Separation costs, job heterogeneity and labor market volatility in the matching model

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Abstract

This paper extends the standard matching model by introducing a gap in separation costs between entrant and incumbent workers. We show that when this gap is omitted from the model, these costs do not improve the labor market volatility without introducing unrealistic unemployment responses to unemployment benefits.

Key Words: Labor Markets, Matching, Insider-Outsider, Separation costs, Business Cycles.

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1 Introduction

Recent papers by Costain and Reiter (2007) and Shimer (2005) have questioned the ability of the simplest equilibrium matching model of unemployment to match the cyclical variations in unemployment and vacancies in response to shocks of reasonable size. For example, Shimer (2005) shows that under a reasonable calibration strategy, the ratio of vacancies to unemployment is less than 10% as volatile as in the U.S. data.

In turn, Costain and Reiter (2007) extend Shimer’s results by arguing that the standard matching model can generate sufficiently large cyclical fluctuations in unemployment, or a sufficiently small response of unemployment to unemployment benefits, but it cannot do both.

One potentially important factor that this standard model abstracts from is the presence of labor turnover costs such as separation costs. In a recent paper, Silva and Toledo (2007) argue that the search and matching model improves the volatility of the labor market if gaps in training and separation costs are introduced in an insider-outsider type model with heterogeneity among matched jobs. This approach generates a two-tier wage structure where workers share the initial training costs and prepay expected firing costs by accepting lower initial wages. Once they become insiders, they have access to higher wages.\(^1\) Mortensen and Nagypal (2007) also study the role of turnover costs on the labor market volatility but in a pure insider model without job heterogeneity. However, when adjusting the training and firing costs, they do not control for the response of unemployment to changes in unemployment benefits.

In this paper we show that when the gap in separation costs is omitted and the

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\(^1\)See Mortensen and Pissarides (1999) for a detailed analysis.
model converges to a one-tier wage structure, the presence of these costs do not improve the labor market volatility without introducing unrealistic unemployment responses to unemployment benefits. From an empirical point of view, this result can be considered important because, as it is well known, fixed-term employment contracts with virtually no separation costs have been introduced in a number of European countries as a way to provide flexibility to economies with high employment protection levels. Because of the direct association between entrants and fixed-term employees, the matching model with a gap in separation costs is particularly suitable to analyze the cyclical implications of these labor market reforms.\footnote{In a recent paper, Sala, Silva and Toledo (2008) study the cyclical behavior of segmented labor markets with flexibility at the margin.}

2 The model

The economy is integrated by a continuum of risk-neutral, infinitely-lived workers and firms, which discount future payoffs at a common rate $r$; capital markets are perfect; and time is continuous.

There is a time-consuming and costly process of matching workers and job vacancies, captured by a standard constant-returns-to-scale matching function $m(u, v) = \kappa u^\alpha v^{1-\alpha}$, where $u$ denotes the unemployment rate, $v$ is the vacancy rate, and $\alpha$ and $\kappa$ are parameters. Unemployed workers find jobs at rate $f(\theta) = m(1, 1/\theta)$ and vacancies are filled at rate $q(\theta) = m(\theta, 1)$, where $\theta = u/v$. This matching function implies that the larger the vacancy-unemployment ratio, the easier to find a job (i.e., $f'(\theta) > 0$), and the more difficult to fill up vacancies ($q'(\theta) < 0$).
Given the state-contingent ratio of vacancies to unemployment $\theta$, the unemployment rate evolves according to the following differential equation:

$$\dot{u} = s(1 - u) - f(\theta)u. \tag{1}$$

As in Shimer (2005), an aggregate shock hits the economy according to a Poisson process with arrival rate $\lambda$, at which point a new productivity $p'$ is drawn from a distribution that depends on the current productivity level $p$.

Workers can be either unemployed or employed. Unemployed workers are considered entrants once they find a job. The latter become incumbent employees at constant rate $\iota$. Both type of workers separate from the firm at constant rate $s$. Their Bellman equations are given by

$$rU = z + f(\theta)(W_e - U) + \lambda(\mathbb{E}_p U' - U), \tag{2}$$

$$rW_e = w_e - s(W_e - U) + \iota(W_i - W_e) + \lambda(\mathbb{E}_p W_e' - W_e), \tag{3}$$

$$rW_i = w_i - s(W_i - U) + \lambda(\mathbb{E}_p W_i' - W_i), \tag{4}$$

where $z$ is the flow value of unemployment, $w_e$ and $w_i$ denote the wage rate of entrants and incumbents, respectively, and $\mathbb{E}_p X'$ represents the expected value of $X$ conditional on $p$ following the next aggregate shock.

