + x_2)x_1 - 2x_1^2 \\ u_2(x_1, x_2) &= (18 + x_1)x_2 - 2x_2^2 \end{aligned}$$

Note that the time spent in cleaning increases the utility of both tenants. That is, cleaning has a positive externality. On the other hand, the time spent cleaning imposes a personal cost.

(a) If both students decide independently the time spent cleaning, how much time will they devote to cleaning? What are their utilities?

Solution: *Agent 1 maximizes* $\max_{x_1} u_1 = (18 + x_2)x_1 - 2x_1^2$. *The first order condition is*

$$\frac{\partial u_1}{\partial x_1} = 18 + x_2 - 4x_1 = 0$$

Note that the second derivative with respect to x_1 is

$$\frac{\partial^2 u_1}{\partial x_1^2} = -4$$

Hence, the first order condition corresponds to a maximum of u_1. The best reply of agent 1 is

$$\mathrm{BR}_1(x_2) = \frac{18 + x_2}{4}$$

Likewise, agent 2 maximizes $\max_{x_2}(18 + x_1)x_2 - 2x_2^2$. *The best reply of agent 2 is*

$$\mathrm{BR}_2(x_1) = \frac{18 + x_1}{4}$$

The NE is the solution to

$$q_1 = \frac{18 + x_2}{4}, \quad q_2 = \frac{18 + x_1}{4}$$

The NE is $x_1^* = x_2^* = 6$. The utilities of the agents are $u_1^* = u_2^* = 72$.

(b) Suppose now they could make a joint agreement on how much time each should spend on cleaning. Which is the amount of time each should spend on cleaning that maximizes their joint welfare?

Solution: *Now, the agents maximize*

$$\max_{x_1, x_2}(18 + x_2)x_1 - 2x_1^2 + (18 + x_1)x_2 - 2x_2^2$$

The solution is $\bar{x}_1 = \bar{x}_2 = 9$. The utilities of the agents are $\bar{u}_1^* = \bar{u}_2^* = 81$.

(c) Would they follow the agreement reached in part (b) or would they have incentives to deviate?

Solution: *The agreement in part (b) is not a NE. For example,*

$$\mathrm{BR}_1(9) = \frac{27}{4} = 6.75 \neq \bar{x}_1$$

with utility

$$u_1\left(\frac{27}{4}, 9\right) = \frac{729}{8} = 91.125$$

Thus, the agents have incentives to deviate.