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On the impossibility of representing infinite utility streams

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Abstract We introduce a new Pareto type criterion on Social Welfare Functions over infinity utility streams which is, roughly speaking, not necessarily sensible to increments in just a finite number of components. We show that there is no Social Welfare Function satisfying, at the same time, this criterion and Diamond's Equity condition. With our result we extend the impossibility theorem by Basu and Mitra. Moreover, we show that, even under a weaker version of Equity, related with Zame's Intergenerational Equity Condition, the impossibility results are obtained as well.
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Introduction

Infinity utility streams are a useful tool to understand economic problems with an infinite time horizon. Introduced by Ramsey in [12], they have been used by Koopmans [10], von Weizsäcker [14] and Gale [9] among others to study optimal growth, saving, taxing or investing models. For instance, Becker and Boyd [6] and Dana et al. [7] are good references about recent developments on these topics.

In the study of infinite utility streams, a challenging problem is the existence of a utility function representing the order given on the set of these streams. Such a function is called a Social Welfare Function (SWF). A SWF is a rule that aggregates the consumptions of all generations into a real number, preserving

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preferences imposed on the set X of all infinite utility streams. In other words, a SWF assigns to an arbitrary sequence of consumptions a level of welfare enjoyed by the whole society.

In Koopmans [10], Diamond [8] and Basu and Mitra [4] it is shown that a SWF preserving the Pareto order is incompatible with an *egalitarian* treatment of all generations, in the sense that if one permutes the consumptions of two generations the level of welfare remains the same. More precisely, Koopmans shows that some form of *impatience* arises when the utility function satisfies recursive properties. Diamond proves that a Paretian and Egalitarian continuous utility function never exists under certain restrictions on the metric chosen on the set X. Finally, Basu and Mitra prove that, independently on the topology chosen on X, it is impossible to construct an utility function that satisfies the *Strong Pareto*[1] axiom and which is egalitarian.

Our starting point is the following observation: the strong Pareto axiom demands that, for a given consumption stream, an increment in consumption of just one generation leads to valuate this new stream better than the original. This seems to be too strict economically because, intuitively, the welfare of a society with infinitely many generations should not be influenced by the utility of just one generation.

Basu and Mitra introduce weaker versions of the Strong Pareto axiom. They call them *Dominance* axiom and *Partial Pareto* axiom in [4] and [5], respectively. In both cases it is proved that, when the consumption of each generation can be chosen in the interval $[0,1]$, then an impossibility result arises if one imposes Diamond's Equity. Both axioms impose that increments in either finitely many or in all components imply more utility. Nevertheless, nothing is imposed on intermediate cases, that is, where infinitely many components are increased and the others remain the same.

It seems to us that, in an economy with infinitely many generations, the idea of Ramsey of not assigning to a particular generation more importance than to others is preserved, if it is considered that the welfare of any generation, in comparison with the welfare of the whole economy, is negligible. In consequence, we propose a criterion, that we call *Infinite Paretian Principle*, where only infinitely many increments affect the value of the SWF. More precisely, a consumption stream is more valued to another one if infinitely many components of the first are greater than the respective ones of the second and the other generations remain the same.

Another point of view to understand our criterion is the following. In terms of intergenerational justice, it seems to us that an increment in the accumulated consumption of all the generations benefits the welfare of the whole society only if infinitely many of them enjoy this increment. In other words, if one can always find generations in the future (infinitely many, but not necessarily all) with a positive increment in its consumption[2].

A second point to ponder is a different look at the idea of equity. We recall that Diamond's Equity means that finite permutations in the utility streams give the same aggregated utility. Our idea is that this condition can be too strong because,

[1] This seems to be the standard name for the axiom, while Basu and Mitra call it just Pareto.

[2] Note the difference between our criterion and the so called *Hammond Equity for the Future*, c.f. [3]: If $x, y \in X$, such that $x_1 > y_1 > u > v$, $x = (x_1, v, v, \dots)$ and $y = (y_1, u, u, \dots)$ then, $W(y) \geq W(x)$.

in many frameworks, we are only interested in preserving the strict preferences by means of finite permutations. Consequently, we consider a *Weak Equity* condition on the SWF[3], that avoids the constraint that Equity imposes in the indifference cases and, at the same time, preserves the spirit of Ramsey and Gale, among others.