From the firms’s side, a job can be either filled or vacant. Before a position is filled, the firm has to open a job vacancy with a flow cost $c$. Firms have a linear production technology that uses only labor. When a match with an entrant is terminated, the firm pays separation cost $\gamma_e$, which is assumed to be fully wasted. Likewise, when incumbents are separated from the job, firms incur in cost $\gamma_i$. 
Thus, the value of vacancies, \( V \), and filled positions, \( J_e \) and \( J_i \), are represented by the following Bellman equations:

\[
rV = \begin{cases} 
    -c + q(\theta)(J_e - V) + \lambda(\mathbb{E}_p V' - V) \equiv \Upsilon & \text{if } \theta > 0 \\
    \max\{0, \Upsilon\} & \text{if } \theta = 0,
\end{cases}
\]  
(5)

\[
rJ_e = p - w_e + s(V - J_e - \gamma_e) + \iota(J_i - J_e) + \lambda(\mathbb{E}_p J_e' - J_e),
\]  
(6)

\[
rJ_i = p - w_i + s(V - J_i - \gamma_i) + \lambda(\mathbb{E}_p J_i' - J_i).
\]  
(7)

To close the model, we need to incorporate two more assumptions. The first one is the free entry condition for vacancies. Therefore, in equilibrium:

\[
V = 0.
\]  
(8)

We assume wages to be the result of bilateral Nash bargaining between the worker and the firm. The first-order conditions for entrants and incumbent employees yield the following two equations:

\[
(1 - \beta)(W_{e} - U) = \beta(J_e - V + \gamma_e),
\]  
(9)

\[
(1 - \beta)(W_{i} - U) = \beta(J_i - V + \gamma_i),
\]  
(10)

where \( \beta \in (0, 1) \) denotes workers’ bargaining power relative to firms’. Note that separation costs are explicitly considered in the wage negotiation.

Using (2)-(10), we can now solve for the equilibrium wages as a function of the current state \( p \) and \( \theta \),

\[
w_e = (1 - \beta)z + \beta c\theta + \beta p + \beta \left[f(\theta) + r\right] \gamma_e - \iota \beta (\gamma_i - \gamma_e)
\]  
(11)

\[
w_i = (1 - \beta)z + \beta c\theta + \beta p + \beta r \gamma_i + \beta f(\theta) \gamma_e.
\]  
(12)
Observe that the two groups of workers differ only in terms of the gap in separation costs \((\gamma_i - \gamma_e)\). Below, we make use of this specific feature of the model to eliminate job heterogeneity.

The equilibrium vacancy-unemployment ratio \(\tilde{\theta}\) is defined as

\[
\tilde{\theta} = \frac{r + s + t + \lambda}{q(\theta)} + \frac{r + s + \lambda + t}{r + s + \lambda} \beta \theta + \frac{f(\theta)}{c} \left[ \frac{1}{r + s + \lambda} \right] + 1 \beta \gamma_e \\
= \frac{1 - \beta}{c} \left[ p - z + \frac{\iota(p - z)}{r + s + \lambda} \right] - \frac{(1 - \beta) \iota \gamma_i}{c} \left[ \frac{s}{r + s + \lambda} \right] + \frac{\lambda \mathbb{E}_p}{q(\theta')} \frac{1}{r + s + \lambda} + \lambda \mathbb{E}_p \left[ \frac{1}{q(\theta')} \right] \\
- \frac{\gamma_e}{c} \left[ \beta r + \iota + s \right] + \frac{\lambda \iota}{c} \left( \frac{\beta \gamma_i + \mathbb{E}_p \iota}{c} \right).
\]

For each state \(p\), there is only one labor market tightness \(\theta\) that satisfies this equation.

### 3 Calibration and simulation results

We calibrate the model in the steady state at quarterly frequency. The numerical analysis accounts for three scenarios: (i) the standard model without separation costs, \(\gamma_i = \gamma_e = 0\); (ii) the model with separation costs and homogeneity among matched workers, \(\gamma_i = \gamma_e > 0\); and (iii) the model with a gap in separation costs, \(\gamma_i > \gamma_e = 0\).

As in Silva and Toledo (2007), we fix the interest rate \(r\) at 1%; the separation rate at \(s = 0.10\); and the average unemployment rate at 5.7%. Substituting this value together with \(s\) in equation (1), for \(\dot{u} = 0\), we find an average job finding rate of \(f^* = 1.65\), which is near the value reported by Shimer (2005) of 1.35. Following Shimer (2005), we set the elasticity of the matching function with respect to unemployment (\(\alpha\)) equal to 0.72. The Saratoga Institute Diagnostic Report estimates hiring costs to be equivalent to 3.6% of

\[\text{Notice that under the first two scenarios, the model converges to a one-tier wage structure, as it can be seen in equations (11) and (12). Thus, } \iota \text{ becomes irrelevant.}\]
the annual employee compensation in the U.S. Thus, the average hiring cost represents 14.5% of the steady-state labor productivity, $p^*$, which we normalize to 1. Considering an average duration of a vacancy of 24 days we set $q^* = 1.25$ and $c = 0.181$. Given the values of $f^*$ and $q^*$, and the Cobb-Douglas matching function, the efficiency of the matching process, $\kappa$, is 1.53. As in Silva and Toledo (2007), we fix the conversion rate from entrant to incumbent at $\iota = 0.25$.

