Trends in distributional characteristics: Existence of global warming *

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Abstract

What type of global warming exists? This study introduces a novel methodology to answer this question, which is the starting point for all issues related to climate change analyses. Global warming is defined as an increasing trend in certain distributional characteristics (moments, quantiles, etc.) of global temperatures, in addition to simply examining the average values. Temperatures are viewed as a functional stochastic process from which we obtain distributional characteristics as time series objects. Here, we present a simple robust trend test and prove that it is able to detect the existence of an unknown trend component (deterministic or stochastic) in these characteristics. Applying this trend test to daily temperatures in Central England (for the period 1772–2017) and to global cross-sectional temperatures (1880–2015), we obtain the same strong conclusions: (i) there is an increasing trend in all distributional characteristics (time series and cross-sectional), and this trend is larger in the lower quantiles than it is in the mean, median, and upper quantiles; (ii) there is a negative trend in the characteristics that measure dispersion (i.e., lower temperatures approach the median faster than higher temperatures do). This type of global warming has more serious consequences than those found by analyzing only the average.

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1 Introduction

According to the Intergovernmental Panel on Climate Change (IPCC), the study of climate change, and particularly global warming (GW), involves a careful analysis of the following four issues or questions, which form a chain (see IPCC, 2014): (i) What type of GW exists?; (ii) causes of GW (is GW caused by human activities?); (iii) economic effects of GW; and (iv) economic policies to mitigate these effects. Obviously, to determine the type of GW is crucial for the next issues in the chain. The purpose of this paper is to offer a complete answer to the first question by analyzing the characteristics of the existent GW. We investigate these characteristics by introducing a novel methodology to analyze trends. This methodology is also valid for quantitative analyses of many other important economic issues that require a thorough study of trend behaviors (e.g., trends in GDP, debt, inequality, etc.).

We start by defining GW as an increasing trend in global temperatures. In this study, a trend is understood in a broader sense than is currently accepted in the literature (see White and Granger, 2011). As such, we look for trends in the characteristics (moments, quantiles, etc.) of the temperature distribution, and do not simply focus on average values. For instance, a random walk has a trend in the variance, but not in the mean. Furthermore, the average temperature might not show any growth pattern, but the lower tail might show a clear increase. According to the standard definition in the literature, this would not be interpreted as GW, but using the proposed methodology, it clearly would. Even when the average shows some growth, having a wider view of the "trending" behavior of the whole distribution will help in the analysis of the remaining three questions in the chain.

There is an extensive body of literature that analyzes the trend behavior (deterministic and stochastic) of the mean of the temperature distribution (see Harvey and Mills, 2003; Hendry and Pretis, 2013; Gay-García et al., 2009; Mills, 2010; Kauffmann et al., 2006, 2010, 2013; Estrada et al., 2013; Chang et al., 2015, etc.) This approach corresponds to the standard popular definition of climate: climate is the average of weather. In contrast, our proposed method for analyzing trends in the distributional characteristics agrees more with the definition adopted by climatologists: climate is the statistics of weather. This definition (see the IPCC 2014 glossary of definitions) includes not just the average, but also statistics on variability, tail behavior, and so on.

For the purpose of this research, global temperatures are treated as a functional stochastic process, $X = (X_t(\omega), t \in T)$, where T is an interval in $\mathbb{R}$, defined on a probability space $(\Omega, \Im, P)$, such that $t \to X_t(\omega)$ belongs to some function space $\mathbf{G}$, for all $\omega \in \Omega$. Here, X defines a $\mathbf{G}$-valued stochastic process. Note that $\mathbf{G}$ can be a Hilbert space, as in Bosq (2000) (AR-H model for sequences of random Hilbert functions $X_1(\omega), X_2(\omega), ..., X_T(\omega)$), Park and Qian (2012), and Chang et al. (2015, 2016) (regression models for sequences of random state densities $f_1(\omega), f_2(\omega), ..., f_T(\omega)$). Alternatively, it can be a Banach space for sequences of random state distributions $(F_1(\omega), F_2(\omega), ..., F_T(\omega))$. Instead of modeling the whole sequence of $\mathbf{G}$ functions, as previous authors do, we present an alternative approach where we model certain characteristics, C_t, of these functions: the state mean, the state variance, the state quantiles, and so on. The main advantage of this approach, apart from its simplicity, is that these characteristics become time series objects. Therefore, we can apply existing tools used in the time series literature for modeling, inference, forecasting, and so on. This alternative proposal resembles the quantile curve estimation approach of Draghicescu et al. (2009), as well as the realized volatility modeling of high-frequency data in financial econometrics (see Andersen et al., 2003, 2006).