We show that, under our weaker hypotheses, Basu and Mitra's result remains true. We remark that, as in [4], our result is independent of the topology chosen on the space of all utility streams.

1 Infinite Utility Streams.

An infinity utility stream is an infinite sequence $x = (x_1, \dots, x_n, \dots)$ where $x_i \in A$, a subset of $\mathbb{R}$. In other words, $x \in A^{\mathbb{N}}$, the infinite cartesian product of A. We will denote $X = A^{\mathbb{N}}$. We will assume from now on that A contains at least two points, if not the problem is tautological.

A Social Welfare Function (SWF) is a function $W : X \to \mathbb{R}$ such that, for a given element $x \in X$, $W(x)$ measures the level of welfare that the sequence of consumptions (x_i) produces in the whole society.

Several principles have been imposed on a SWF. For instance, the Strong Pareto and Weak Pareto, that we recall here. Moreover, we introduce a new intermediate principle that seems to us to be very natural.

Definition 1.1 *(Strong Pareto principle)* We say that W, a SWF, is *Strong Paretian* if, given x and y in X, satisfying:

1. $x_i \geq y_i$ for all $i \in \mathbb{N}$, and
2. there exists $j \in \mathbb{N}$ such that $x_j > y_j$,

then $W(x) > W(y)$.

If x and y satisfy conditions (1) and (2) in the above definition we will write $x \succ_{\mathcal{P}} y$.

Definition 1.2 *(Weak Pareto principle)* We say that W, a SWF, is *Weak Paretian* if given x, $y \in X$ such that if $x_i > y_i$ $\forall i \in \mathbb{N}$, then $W(x) > W(y)$.

When x and y satisfy this condition we will use the notation $x \succ_{\mathcal{W}} y$.

A Strong Paretian SWF is sensible to increments in *at least* one component. As discussed in the Introduction, one unit of consumption should not affect, in principle, the global utility of an economy where infinitely many consumers are assumed[4]. On the contrary, a Weak Paretian SWF is only sensible to increments in *all* the components. For this reason we introduce the following intermediate Pareto axiom.

Definition 1.3 *(Infinite Pareto principle)* We say that W, a SWF, is *Infinite Paretian* if given x, $y \in X$ such that:

[3] Zame [15] uses strict preferences that *display Intergenerational Equity.*

[4] This point of view agrees with Aumann's [2], where an individual in an economy with a continuum of agents is negligible. This fact has also been pointed out by Lauwers [11]

1. $x_i \geq y_i \ \forall i \in \mathbb{N}$, and
2. There exists $M \subset \mathbb{N}$ with $\sharp M = \infty$ such that $x_j > y_j \ \forall j \in M$,

then $W(x) > W(y)$.

If x and y satisfy conditions (1) and (2) in the above definition we will write $x \succ_{\mathcal{I}} y$.

An alternative way to define this new principle is to say that for x and y satisfying:

1. $x_j \geq y_j$, $\forall j$ and
2. there exists $(s_n)_{n \geq 1}$, a strictly increasing sequence in $\mathbb{N}$, such that $x_{s_j} > y_{s_j}$ then $W(x) > W(y)$.

An Infinite Paretian SWF is sensible to increments in infinitely many components but, a priori, increments in just a finite number of them do not imply a variation in the value of the SWF.

Obviously, if W is Strong Paretian, then W is Infinite Paretian and, analogously, if W is Infinite Paretian, then W is Weak Paretian, but the converses are not true as the following SWFs show.

Example 1.4

a) Consider $A \subset \mathbb{R}$ and let

$$W(x) = \sum_{n>1} \frac{1}{2^n} \arctan(x_n).$$

This is obviously an Infinite Paretian SWF but it does not satisfy the Strong Pareto principle.

b) Suppose $A \subset \mathbb{R}$ any lower bounded, closed and discrete set and let

$$W(x) = \inf_{n \geq 1} x_n$$

the Rawlsian SWF. This is obviously a Weak Paretian SWF but it does not satisfy the Infinite Pareto principle.

