MICROECONOMICS II
Midterm Exam
Universitat Pompeu Fabra – Winter quarter 2004
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1. A mixed-strategy profile σ^* of a strategic-form game is a *strong* equilibrium if there is no other strategy profile σ' such that $u_j(\sigma') > u_j(\sigma^*)$ for all j with $\sigma'_j \neq \sigma^*_j$.

   (a) (5) What is the relationship between the set of Nash equilibria and the set of strong equilibria (a subset, a superset, neither)? Why?

   (b) (10) Give an example of a game with no strong equilibria.

   (c) (10) Give an example of a game which has at least one strong equilibrium, and where the sets of strong equilibria and Nash equilibria are different.

2. There are ten locations, with respective values $a_1 < a_2 < ... < a_{10}$. Player i ($i = 1, 2$) is endowed with n_i soldiers ($n_i < 10$) and must allocate them among the locations. To each particular location he can allocate no more than one soldier. The payoff at location p is a_p to the player whose soldier is unchallenged, and $-a_p$ to his opponent, unless both have a soldier at p or noone has, in which case the payoff is zero to both. The total payoff is obtained by summing up local payoffs.

   (a) (25) Show that in this game both players have a unique strategy which weakly dominates all others. Does the strategy profile where both agents use their dominating strategy constitute a Nash equilibrium (please justify your answer, positive or negative)?

3. Consider the following two-player game. First player 1 can choose either *Stop* or *Continue*. If she chooses *Stop* then the game ends with the pair of payoffs (1, 1). If she chooses *Continue* then the players simultaneously announce nonnegative integers and each player's payoff is the product of the numbers (notice that "infinity" is NOT an integer number).

   (a) (25) What are the subgame-perfect equilibria of this game (in pure and mixed strategies)?

4. The following is the game of the "crazy crab" we analyzed in class.Assume $v > c$ and write the payoff matrix of the normal form. If in part b) or c) there is a mixed-strategy equilibrium, I only need a proof that such an equilibrium exists, and an indication of the system of equations that need to be solved (and not necessarily the solution for the system of equations).

   (a) (5) What strategies are strictly dominated?

   (b) (10) What are the Nash equilibria when $0.4v - 0.6k < 0$?

   (c) (10) What are the Nash equilibria when $0.4v - 0.6k > 0$?