We assume that at each period t, we have N observations from higher-frequency time series or from cross-sectional units. From these observations, we obtain relevant characteristics, which we convert into time series objects. In order to detect trend behavior in these characteristics, we test $\beta = 0$ in the following simple least squares (LS) regression: $C_t = \alpha + \beta t + u_t$. This regression needs to be understood as the best linear LS approximation to an unknown trend function (see White, 1980). We prove that the *t-test* ($\beta = 0$) is able to detect the standard deterministic trends used in the literature (see Davis, 1941), as well as stochastic trends generated by long-memory, near-unit-root, and local-level models (see Müeller and Watson, 2008).

In order to show the generality of our results, we implement two applications: one with N time series observations for each year t, and another with N cross-sectional observations, also for each year t. The first application studies the trend behavior of the distributional characteristics of temperature in Central England from January 1, 1772, to October 31, 2017. To ensure the robustness of our findings, we also present results for temperatures in other locations (Stockholm, Cadiz, and Milan). In the second application, we analyze global temperatures across different stations in the Northern and Southern Hemispheres for the period 1880–2015. The two applications lead to similar trend results, which can be summarized as follows. First, there exists a trend in most of the characteristics considered. The trend in the lower quantiles is stronger than those in the mean and upper quantiles of the temperature distribution (the IPCC 2014 reports a decrease in cold temperature extremes and an increase in warm temperature extremes). Second, dispersion measures such as the interquartile range (iqr), standard deviation (std), and $range$ ($max - min$) show a negative trend (a possible cause for this fact is suggested in Arrhenius, 1896). Therefore, we conclude that GW is not only a phenomenon of an increase in the average temperature, but also of a larger increase in lower temperatures, leading to decreased dispersion. Ignoring these facts could have serious consequences for climate analyses (e.g., an acceleration in global ice melting) and, therefore, they should be considered in all future international climate agreements. Present agreements focus only on the mean characteristic.

The rest of the paper is organized as follows. In Section 2, we define GW and the trends we use to investigate GW. In Section 3, we present our basic framework for the time series analysis. In Section 4, we introduce and analyze our proposed trend test (TT) to detect a general unknown trend behavior in any distributional characteristic. Section 5 provides two empirical applications: using a purely temporal dimension (local daily temperature on an annual basis), and using a cross-sectional dimension (global temperatures measured annually, by station). Finally, Section 6 concludes the paper. The Appendix contains detailed proofs of the main results, as well as the finite-sample performance of our proposed test and additional empirical results.

2 Global Warming and Trends

In this section, we introduce our definition of GW, as well as the definition of trend that we use to characterize the type of existent GW.

Definition 1. *(Global warming)*: Global warming is defined as the existence of an increasing trend in some of the characteristics measuring the central tendency or position (quantiles) of the global temperature distribution.

Under this definition, the existence of a trend in other types of characteristics like, for instance, those measuring dispersion or symmetry will not constitute warming but clearly can help to describe it. As mentioned in the Introduction, this definition agrees with the climatologist definition of climate: the statistics of weather. As such, it includes not just the average temperature, but also its distributional behavior. The key issue is to find a useful definition and characterization of a trend. Surprisingly, not many statistical or econometric books dedicate a chapter to this topic. As noted by Phillips (2005), this may be because "[n]o one understands trends, but everyone sees them in the data." Exceptions to this include books by Davis (1941), Anderson (1971), and Kendall and Stuart (1983). However, even these do not provide a definition or characterization of a trend that would be useful for our GW analysis. Instead, a useful definition is provided in White and Granger (2011) (WG): (i) a trend should have a direction; (ii) a trend should be basically smooth; (iii) a trend does not have to be monotonic throughout; and (iv) a trend can be a local behavior (observed trends can be related to a particular section of data). These characterizations are formalized by WG in the following two definitions, one for deterministic trends and the other for stochastic trends.