Now we recall the concept of Equity, as introduced by Diamond [8], to ensure intergenerational justice. Observe the contrast with Pareto axioms, that deal with efficiency.

Definition 1.5 Let W be a SWF. We say that W is *Egalitarian* (or that W satisfies the *Equity* condition) if, for all x, y in X such that:

1. x and y differ only in periods i, j, and
2. $x_i = y_j$, $x_j = y_i$,

then $W(x) = W(y)$.

This is equivalent to say that the value of $W(x)$ is not affected by finite permutations of the components of x. For instance, the SWF in Example 1.4 b) is Egalitarian, but the one in Example 1.4 a) is not.

Remark 1.6 Assume that W is Infinite Paretian and Egalitarian. If $x \succ_{\mathcal{I}} y$, then $W(\sigma x) > W(\tau y)$, where σx and τy are obtained applying the finite permutations σ and τ to the components of x and y, respectively.

2 The Impossibility Theorem.

As it is shown in Example 1.4 b), there are Weak Paretian and Egalitarian SWFs under some mild conditions on the consumption domain. Basu and Mitra [4] prove that there does not exist any Strong Paretian and Egalitarian SWF without restrictions on the domain. In this section we will prove that it is also impossible to construct an Infinite Paretian and Egalitarian SWF for any domain.

Let us recall the immersion of the interval $(0,1)$ in $\{0,1\}^{\mathbb{N}}$, given by Sierpinski in [13], which was used in Basu and Mitra [4] to prove their impossibility theorem. Set $q_1,\dots,q_n,\dots$ an enumeration of $\mathbb{Q}\cap(0,1)$. For $r\in(0,1)$, let the (infinite) sequence $i(r)\in\{0,1\}^{\mathbb{N}}$ be defined as follows

$$i(r)_n = \begin{cases} 1 & \text{if } q_n < r \\ 0 & \text{if } q_n \geq r \end{cases} .$$

Remark 2.1

1. $i(r)$ contains infinitely many ones and infinitely many zeros. This is because there are infinitely many rationals q_n in the interval $(0,r)$ (so $i(r)_n = 1$) as well as infinitely many q_m in $[r,1)$ (and hence $i(r)_m = 0$).
2. If $r < s$ are two numbers in $(0,1)$ then $i(r) \prec_{\mathcal{I}} i(s)$ in $\{0,1\}^{\mathbb{N}}$. Indeed, if $i(r)_n = 1$ one has $q_n < r < s$ and hence $i(s)_n = 1$. On the other hand, the interval $[r,s)$ contains infinitely many rationals q_m, so in the components associated to those rationals $i(r)_m = 0$ while $i(s)_m = 1$.

Proposition 2.2 *Let W be an Infinite Paretian SWF on X. Then, the composite $f = W\circ i : (0,1)\to(0,1)$ has, at most, a countable set of discontinuities.*

Proof The composite $f = W\circ i$ is an strictly increasing function from $(0,1)$ to $(0,1)$ because, if $r<s$, then $i(r)\prec_{\mathcal{I}} i(s)$ by Remark 2.1 (2) and, if $x\prec_{\mathcal{I}} y$, then $W(x) < W(y)$ since W is Infinite Paretian. Hence, f could have at most a countable set of discontinuities.

Our main result extends to our framework the Impossibility Theorem of Basu and Mitra [4]. In fact our proof is inspired by theirs, in the sense that we use Sierpinski's construction to produce an strictly increasing function, with a non-countable number of discontinuities.

Theorem 2.3 *There does not exist an Egalitarian and Infinite Paretian SWF.*

Proof Fist of all, observe that it is enough to prove the result for $X = \{0,1\}^{\mathbb{N}}$. Indeed, if X has at least two elements and W is a SWF satisfying the hypotheses of the Theorem, then W has to satisfy them when we restrict to sequences with components taken in these two points.

