UNIVERSITY CARLOS III
Master in Industrial Economics and Markets
Game Theory

TEST 2 –November 2017- SOLUTION

NAME:

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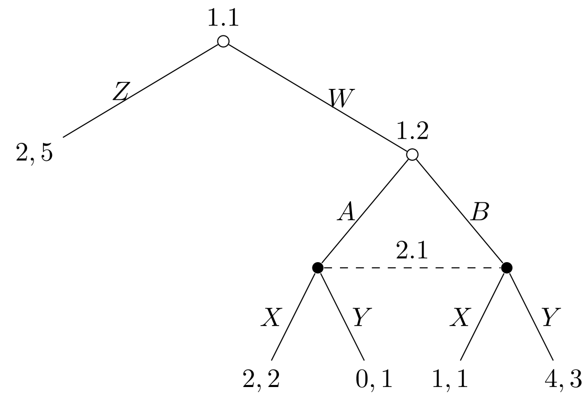
(1) *Consider the following game in extensive form*



(a) *Write the normal form of the sub-game that starts at at node* 1.2. *Find all the Nash equilibria of this sub-game, in pure as well as in mixed strategies. Compute the payoffs of the players* **(4 points)**

Solution: The normal form of the sub-game that starts at at node 1.1 is,

	X	Y
A	$\underline{2},\underline{2}$	$0,1$
B	$1,1$	$\underline{4},\underline{3}$

There are two NE in pure strategies: (A, X) with payoffs $(2, 2)$ and (B, Y) with payoffs $(4, 3)$. Now we look for a mixed strategy equilibrium of the form

$$\begin{aligned} \sigma_1 &= pA + (1-p)B \\ \sigma_2 &= qX + (1-q)Y \end{aligned}$$

The expected utilities of player 1 are

$$\begin{aligned} u_1(A, \sigma_2) &= 2q \\ u_1(B, \sigma_2) &= q + 4(1-q) = 4 - 3q \end{aligned}$$

So, player 1 is indifferent between the strategies X and Y if and only if $2q = 4 - 3q$, that is if and only if $q = \frac{4}{5}$. On the other hand, the expected utilities of player 2 are

$$\begin{aligned} u_2(\sigma_1, X) &= 2p + 1 - p = 1 + p \\ u_2(\sigma_1, Y) &= p + 3(1-q) = 3 - 2p \end{aligned}$$

So, player 2 is indifferent between the strategies A and B if and only if $1 + p = 3 - 2p$, that is if and only if $p = \frac{2}{3}$. We conclude that there is a mixed strategy NE of the form

$$\sigma = \left(\frac{2}{3}A + \frac{1}{3}B, \frac{4}{5}X + \frac{1}{5}Y\right)$$

with payoffs $u_1(\sigma) = \frac{8}{5}$ and $u_2(\sigma) = \frac{5}{3}$.

(b) *Find all the sub-game perfect Nash equilibria of the complete game.* **(6 points)**

Solution: We use the notation

$$((1.1, 1.2), 2.1)$$

to the denote the strategies followed by the players. For each of the NE of the sub-game that starts in node 1.2 there is a corresponding SPNE. In particular, we obtain the following two NE,

$$((W, B), Y) \quad \text{with payoffs} \quad u_1 = 4, u_2 = 3$$

and

$$\left(\left(Z, \frac{2}{3}A + \frac{1}{3}B\right), \frac{4}{5}X + \frac{1}{5}Y\right) \quad \text{with payoffs} \quad u_1 = 2, u_2 = 5$$

Note that for player 1 the payoff of the NE (A, X) is 2 which coincides with the payoff for this player when he chooses W. In other words, if player 1 anticipates the NE (A, X) in the sub-game that starts in node 1.2, then this player is indifferent between the strategies Z and W. Thus, there is continuum of SPNE of the form

$$((pZ + (1-p)W, A), X), 0 \leq p \leq 1, \quad \text{with payoffs} \quad u_1 = 2, u_2 = 5p + 2(1-p) = 2 + 3p.$$