NAME:

Consider the following normal form game:

	X	Y	Z
A	1,0	0,2	4, 5
B	2, 5	2, -1	4,0
C	0, 3	-1,3	3,-2

(a) What are the strategies that survive the iterated elimination of strictly dominated strategies?

Solution: Strategy C is dominated by strategy A for player 1. After eliminating this strategy we obtain the following game

	X	Y	Z
A	1,0	0,2	4, 5
B	2, 5	2, -1	4,0

Strategy Y is dominated by strategy Z for player 2. After eliminating this strategy we obtain the following game

	X	Z
A	1,0	4, 5
B	2, 5	4,0

The rationalizable strategies are $\{A, B\} \times \{X, Z\}$.

(b) Find all pure strategy Nash equilibria and the payoffs of these equilibria.

Solution: Let us look for a NE of the form

$$
\begin{aligned}
\sigma_1 &= xA + (1-x)B \\
\sigma_2 &= yX + (1-y)Z
\end{aligned}
$$

We compute the expected utilities of the players

$$
\begin{aligned}
u_1\,(A, \sigma_2) &= y + 4(1 - y) = 4 - 3y \\
u_1\,(B, \sigma_2) &= 2y + 4(1 - y) = 4 - 2y \\
u_2\,(\sigma_1, X) &= 5(1 - x) \\
u_2\,(\sigma_1, Z) &= 5x
\end{aligned}
$$

Note that

(a) $4 - 3y < 4 - 2y$ for $0 < y \le 1$.

(b) $4 - 3y = 4 - 2y$ for $y = 0$.

Thus, we have that best reply of player 1 is

$$\text{BR}_1(\sigma_2) = \begin{cases} [0,1] & \text{if} \quad y = 0 \\ x = 0 & \text{if} \quad 0 < y \le 1 \end{cases}$$
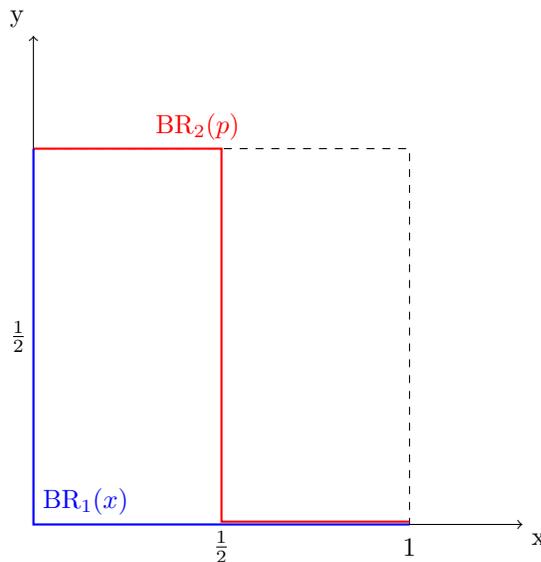
Note that

(a) $5(1 - x) < 5x$ for $x > \frac{1}{2}$.

(b) $5(1 - x) > 5x$ for $x < \frac{1}{2}$.

(c) $5(1 - x) = 5x$ for $x = \frac{1}{2}$.

Thus, we have that best reply of player 2 is

$$\text{BR}_1(\sigma_2) = \begin{cases} y = 1 & \text{if} \quad x < \frac{1}{2} \\ [0,1] & \text{if} \quad x = \frac{1}{2} \\ y = 0 & \text{if} \quad x > \frac{1}{2} \end{cases}$$

Graphically,

We obtain two NE in pure strategies: $x = 0$, $y = 1$ and $x = 1$, $y = 0$. That is we obtain the NE

$$(B, X) \quad \text{and} \quad (A, Z)$$

(c) *Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.*

Solution: From the previous part we see that the NE in mixed strategies correspond to $0 \le x \le \frac{1}{2}$, $y = 0$. That is,

$$\begin{aligned} \sigma_1 &= xA + (1 - x)B \quad 0 \le x \le \frac{1}{2} \\ \sigma_2 &= Z \end{aligned}$$