We will show that, if such a W exists, then the composite $f = W\circ i$ is discontinuous at every $r\in(0,1)$, contradicting Proposition 2.2.

Fix $r\in(0,1)$ and consider $i(r)$. We are going to construct an element of X, $i(r)^+$, such that $W(i(r)) < W(i(r)^+) < W(i(s))$ for all $s\in(r,1)$.

Let $q_1,\dots,q_k,\dots$ be the enumeration of the rationals in $(0,1)$ used to define the immersion i. Consider a strictly decreasing sequence of rational numbers $(z_j)_{j\geq 1}$

in $(0,1)$, converging to r, and define $i(r)^+$ as follows:

$$i(r)^+_n = \begin{cases} 1 \text{ if } q_n < r, \\ 1 \text{ if } q_n = z_j \text{ for some } j, \\ 0 \text{ otherwise.} \end{cases}$$

So, if $q_n < r$, we have $i(r)^+_n = 1 = i(r)_n$. If $q_n = z_j$ for some j, then $i(r)_n = 0 < 1 = i(r)^+_n$. Finally, if $q_n \geq r$, $q_n \neq z_j$ for all j, then $i(r)_n = 0 = i(r)^+_n$. In other words, $i(r) \prec_{\mathcal{I}} i(r)^+$, because $(z_j)_{j\geq 1}$ is an infinite sequence of rational numbers bigger than r.

On the other hand, let $r < s < 1$ and consider $i(s)$. Then $W(i(r)^+) < W(i(s))$. Indeed, as $(z_j)_{j\geq 1}$ converges to r, there exists n_0 such that $z_n < s$ for all $n > n_0$, hence there is only a finite number of components n such that $i(r)^+_n = 1$ and $i(s)_n = 0$, those n such that q_n are equal to $z_1, \dots, z_{n_0}$. Denote these components by $i(s)_{a_1}, \dots, i(s)_{a_{n_0}}$. Moreover, there are infinitely many rational numbers in (r, s) that are not in $(z_j)_{j\geq 1}$, hence there are infinitely many components n such that $i(s)_n = 1$ and $i(r)^+_n = 0$. Denote these components by $i(s)_{b_j}$ with $j \in \mathbb{N}$. Now permute the components $i(s)_{a_j}$ with the components $i(s)_{b_j}$ for $j = 1, \dots, n_0$ and call this new vector $\sigma i(s)$. By construction $i(r)^+ \prec_{\mathcal{I}} \sigma i(s)$ and, using first the Infinite Paretian principle and second the Equity condition, one obtains $W(i(r)^+) < W(\sigma i(s)) = W(i(s))$.

Observe that the construction of $i(r)^+$ does not depend on s. So, for any given $s \in (0,1)$, with $r < s$, one has $W(i(r)) < M < W(i(s))$, where $M = W(i(r)^+)$. Then, $W \circ i$ is discontinuous at every $r \in (0,1)$.

Corollary 2.4 *(Basu–Mitra 2003). There does not exist a Egalitarian and Strong Paretian SWF.*

We can extend the previous results to other criteria, such as the von Weiszächer's overtaking criterion. Following Asheim and Tungodden [1], we recall only the strict preference of this criterion. We say that W preserves the overtaking criterion if $x \prec_O y$ implies $W(x) < W(y)$, where

$$x \prec_O y \Leftrightarrow \exists\, n_0 \text{ such that } \sum_{k=1}^{n} x_k < \sum_{k=1}^{n} y_k, \forall n > n_0.$$

Observe that $x \prec_{\mathcal{P}} y$ implies $x \prec_O y$. Hence, if an Egalitarian SWF preserves the overtaking criterion, then it has to be Strong Paretian, which is impossible, by Corollary 2.4. Then, we have obtained the following result.

