UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

Quiz 1–October 7th, 2022

NAME:

PLEASE, CHOOSE ONE (AND ONLY ONE) OF THE FOLLOWING EXERCISES

Exercise 1: *Assume that two firms, 1 and 2, produce heterogeneous products and the quantities demanded by the market, when these firms fix prices p_1 and p_2, are, respectively:*

$$\begin{aligned} x_1(p_1, p_2) &= 60 - p_1 + \frac{p_2}{2} \\ x_2(p_1, p_2) &= 60 - p_2 + \frac{p_1}{2} \end{aligned}$$

These demand functions describe a situation in which products are not perfectly homogenous. Suppose that both firms have constant marginal costs MC= 30 and MC2 = 30 and that the features of the market are such that both firms have to set prices simultaneously.

(a) *Describe the situation as a game. Write the profit functions, $\pi_1(p_1, p_2)$ and $\pi_2(p_1, p_2)$, of the firms.*

Solution: There are two players, $N = \{1, 2\}$. The strategy sets are $S_1 = S_2 = [0, \infty)$. The payoffs of the firms are the profits.

$$\begin{aligned} \pi_1(p_1, p_2) &= \left(60 - p_1 + \frac{p_2}{2}\right)(p_1 - 30) = \frac{p_1 p_2}{2} - p_1^2 + 90p_1 - 15p_2 - 1800 \\ \pi_2(p_1, p_2) &= \left(60 - p_2 + \frac{p_1}{2}\right)(p_2 - 30) = \frac{p_1 p_2}{2} - 15p_1 - p_2^2 + 90p_2 - 1800 \end{aligned}$$

(b) *Compute the best reply of each firm.*

Solution: Firm 1's best response to a price set by firm 2 is

$$\mathrm{BR}_1(p_2) = \frac{p_2 + 180}{4}$$

Fiirm 2's best response to a price set by firm 1 is

$$\mathrm{BR}_2(p_1) = \frac{p_1 + 180}{4}$$

(c) *Find the Nash equilibrium (p_1^*, p_2^*). Determine equilibrium profits, $\pi_1(p_1,^* p_2^*)$ and $\pi_2(p_1^*, p_2^*)$.*

Solution: The Nash equilibrium (p_1^*, p_2^*) is the solution of the following system of equations

$$\begin{aligned} p_1 &= \frac{p_2 + 180}{4} \\ p_2 &= \frac{p_1 + 180}{4} \end{aligned}$$

The solution is

$$p_1^* = p_2^* = 60$$

The equilibrium profits are

$$\pi_1(p_1^*, p_2^*) = \pi_2(p_1^*, p_2^*) = 900$$

(d) *Suppose now that firm 1 considers the possibility of acquiring firm 2. In such case firm 1 can choose p_1 and p_2 so that it maximimes $\pi_1(p_1,p_2)+\pi_2(p_1,p_2)$.*

- *What is the maximum price firm 1 is willing to pay to acquire firm 2?*

  Solution: The firm maximizes

$$\pi(p_1,p_2)=\pi_1(p_1,p_2)+\pi_2(p_1,p_2)=-p_1^2+p_1p_2+75p_1-p_2^2+75p_2-3600$$

  with respect p_1,p_2. The first order conditions are

$$\begin{aligned} -2p_1+p_2+75 &= 0 \\ p_1-2p_2+75 &= 0 \end{aligned}$$

  The solution is $\bar{p}_1=\bar{p}_2=75$. And the new profit is $\bar{\pi}=2025$. Thus, the maximum prize firm 1 is willing to pay for the acquisition of firm 2 is $2025-900=1125$.

- *What is the minimum price firm 2 is willing to accept for the acquisition?*

  Solution: The minimum prize firm 2 is willing to accept is 900.

- *Is it possible that the purchase takes place?*

  Solution: Any price between 900 and 1125 will be acceptable for both firms.

Exercise 2: *Consider the following normal form game:*

	X	*Y*	*Z*	*U*
A	*1,0*	*0,2*	*2, 10*	*1,5*
B	*0,10*	*-1, 7*	*1, 4*	*10,1*
C	*2, 15*	*1, 10*	*1 , 5*	*1 , 2*

(a) *What are the strategies that survive the iterated elimination of strictly dominated strategies?*

Solution: Strategy U is dominated by strategy Z for player 2. After eliminating this strategy we obtain the following game

	X	*Y*	*Z*
A	1,0	0,2	2, 10
B	0,10	-1, 7	1, 4
C	2, 15	1, 10	1 , 5

Now strategy B is dominated by strategy A for player 1. After eliminating this strategy we obtain the following game

	X	*Y*	*Z*
A	1,0	0,2	2, 10
C	2, 15	1, 10	1 , 5

Note also that the mixed strategy $\sigma_2=\frac{2}{3}X+\frac{1}{3}Z$ dominates Y, because

$$\begin{aligned} u_2(A,\sigma_2) &= \frac{2}{3}\times 0+\frac{1}{3}\times 10=\frac{10}{3}>2=u_2(A,Y) \\ u_2(C,\sigma_2) &= \frac{2}{3}\times 15+\frac{1}{3}\times 5=\frac{35}{3}>10=u_2(C,Y) \end{aligned}$$

(See also the argument in part (c) below).