The remaining two parameters are the employment opportunity cost, $z$, and the workers' bargaining power, $\beta$. Following Constain and Reiter (2005), we calibrate these parameters to simultaneously satisfy the equilibrium condition (13) in the steady state (i.e., for $\lambda = 0$), and their estimated semielasticity of unemployment with respect to unemployment benefits equal to 2. Thus, we prevent an excessive sensitivity of unemployment to changes in unemployment compensation in each scenario.

As in Shimer (2005), the labor productivity satisfies $p = z + e^y(p^* - z)$, where $y$ is an Ornstein-Uhlenbeck process with persistence parameter $\zeta$ and diffusion parameter $\sigma$. We choose $\zeta$ and $\sigma$ in order to reproduce the cyclical behavior of $p$ in the U.S. data, with standard deviation of 0.02 and autocorrelation of 0.87.

We simulate the model and detrend the generated data using an HP filter with $10^5$ smoothing parameter.\footnote{For our simulations, we set $\lambda = n\zeta$, where $n$ is given by the size of the discrete grid $(2n + 1) = 201$ we use. See Shimer (2005) for a discussion.} Finally, we calculate the standard deviations of our filtered data. In particular, we focus on unemployment $u$, vacancies $v$ and labor market tightness $\theta$. Table 1 shows the simulated standard deviations without separation costs (scenario 1), with separation costs and job homogeneity (scenario 2), and with a gap in these costs.
The first two scenarios without gap in separation costs yield relatively low variability of the relevant variables, with standard deviations that are at most 40 percent as volatile as in the third scenario where the gap in separation costs is positive.

To understand why cyclical fluctuations of vacancies and unemployment increase when a gap in firing costs \((\gamma_i - \gamma_e)\) is introduced into the model we need to consider the total match surplus of a new entrant. Let us define this surplus as

\[ S_e = J_e - V + \gamma_e + W_e - U. \]

After some substitutions we obtain the following expression for its flow value:

\[ (r + s)S_e = p - z + \gamma_e - \iota(\gamma_i - \gamma_e) + \iota(S_i - S_e) - f(\theta)\beta S_e + \lambda(ES_e' - S_e), \]

where \( S_i = J_i - V + \gamma_i + W_i - U \) represents the incumbent’s match surplus. Notice that for every productivity level \( p \), the match surplus of a new entrant is lower when we account for the gap in separation costs. Figure 1 shows that \( S_e \) is lower for every level of \( p \) when the gap in separation costs is positive. This implies that productivity shocks have a relatively greater impact in \( S_e \) and, consequently, in the value of a newly filled position \( J_e \). More in detail, given the equilibrium job creation condition, \( J_e = c/q(\theta) \) for all \( \theta > 0 \), a larger percentage change in \( J_e \) induces a greater response in labor market tightness \( \theta \). Hence a higher volatility of vacancies and unemployment. To conclude, notice that the model with a gap in separation costs is able to improve the volatility of vacancies and unemployment even though we are adjusting the employment opportunity cost, \( z \), and the wage bargaining parameter, \( \beta \), in order to avoid an excessive response of unemployment to changes in unemployment insurance. If we do not recalibrate these parameters, the semielasticity of unemployment with respect to unemployment compensation jumps to 8.
References


<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\gamma_i = \gamma_e = 0$ ($z = 0.858$; $\beta = 0.33$)</th>
<th>$\sigma(u)$</th>
<th>$\sigma(v)$</th>
<th>$\sigma(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1: $\gamma_i = \gamma_e = 0$ ($z = 0.858$; $\beta = 0.33$)</td>
<td></td>
<td>0.033</td>
<td>0.098</td>
<td>0.129</td>
</tr>
<tr>
<td>Scenario 2: $\gamma_i = \gamma_e = 1$ ($z = 0.536$; $\beta = 0.147$)</td>
<td></td>
<td>0.039</td>
<td>0.118</td>
<td>0.155</td>
</tr>
<tr>
<td>Scenario 3: $\gamma_i = 1, \gamma_e = 0$ ($z = 0.789$; $\beta = 0.33$)</td>
<td></td>
<td>0.057</td>
<td>0.383</td>
<td>0.433</td>
</tr>
</tbody>
</table>
Figure 1: Total surplus of new matches versus $p$