Corollary 2.5 *There does not exist an Egalitarian SWF preserving the von Weiszächer's overtaking criterion.*

3 Weakening Equity.

Our main result in this paper (as well as Theorems 1 and 2 in Basu-Mitra's paper) shows that Diamond's Equity is a too strong condition to be compatible with the Infinite Paretian principle as well as with the Strong Paretian one. One can think that these impossibility results arise because equity among generations deals with indifference relations, while the paretian principles deal with strict preferences. Zame in [15] works with a notion of intergenerational equity applied to the case of strict preferences. Precisely, a strict preference relation $\succ$ *displays intergenerational equity* if, given $x \succ y$, then $\sigma x \succ \tau y$ for any finite permutations σ and τ of the components of x and y. In a parallel way, we define a weaker version of intergenerational equity than the one defined in Section 1.

Definition 3.1 Let $W : X \to \mathbb{R}$ be a SWF and $\succ$ an irreflexive preference on X. We say that W is *Weak Egalitarian* (with respect to $\succ$) if, for every x and y in X with $x \succ y$, then $W(\sigma x) > W(\tau y)$, for any σ and τ finite permutations of the components of x and y.

Observe that an Egalitarian Strong (respectively Infinite) Paretian SWF satisfies this condition for the irreflexive preference $\prec_{\mathcal{P}}$ (respectively $\prec_{\mathcal{I}}$) but, obviously, the converse is not true. In case that we have W, a SWF which is Strong Paretian and Weak Egalitarian w.r.t $\prec_{\mathcal{P}}$, we will just say that W is a *Weak Egalitarian Strong Paretian* SWF. In the same, way we will talk about a *Weak Egalitarian Infinite Paretian* SWF.

The goal of this section is to prove that, even under this weak version of equity, the impossibility results appear both for Strong or Infinite Paretian SWFs.

First of all, we are going to show a set of sequences that are comparable by means of any SWF which is either Weak Egalitarian Strong Paretian or Weak Egalitarian Infinite Paretian. Hence, this is true in particular if we assume Diamond's equity on W and this SWF is either Strong Paretian or Infinite Paretian.

Lemma 3.2 *Let W be a SWF over X. Assume that W is either a Weak Egalitarian Strong Paretian or Weak Egalitarian Infinite Paretian SWF. Let $a, b \in A$ and $x, y \in \{a, b\}^{\mathbb{N}}$ satisfying:*

1. *$y_i < x_i$ for, at most, finitely many i, and*
2. *$x_j < y_j$ for infinitely many j in $\mathbb{N}$;*

then $W(x) < W(y)$.

Proof Assume $a < b$. Under the assumptions of the Lemma, there exists $(s_n)_{n \geq 1}$, an strictly increasing sequence in $\mathbb{N}$ such that $x_{s_j} = a < b = y_{s_j}$ and, only for certain $r_1, r_2, \ldots, r_k$ in $\mathbb{N}$, one has $y_{r_j} = a < b = x_{r_j}$. For $j = 1, \ldots, k$, rearranging the components x_{r_j} into the components x_{s_j} and viceversa is just a finite permutation of the components of x. Call this new vector $\sigma x = ((\sigma x)_1, \ldots, (\sigma x)_n, \ldots)$. By construction $y_i - (\sigma x)_i \geq 0$ for all i and $y_{s_j} - (\sigma x)_{s_j} = b - a$ for $j = k+1, \ldots$, hence $\sigma x \prec_{\mathcal{I}} y$ (and also $\sigma x \prec_{\mathcal{P}} y$). Then, using the Weak Equity condition $W(x) < W(y)$, because σx is a finite permutation of x.

If two sequences x, y in $\{a, b\}^{\mathbb{N}}$ satisfy conditions (1) and (2) in Lemma 3.2 we will write $x \prec_{\mathcal{Q}} y$.

The converse of this result is also true when we restrict the domain to a set with only two points.

Lemma 3.3 *Let a, b two real numbers and $X = \{a, b\}^{\mathbb{N}}$. Let W be a SWF on X such that, if x and y satisfy $x \prec_{\mathcal{Q}} y$, one has $W(x) < W(y)$. Then, W is a Weak Egalitarian Infinite Paretian SWF.*

Proof Under the conditions of the Lemma W is obviously Infinite Paretian. Now consider $x \prec_{\mathcal{I}} y$. It is also evident that $\sigma x \prec_Q \tau y$, for any pair of finite permutations σ and τ. Hence $W(\sigma x) < W(\tau y)$.