The rationalizable strategies are $\{A,C\}\times\{X,Z\}$.

(b) *Find all pure strategy Nash equilibria and the payoffs of these equilibria.*

(c) *Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.*

Solution:

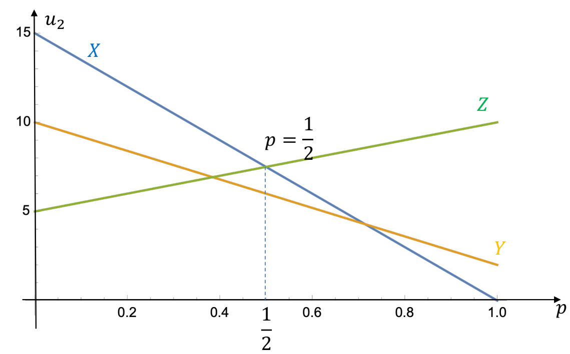
Let us look for a NE of the form

$$\begin{aligned}\sigma_1 &= pA + (1-p)B \\ \sigma_2 &= xX + yY + (1-x-y)Z\end{aligned}$$

We compute the expected utilities of the players

$$\begin{aligned}u_1(A, \sigma_2) &= 2 - x - 2y \\ u_1(C, \sigma_2) &= 1 + x \\ u_2(\sigma_1, X) &= 15 - 15p \\ u_2(\sigma_1, Y) &= 10 - 8p \\ u_2(\sigma_1, Z) &= 5 + 5p\end{aligned}$$

We graph the utilities of player 2.



We see that Y is not part of any best reply for player 2. Hence, we may assume $y = 0$. And

$$\begin{aligned}\sigma_1 &= pA + (1-p)C \\ \sigma_2 &= xX + (1-x)Z\end{aligned}$$

Also, from the picture we see that best reply of player 2 is

$$\text{BR}_2(\sigma_1) = \begin{cases} X \quad (x = 1) & \text{if} \quad 0 \leq p = 1/2 \\ \{X, Z\} \quad (0 \leq x \leq 1) & \text{if} \quad 0 \leq p = 1/2 \\ \text{Z} \quad (x = 0) & \text{if} \quad 1/2 < p \leq 1 \end{cases}$$

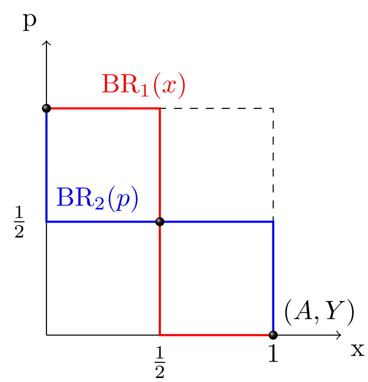
Since,

$$\begin{aligned}u_1(A, \sigma_2) &= 2 - x \\ u_1(Z, \sigma_2) &= 1 + x\end{aligned}$$

we have that $u_1(A, \sigma_2) \geq u_1(Z, \sigma_2)$ iff $x \leq 1/2$. Thus, best reply of player 1 is

$$\text{BR}_1(\sigma_2) = \begin{cases} A \quad (p = 1) & \text{if} \quad 0 \leq x < \frac{1}{2} \\ \{A, C\} \quad (p \in [0, 1]) & \text{if} \quad x = \frac{1}{2} \\ C \quad (y = 0) & \text{if} \quad 1 \geq x > \frac{1}{2} \end{cases}$$

Graphically,



We obtain the NE

$$\begin{aligned} (x=0, p=1), (A, Z), &\qquad \text{with payoffs } u_1 = 2, u_2 = 10. \\ (x=1, p=0), (C, X), &\qquad \text{with payoffs } u_1 = 2, u_2 = 15. \\ \left(\frac{1}{2}A + \frac{1}{2}C, \frac{1}{2}X + \frac{1}{2}Z\right), &\qquad \text{with payoffs } u_1 = \frac{3}{2}, u_2 = \frac{15}{2}. \end{aligned}$$