Definition 2. *(Deterministic trend (WG, 2011))*: Let $\{C_t\} = \{C_t : t = 0, 1, ...\}$ be a sequence of real numbers. If $C_t < C_{t+1}$ for all t, then $\{C_t\}$ is a strictly increasing trend. If $C_t \leq C_{t+1}$ for all t and there exists a countable subsequence C_{t_j} such that C_{t_j} is a strictly increasing trend, then C_t is an increasing trend. If $\{-C_t\}$ is a strictly increasing (an increasing) trend, then $\{C_t\}$ is a strictly decreasing (a decreasing) trend.

If definition 2 is only satisfied for all $t_1 \leq t < t_2$ then C_t is a strictly increasing (or an increasing) local trend in $[t_1, t_2]$.

Example of a deterministic trend: A polynomial trend for certain values of the β parameters $C_t = \beta_0 + \beta_1 t + \beta_2 t^2 + ... + \beta_k t^k$.

Additional examples can be found in Chapters 1 and 6 of Davis (1941) and in WG.

Definition 3. *Stochastic trend (WG, 2011))*: Let X_t be a stochastic process.

- Consider $C_t = E(X_t)$. If C_t is a strictly increasing (an increasing) trend, then $\{X_t\}$ has a strictly increasing (an increasing) trend in the mean.
- Let $C_t = E(|X_t - E(X_t)|^k)$, for finite positive real k. If C_t is a strictly increasing (an increasing) trend, then $\{X_t\}$ has a strictly increasing (an increasing) trend in the k^{th} absolute central moment.
- Let $C_t(p) = inf\{x \epsilon \mathbb{R} : F_t(x) \geq p\}$ be the quantile $p \in (0, 1)$ of the distribution function $F_t(x) = P(X_t \leq x)$. If $C_t(p)$ is strictly increasing (an increasing) trend, then $\{X_t\}$ has a strictly increasing (an increasing) trend in quantile p.

Examples of stochastic trends:

- A random walk $X_t = X_{t-1} + u_t$ has a trend in the variance, but not in the mean.

- A random walk with drift $X_t = \alpha + X_{t-1} + u_t$ has a trend in the mean and in the variance.

More examples can be found in Müeller and Watson (2008).

Note that, from Definition 3, the concept of a stochastic trend considered in the econometrics literature now becomes a pure deterministic trend in the second moment of the distribution. This implies that by developing a method able to detect deterministic trends, and applying this method to different distributional characteristics, we can detect any type of trend. This method is introduced in Section 4. First, in the next section, we present the basic framework for our proposed time series analysis, which we use to obtain the distributional characteristics as time series objects.

3 Basic Framework for a Time Series Analysis

In this study, temperature is viewed as a functional stochastic process, $X = (X_t(\omega), t \in T)$, where T is an interval in $\mathbb{R}$, defined on a probability space $(\Omega, \Im, P)$, such that $t \to X_t(\omega)$ belongs to some function space $\mathbf{G}$, for all $\omega \in \Omega$. Here, X defines a $\mathbf{G}$-valued stochastic process.

This function space $\mathbf{G}$ is equipped with a scalar product $< ., . >$ and/or a norm $\| . \|$, and a Borel σ-algebra, B_G. It is separable and complete. Thus, $\mathbf{G}$ can be a Hilbert space, as in Bosq (2000) (AR-H model for sequences of random Hilbert functions $X_1(\omega), X_2(\omega), ..., X_T(\omega)$), Park and Qian (2012), and Chang et al. (2015, 2016) (regression models for sequences of random state densities $f_1(\omega), f_2(\omega), ..., f_T(\omega)$), a Banach space for a sequence of random state distributions $(F_1(\omega), F_2(\omega), ..., F_T(\omega))$, etc.

