CORRIGENDUM TO "EXISTENCE AND UNIQUENESS OF SOLUTIONS TO THE BELLMAN EQUATION IN THE UNBOUNDED CASE"

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We thank Janusz Matkowski and Andrzej S. Nowak for pointing out an error in Proposition 1 and providing a counterexample[1]. In the aforementioned result, the metric $d(f,g)=\sum_{j=1}^{\infty}2^{-j}\frac{d_j(f,g)}{1+d_j(f,g)}$ considered is not a contraction on A, a bounded subset of $\mathcal{C}(X)$. However, a slight modification of d restores the result, once the condition $\sup_{j\geq 1}\beta_j=\beta<1$ is imposed. This last condition is harmless in our setting, since all the applications we give satisfy this assumption.

Given a bounded set A with $\sup_{f,g\in A} d_j(f,g)\leq m_j$ (without loss of generality it can be considered $m_j>0$ for each j), consider the metric

$$d_A(f,g)=\sum_{j=1}^{\infty}2^{-j}\frac{d_j(f,g)}{m_j+d_j(f,g)}.$$

This metric turns A into a complete metric space. The correct statement of Proposition 1 is then as follows.

Proposition 1: Let $T: C(X)\longmapsto C(X)$ be a 0–LC relative to A, a bounded subset of $C(X)$ with bounds $\{m_j\}$, such that $\sup\beta_j=\beta<1$. Then there exists $\alpha\in[0,1)$ such that

$$d_A(Tf,Tg)\leq\alpha\, d_A(f,g)\qquad\text{for all } f,g\in A.$$

Proof. For $f,g\in A$, it follows

$$d_A(Tf,Tg)\leq\sum_{j=1}^{\infty}2^{-j}\,\frac{\beta_j d_j(f,g)}{m_j+\beta_j d_j(f,g)}\leq\sum_{j=1}^{\infty}2^{-j}\,a_j\,\frac{d_j(f,g)}{m_j+d_j(f,g)},$$

where $a_j=\beta_j(m_j+d_j(f,g))/(m_j+\beta_j\, d_j(f,g))$. Obviously, $a_j\leq 2\beta_j/(1+\beta_j)$ for each j, since A is bounded and $(m_j+x)/(m_j+\beta_j x)$ is increasing with respect to x. Thus, $a_j\leq 2\beta/(1+\beta)<1$ for each j, and hence $d_A(Tf,Tg)\leq\alpha\, d_A(f,g)$ for $\alpha=2\beta/(1+\beta)$. *Q.E.D.*

In the reading of the proofs of Theorems 1, 3 and 6 and Proposition 3, the metric d should be changed by the metric d_A for the results to hold. None of the remaining results are affected.

[1] Matkowski and Nowak (2008) apply our concept of *k-Local Contraction* to stochastic dynamic programming.

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