MICROECONOMICS II
Problem set 4
Universitat Pompeu Fabra – Winter 2006
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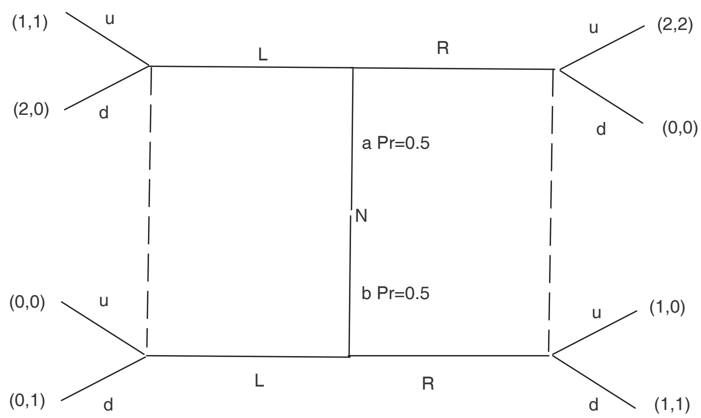
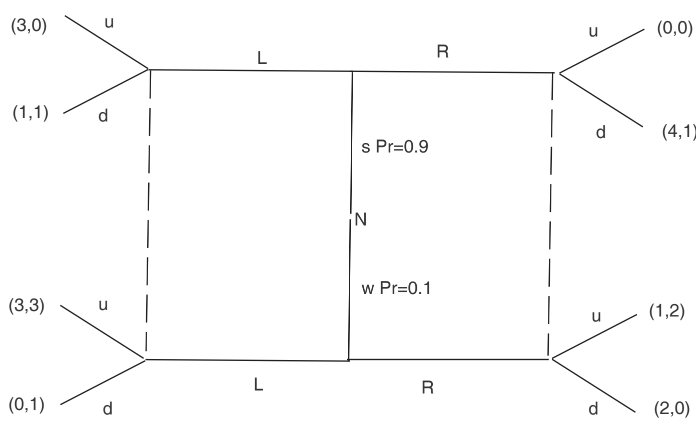
1. Describe all the pure-strategy perfect Bayesian equilibria of the following signaling games.

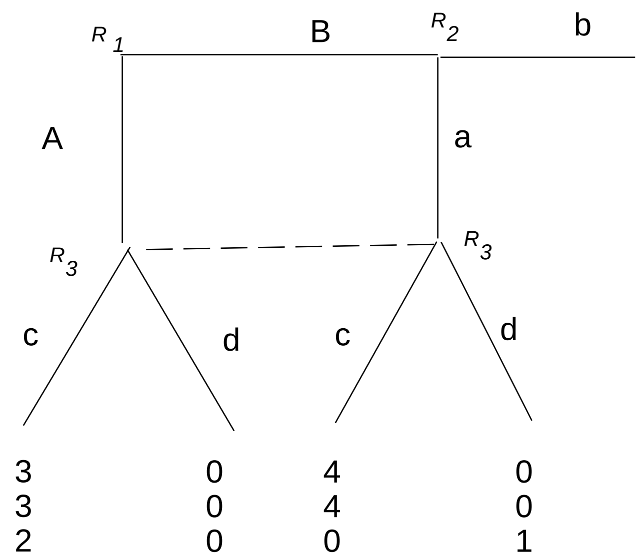




2. Two partners must dissolve their partnership. Partner 1 currently owns s share of the partnership, partner 2 owns $1-s$. The partners agree to play the following game: partner 1 names a price, p, for the whole partnership, and partner 2 then chooses either to buy 1's share for ps

or to sell her share to 1 for $p(1 - s)$. Suppose it is common knowledge that the partners' valuations for owning the whole partnership are independently and uniformly distributed on [0, 1], but that each partner's valuation is private information. What is the perfect Bayesian equilibrium?

3. The following game has two classes of Nash equilibria.



   (a) Find these Nash equilibria.

   (b) Show that the equilibria of one of these classes are not sequential, but those of the other class are.

4. Players 1 and 2 bargain over an item whose value for player 1 is either 0 or 3, with equal probabilities. Player 1 knows the value of the object, while player 2 is informed of this value only after he purchases it. The value of the object to player 2 is its value to player 1 plus 2. The bargaining procedure is the following: player 1 makes an offer, which player 2 either accepts or rejects; in the event of rejection player 1 makes another offer, which player 2 either accepts or rejects. If no offer is accepted then player 1 is left with the object and obtains a payoff equal to its value; player 2's payoff is 0. Show that there is a sequential

equilibrium in pure strategies in which there is no deal when player 1's valuation is 3, while the object is sold at the price of two in the first period when player 1's valuation is 0.

5. Consider the three-player symmetric infinitely repeated game in which each player's preferences are represented by the discounting criterion and the stage game is $(\{1,2,3\},(A_i),(u_i))$ where for $i = 1,2,3$ we have $A_i = [0,1]$ and $U_i(a_1,a_2,a_3) = a_1a_2a_3 + (1-a_1)(1-a_2)(1-a_3)$ for all $(a_1,a_2,a_3) \in A_1 \times A_2 \times A_3$.

   (a) Find the set of feasible, individually rational payoffs of the stage game.

   (b) Show that for any discount factor $\delta \in (0,1)$ the payoff of any player in any subgame perfect equilibrium of the repeated game is at least $\frac{1}{4}$.

   (c) Relate this result to the Folk Theorem of Fudenberg and Maskin.

6. Consider the following infinite-horizon game between a single firm and a sequence of workers each of whom lives for one period. In each period the worker chooses either to expend effort and so produce output y at effort cost c or to expend no effort, produce no output, and incur no cost. Assume that at the beginning of the period the worker has an alternative opportunity worth zero (net of effort cost) and that the worker cannot be forced to accept a wage less than zero. Assume also that $y > c$ so that expending effort is efficient.

   Within each period, the timing of events is as follows: first the worker chooses an effort level, then output is observed by both the firm and the worker, and finally the firm chooses a wage to pay the worker. Assume that no wage contracts can be enforced: the firm's choice of a wage is completely unconstrained.

   Now consider the infinite-horizon problem. Assume that at the beginning of period t, the history of the game through period $t-1$ is observed by the worker who will work in period t. Suppose the firm discounts the future according to the discount factor δ per period.

   (a) What is the subgame perfect equilibrium of the stage game?

(b) Give discount factors and describe strategies for firm and workers such that each worker expends effort and gets a salary ω for all $\omega \in (0, y)$ in a subgame-perfect equilibrium

(c) Are there subgame-perfect equilibria in which some workers are paid even if they don't work? If not, explain why; if so, describe the equilibria.