A convenient example of an infinite-dimensional discrete-time process is that of associating a sequence of random variables with values in an appropriated function space, where $\xi = (\xi_n, n \in \mathbb{R}_+)$. This may be obtained by setting

$$X_t(n) = \xi_{tN+n},\ 0 \leq n \leq N,\ t = 0, 1, 2, ..., T. \tag{1}$$

Thus, $X = (X_t, t = 0, 1, 2, ..., T)$. If the sample paths of ξ are continuous, then we have a sequence $X_0, X_1, ...$ of random variables in the space $C[0, N]$. The choice of the period or segment t is compelling in many specific situations. In our case, t denotes the year, and N can represent temporal or cross-sectional observations.

We may wish to model the whole sequence of **G** functions, for instance, the sequence of state densities $(f_1(\omega), f_2(\omega), ..., f_T(\omega))$, as in Chang et al. (2015, 2016). Alternatively, we may wish to model only certain characteristics $(C_t(w))$ of these **G** functions, for instance, the state mean, state variance, state quantile, and so on. These characteristics can be considered as time series objects, which means we can apply existing econometrics tools to $C_t(w)$. For this reason, we follow the second option, which resembles the quantile curve estimation analyzed in Draghicescu et al. (2009) and in Zhou and Wu (2009). In terms of the variance characteristic, it also resembles the literature on realized volatility (Andersen et al., 2003, 2006). Using this characteristic approach, we move from Ω to $\mathbb{R}^T$, as in a standard stochastic process, passing through a **G** functional space:

$$\underset{(w)}{\Omega} \xrightarrow{X} \underset{X_t(w)}{\mathbf{G}} \xrightarrow{C} \underset{C_t(w)}{\mathbb{R}}.$$

Returning to the convenient example and abusing the notation, the stochastic structure can be summarized in the following array:

$X_{10}(w) = \xi_0(w)$	$X_{11}(w) = \xi_1(w)$	...	$X_{1N}(w) = \xi_N(w)$	$C_1(w)$
$X_{20}(w) = \xi_{N+1}(w)$	$X_{21}(w) = \xi_{N+2}(w)$	...	$X_{2N}(w) = \xi_{2N}(w)$	$C_2(w)$
.	.	...	.	.
.	.	...	.	.
.	.	...	.	.
$X_{T0}(w) = \xi_{(T-1)N+1}(w)$	$X_{T1}(w) = \xi_{(T-1)N+2}(w)$	...	$X_{TN}(w) = \xi_{TN}(w)$	$C_T(w)$.

(2)

Throughout this paper, similarly to the assumptions made in Park and Qian (2012) and Chang et al. (2016), we assume that in each period, t, there are sufficient temporal or cross-sectional observations $(N \to \infty)$ for these characteristics to be estimated consistently.

Assumption 3.1. In each period, t, the stochastic functional process $X = (X_t(\omega), t \in T)$ satisfies certain regularity conditions, such that the state densities, distribution and, therefore, quantiles are estimated consistently.

In the temporal framework, local stationarity (Dahlhaus, 2009) plus some strong mixing conditions (Hansen, 2008) are sufficient to obtain uniform ““strong”” consistency for suitable regular kernel estimators of the state densities. Local stationarity plus some φ-mixing conditions (see Degenhardt et al. 1996) are sufficient for the central limit theorem to hold for smoothed empirical distribution functions and for smoothed sample quantiles for each period or segment, t. For the cross-sectional situation, similar results hold (e.g., Silverman, 1978) if the state distributions are defined as cross-sectional distributions, and if independent and identically distributed observations are available to estimate them for each period (for dependency among N observations, see Bosq (1998, Thm 2.2) and for clustered data, see Breunig (2001)).

4 Testing for a Trend

The objective of this section is to provide a simple test to detect the existence of a general unknown trend component in a given characteristic C_t of X_t. To do this, we need to convert Definition 3 into a more practical definition.

Definition 4. *(Practical definition 1)*: Let $h(t)$ be an increasing function of t. A characteristic C_t of a functional stochastic process X_t contains a trend if $\beta \neq 0$ in the regression

$$C_t = \alpha + \beta h(t) + u_t, \ t = 1, ..., T. \tag{3}$$

This definition has its natural local trend version. From this definition, two questions arise that need to be answered. First, we need to specify which function $h(t)$ to use in regression (3), and second, we have to design a proper test for the null hypothesis of interest, $\beta = 0$. Before resolving these two questions, the practical Definition 4 requires some preliminary concepts and results, in particular, the concept of *summability* (see Berenguer-Rico and Gonzalo 2014, for a stochastic version).