Putting together the above results, one has the following characterization of Weak Egalitarian Infinite Paretian SWFs defined over binary sets, by means of the irreflexive preference relation $\prec_Q$.

Proposition 3.4 *Let a, b two real numbers and $X = \{a, b\}^{\mathbb{N}}$. Let W be a SWF on X. Then, W is a Weak Egalitarian Infinite Paretian SWF if and only if, for any x and y with $x \prec_Q y$, one has $W(x) < W(y)$.*

Finally, we show that, even under the weakest egalitarian hypotheses introduced in this Section, the impossibility result also holds.

Theorem 3.5 *There does not exist a Weak Egalitarian and Infinite Paretian SWF.*

Proof Suppose that W satisfies the hypotheses of the Theorem. Call $\widetilde{W}$ the restriction of W to the set $\{a, b\}^{\mathbb{N}}$, where $a < b$ are two arbitrary points in A. Then, $\widetilde{W}$ has to satisfy as well the conditions of the Theorem. Now Proposition 3.4 implies that, for any x and y in $\{a, b\}^{\mathbb{N}}$, if $x \prec_Q y$ one has $\widetilde{W}(x) < \widetilde{W}(y)$. We will see that this is impossible. Assuming without loss of generality that $a = 0$ and $b = 1$, then $X = \{0, 1\}^{\mathbb{N}}$.

The proof of Theorem 2.3 follows as well in this case, in fact, in an easier way. Recall that, using Sierpinski's immersion, we assigned to every $r \in (0, 1)$ a sequence $i(r) \in X$. We then constructed a sequence $i(r)^+$ satisfying $i(r) \prec_{\mathcal{I}} i(r)^+$ and hence $i(r) \prec_Q i(r)^+$. Now, for any s with $r < s < 1$, we proved that $i(r)^+_m > i(s)_m$ for just finitely many values of m. Moreover, $i(r)^+_j < i(s)_j$ for infinitely many j. Then, $i(r)^+ \prec_Q i(s)$ and, hence, $\widetilde{W}(i(r)) < \widetilde{W}(i(r)^+) < \widetilde{W}(i(s))$, arriving to a contradiction.

As any Weak Egalitarian Strong Paretian SWF is automatically Weak Egalitarian Infinite Paretian, the following result follows.

Corollary 3.6 *There does not exist a Weak Egalitarian and Strong Paretian SWF.*

4 Concluding Remarks.

It is proved in Basu and Mitra [4] that there does not exist any social welfare function which satisfies the Strong Pareto and intergenerational equity axioms. When they weaken the Strong Pareto axiom and arrive to a weaker version which they call the Dominance axiom, they recognize in page 1561 that *"It is not as if we wish to recommend the use of such a weak form of the Pareto condition..."*. So, it was left open the following problem: is it possible to have another weaker and economically acceptable version of Pareto axiom for which the impossibility result follows? Inspired in Aumann's model of competitive markets with a continuum of agents, we propose a modification of the Pareto axiom aiming the negligibility of the utility of a single generation. In the same way, Lauwers [11] criticizes the relevance of a single generation, which is the key point of the strong Pareto axiom.

On the other hand, Zame [15] introduces a notion of equity for strict preferences. We follow this concept in order, first, to weaken the hypothesis of Basu–Mitra's result and, second, to give an idea of equity which is compatible with the impossibility of a single generation having veto power.

In consequence, our aim is to reflect in the axioms an ordering where the welfare of the whole society is not necessarily modified by a change in a single generation (and for extension, of any finite number of generations), but increments in infinitely many generations imply to increase the level of welfare if no generation has a loss.

Our line of attack is twofold: (a) to introduce a weaker version of the Strong Pareto principle (that we call the Infinite Pareto principle), and (b) to adapt Zame's weak equity condition. We have shown that, even under these hypotheses, the impossibility result of Basu and Mitra [4] remains. We hope that our hypotheses will be mathematically simpler and economically more useful than Basu–Mitra's Dominance axiom.

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