UNIVERSITY CARLOS III

Master in Economics **Master in Industrial Economics and Markets**

Game Theory

TEST 1–OCTOBER 2017

NAME:

Consider the following normal form game:

	X	Y	Z
A	1,0	0,2	4,10
B	2,25	2,20	4,0
C	0,10	-1,3	3,-2

(a) *What are the strategies that survive the iterated elimination of strictly dominated strategies?*

Strategy C is dominated by strategy A for player 1. After eliminating this strategy we obtain the following game

	X	Y	Z
A	1,0	0,2	4,10
B	2,25	2,20	4,0

(b) *Find all pure strategy Nash equilibria and the payoffs of these equilibria.* The best responses of the players are

	X	Y	Z
A	1,0	0,2	$\underline{4},\underline{10}$
B	$\underline{2},\underline{25}$	$\underline{2}$,20	$\underline{4}$,0

Hence, the NE are (A, Z), with payoffs $u_1 = 4$, $u_2 = 10$ and (B, X), with payoffs $u_1 = 2$, $u_2 = 25$.

(c) *Compute the mixed strategy Nash equilibria and the expected payoffs of these equilibria.* Let us look for a NE of the form (σ_1, σ_2) with

$$\sigma_1 = pA + (1-p)B, \quad \sigma_2 = q_1 X + q_2 Y + (1 - q_1 - q_2)Z$$

We compute the expected utilities of the players

$$\begin{aligned} u_1(A, \sigma_2) &= 4 - 3q_1 - 4q_2 \\ u_1(B, \sigma_2) &= 4 - 2q_1 - 2q_2 \\ u_2(\sigma_1, X) &= 25 - 25p \\ u_2(\sigma_1, Y) &= 20 - 18p \\ u_2(\sigma_1, Z) &= 10p \end{aligned}$$

Suppose first that $p = 0$ or $p = 1$. If $p = 0$, then the best reply of player 2 is to choose X and if $p = 1$, then the best reply of player 2 is to choose Z. These NE correspond to the pure strategy NE found in part (b).

Let us look now for a NE in which player 1 uses a completely mixed strategy, $0 < p < 1$. Note that $u_1(B, \sigma_2) - u_1(A, \sigma_2) = 2q_2 + q_1 \geq 0$. Hence, $u_1(B, \sigma_2) \geq u_1(A, \sigma_2)$ for every $0 \leq q_1, q_2 \leq 1$ and $u_1(B, \sigma_2) > u_1(A, \sigma_2)$, unless $q_1 = q_2 = 0$. We conclude that player 1 can be indifferent between strategies A and B only if player 2 is using strategy Z. That is only if $q_1 = q_2 = 0$.

On the other hand, strategy Z is a best reply of player 2 to the strategy σ_1 of player 1 if and only if $u_2(\sigma_1, Z) \geq u_2(\sigma_1, X)$ and $u_2(\sigma_1, Z) \geq u_2(\sigma_1, Y)$. That is, only if

$$10p \geq 25 - 25p \quad \text{and} \quad 10p \geq 20 - 18p$$

which occurs if and only if

$$p \geq \frac{5}{7}$$

Hence, there are infinitely many mixed strategy equilibria of the form

$$(pA + (1-p)B, Z) \quad \frac{5}{7} \leq p \leq 1$$

The payoffs are $u_1 = 4$, $u_2 = 10p$.