Definition 5. *(Order of Summability)*: A trend $h(t)$ is said to be summable of order "δ" ($S(\delta)$) if there exists a slowly varying function $L(T)$,[1] such that

$$S_T = \frac{1}{T^{1+\delta}} L(T) \sum_{t=1}^{T} h(t) \tag{5}$$

is $O(1)$, but not $o(1)$.

The following examples illustrate the order of summability:

[1] A positive Lebesgue measurable function, L, on $(0, \infty)$ is slowly varying (in Karamata's sense) at ∞ if

$$\frac{L(\lambda n)}{L(n)} \to 1 \ (n \to \infty) \ \forall \lambda > 0. \tag{4}$$

(See Embrechts et al., 1999, p. 564).

Example 4.1. Let $h(t) = c(\neq 0)$. Then, $\frac{1}{T}\sum_{t=1}^{T} c = c$. Therefore, $\delta = 0$.

Example 4.2. Let $h(t) = t^k$. Then, $\frac{1}{T^{1+k}}\sum_{t=1}^{T} t^k = O(1)$. Therefore, $\delta = k$.

Example 4.3. Let $h(t) = e^{\lambda t}$. Then, $\frac{1}{e^{\lambda T}}\sum_{t=1}^{T} e^{\lambda t} = O(1)$. Therefore, $\delta_T = \frac{\lambda T}{Log(T)} - 1 \to \infty$, as $T \to \infty$.

Example 4.4. Let $h(t) = \frac{K}{1+Be^{-\lambda t}}$. Then, $\frac{1}{T}\sum_{t=1}^{T} \frac{K}{1+Be^{-\lambda t}} = O(1)$. Therefore, $\delta = 0$.

Example 4.5. Let $h(t) = Log(t)$. Then, $\frac{1}{TLog(T)}\sum_{t=1}^{T} Log(t) = O(1)$. Therefore, $\delta = 0$.

Example 4.6. Let $h(t) = \frac{1}{t}$. Then, $\frac{1}{Log(T)}\sum_{t=1}^{T} \frac{1}{t} = O(1)$. Therefore, $\delta = -1$.

The properties of the OLS estimator $\widehat{\beta}$ in regression (3) depend on the balance between the trend components of the dependent variable C_t and the regressor $h(t)$. To characterize this balance, we need the following definition.

Definition 6. *(Trend strength)*: A trend function $h(t)$ is said to be stronger than another trend function $g(t)$ if $\delta_h > \delta_g$.

Now, using Definitions 5 and 6, we have all the necessary elements to present our *trend test* to detect a general trend component in a given characteristic $C_t = h(t) + I(0)$,[2] with $h(t)$ unknown. First, we recall a well-known related result (see Hamilton, 1994, Chapter 16):

Proposition 1. *Let $C_t = I(0)$. In the LS regression*

$$C_t = \alpha + \beta t + u_t, \tag{6}$$

the OLS estimator satisfies

$$T^{3/2}\widehat{\beta} = O_p(1) \tag{7}$$

and asymptotically ($T \to \infty$)

$$t_{\beta=0} \text{ is } N(0,1).$$

Proposition 2. *Let $C_t = h(t) + I(0)$, such that $h(t)$ is an increasing $S(\delta)$ function with $\delta \geq 0$, and the function $g(t) = h(t)t$ is $S(\delta + 1)$. In the LS regression*

$$C_t = \alpha + \beta t + u_t, \tag{8}$$

[2] Our definition of an I(0) process follows Johansen (1995). A stochastic process Y_t that satisfies $Y_t - E(Y_t) = \sum_{i=1}^{\infty}\Psi_i\varepsilon_{t-i}$ is called I(0) if $\sum_{i=1}^{\infty}\Psi_i z^i$ converges for $|z| < 1+\delta$, for some $\delta > 0$ and $\sum_{i=1}^{\infty}\Psi_i \neq 0$, where the condition $\varepsilon_t \sim$ iid(0,σ^2) with $\sigma^2 > 0$ is understood.

