UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

TEST 1–October 9th, 2019

NAME:

Consider the following normal form game:

		Player 2		
		X	*Y*	*Z*
Player 1	*A*	*2,2*	*6,4*	*10,1*
	B	*4,6*	*2,2*	*15,0*
	C	*2,2*	*3,7*	*10,-1*

(a) *What are the strategies that survive the iterated elimination of strictly dominated strategies? Hint: One of the players has a strategy which is strictly dominated by a mixed strategy composed by the other two strategies.*

Solution: Strategy Z is dominated by strategy Y for player 2. After eliminating this strategy we obtain the following game

		Player 2	
		X	Y
Player 1	A	2,2	6,4
	B	4,6	2,2
	C	2,2	3,7

Strategy C is dominated by the mixed strategy

$$\sigma = \frac{1}{2}A + \frac{1}{2}B$$

for player 1, because

$$\begin{aligned} u_1(\sigma, X) &= \frac{1}{2} \times 2 + \frac{1}{2} \times 4 = 3 > u_1(C, X) \\ u_1(\sigma, Y) &= \frac{1}{2} \times 6 + \frac{1}{2} \times 2 = 4 > u_1(C, Y) \end{aligned}$$

After eliminating this strategy we obtain the following game

		Player 2	
		X	Y
Player 1	A	2,2	6,4
	B	4,6	2,2

The rationalizable strategies are $\{A, B\} \times \{X, Y\}$.

(b) *Find all pure strategy Nash equilibria and the payoffs of these equilibria.*

Solution: Let us look for a NE of the form

$$\begin{aligned} \sigma_1 &= xA + (1-x)B \\ \sigma_2 &= yX + (1-y)Y \end{aligned}$$

We compute the expected utilities of the players

$$\begin{aligned}
u_1(A, \sigma_2) &= 2y + 6(1-y) = 6 - 4y \\
u_1(B, \sigma_2) &= 4y + 2(1-y) = 2 + 2y \\
u_2(\sigma_1, X) &= 2x + 6(1-x) = 6 - 4x \\
u_2(\sigma_1, Z) &= 4x + 2(1-x) = 2 + 2x
\end{aligned}$$

Note that

(a) $6 - 4y < 2 + 2y$ iff $\frac{2}{3} < y \leq 1$.
(b) $6 - 4y = 2 + 2y$ for $y = \frac{2}{3}$.

Thus, we have that best reply of player 1 is

$$\mathrm{BR}_1(\sigma_2) = \begin{cases} A(x=1) & \text{if } \quad 0 \leq y < \frac{2}{3} \\ [0,1] & \text{if } \quad y = \frac{2}{3} \\ B(x=0) & \text{if } \quad \frac{2}{3} < y \leq 1 \end{cases}$$

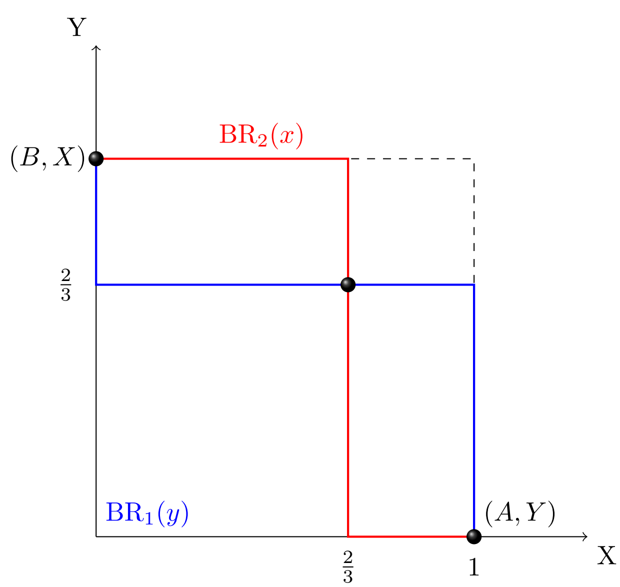
Note that

(a) $6 - 4x < 2 + 2x$ iff $\frac{2}{3} < x \leq 1$.
(b) $6 - 4x = 2 + 2x$ for $x = \frac{2}{3}$.

Thus, we have that best reply of player 2 is t

$$\mathrm{BR}_2(\sigma_1) = \begin{cases} X(y=1) & \text{if } \quad 0 \leq x < \frac{2}{3} \\ [0,1] & \text{if } \quad x = \frac{2}{3} \\ B(y=0) & \text{if } \quad \frac{2}{3} < x \leq 1 \end{cases}$$

Graphically,



We obtain two NE in pure strategies: $x = 0$, $y = 1$ and $x = 1$, $y = 0$. That is we obtain the NE

$$(B, X) \quad \text{with payoffs} \quad u_1 = 2, u_2 = 4$$

and

$$(A, Y) \quad \text{with payoffs} \quad u_1 = 4, u_2 = 2$$

(c) *Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.*

Solution: From the previous part we see that the NE in mixed strategies correspond to $x = y = \frac{2}{3}$. That is,

$$\begin{aligned}\sigma_1 &= \frac{2}{3}A + \frac{1}{3}B \\ \sigma_2 &= \frac{2}{3}X + \frac{1}{3}Y\end{aligned}$$

with payoffs $u_1 = u_2 = \frac{10}{3}$.