UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

TEST 1. October 16th, 2020

NAME:

Consider the following normal form game

		Player 2		
		X	*Y*	*Z*
	A	*2,1*	*0,1*	*5,3*
Player 1	*B*	*3,6*	*2,1*	*4,3*
	C	*2,10*	*1,15*	*2, 10*

(a) *What are the strategies that survive the iterated elimination of strictly dominated strategies?*

Strategy C is dominated by strategy B for player 1. After eliminating this strategy we obtain the following game

		Player 2		
		X	Y	Z
Player 1	A	2,1	0,1	5,3
	B	3,6	2,1	4,3

Now strategy Y is dominated by strategy Z for player 2. After eliminating this strategy we obtain the following game

		Player 2	
		X	Z
Player 1	A	2,1	5,3
	B	3,6	4,3

The rationalizable strategies are $\{A, B\} \times \{X, Z\}$.

(b) *Find all pure strategy Nash equilibria and the payoffs of these equilibria.* The best responses of the players are

		Player 2	
		X	Z
Player 1	A	2,1	$\underline{5}$, $\underline{3}$
	B	$\underline{3}$, $\underline{6}$	4, 3

Hence, the NE are:

- (A, Z) with payoffs $(5, 3)$; and
- (B, X) with payoffs $(3, 6)$.

(c) *Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.* Let us look for a NE of the form

$$(pA + (1-p)B, qX + (1-q)Z)$$

We compute the expected utilities of the players

$$
\begin{aligned}
u_1\left(A, qX+(1-q)Z\right) &= 2q+5-5q=5-3q \\
u_1\left(B, qX+(1-q)Z\right) &= 3q+4-4q=4-q \\
u_2\left(pA+(1-p)B, X\right) &= p+6-6p=6-5p \\
u_2\left(pA+(1-p)B, Z\right) &= 3
\end{aligned}
$$

Thus, we have that $5-3q=4-q$, so $q=1/2$. And $6-5p=3$, so $p=3/5$. Thus,

$$
\left(\frac{3}{5}A+\frac{2}{5}B, \frac{1}{2}X+\frac{1}{2}Z\right)
$$

is a mixed strategy NE. The payoffs of the players are $\left(\frac{7}{2}, 3\right)$.