the OLS $\widehat{\beta}$ estimator satisfies

$$T^{(1-\delta)}\widehat{\beta} = O_p(1). \tag{9}$$

In order to analyze the behavior of the t-statistic $t_\beta = 0$, we assume that the function $h(t)^2$ is $S(1+2\delta-\gamma)$, with $0 \leq \gamma \leq 1+\delta$. Then, the t-statistic diverges at the following rates

$$t_{\beta=0} = \begin{cases} O_p(T^{\gamma/2}) \text{ for } 0 \leq \gamma \leq 1 \\ O_p(T^{1/2}) \text{ for } 1 \leq \gamma \leq 1+\delta. \end{cases} \tag{10}$$

Proof in Appendix A.

The following examples illustrate how to use Proposition 2:

Example 4.7. Let $C_t = h(t) + I(0)$, with $h(t) = t^2$. The summability parameters are $\delta = 2$ and $\gamma = 1$. Then, in regression (8), $\widehat{\beta}$ and $t_{\beta=0}$ diverges as $T \to \infty$.

Example 4.8. Let $C_t = h(t) + I(0)$, with $h(t) = t$. The summability parameters are $\delta = 1$ and $\gamma = 1$. Then, in regression (8), $\widehat{\beta} = O_p(1)$; but, $t_{\beta=0}$ diverges as $T \to \infty$.

Example 4.9. Let $C_t = h(t) + I(0)$, with $h(t) = t^{1/2}$. The summability parameters are $\delta = 1/2$ and $\gamma = 1$. Then, in regression (8), $\widehat{\beta} \xrightarrow{p} 0$; but, $t_{\beta=0}$ diverges as $T \to \infty$.

Example 4.10. Let $C_t = h(t) + I(0)$, with $h(t) = log(t)$. The summability parameters are $\delta = 0$ and $\gamma = 1$. Then, in regression (8), $\widehat{\beta} \xrightarrow{p} 0$; but, $t_{\beta=0}$ diverges as $T \to \infty$.

Example 4.11. Let $C_t = h(t) + I(0)$, with $h(t) = e^{\lambda t}$. The summability parameters are $\delta_T = \frac{\lambda T}{Log(T)} - 1$ and $\gamma = 0$. Then, in regression (8), $\widehat{\beta}$ *diverges* and $t_{\beta=0} = O_p(1)$ as $T \to \infty$. It can be proved that, asymptotically, $t_{\beta=0} \geqslant \mathbf{z}_{0.95}$ for $\lambda \in (0, 2.095)$.

Example 4.12. Let $C_t = h(t) + I(0)$, with $h(t) = \frac{K}{1+Be^{-\lambda t}}$. The summability parameters are $\delta = 0$ and $\gamma = 1$. Then, in regression (8), $\widehat{\beta} \xrightarrow{p} 0$; but, $t_{\beta=0}$ diverges as $T \to \infty$.

A question of great empirical importance is how does our trend test of Proposition 2 behave when $C_t = I(1)$. Following Durlauf and Phillips (1988), $T^{1/2}\widehat{\beta} = O_p(1)$; however, $t_{\beta=0}$ diverges as $T \to \infty$. Therefore, our trend test can detect the stochastic trend generated by an I(1) process. In fact, our test will detect trends generated by any of the three standard persistent processes considered in the literature (see Muller and Watson, 2008): (i) fractional or long-memory models; (ii) near-unit-root AR models; and (iii) local-level models. Let

$$C_t = \mu + z_t,\ t = 1, ..., T. \tag{11}$$

In the first model, z_t is a fractional process with $1/2 < d < 3/2$. In the second model, z_t follows an AR, with its largest root close to unity, $\rho_T = 1 - c/T$. In the third model, z_t is decomposed into an I(1) and an I(0) component. Its simplest format is $z_t = \upsilon_t + \epsilon_t$ with $\upsilon_t = \upsilon_{t-1} + \eta_t$, where ϵ_t is $ID(0, q * \sigma^2)$, η_t is $ID(0, \sigma^2)$, $\sigma^2 > 0$ and both disturbances are serially and mutually independent. Note that the pure unit-root process is nested in all three models: $d = 1$, $c = 0$, and $q = 0$.

The long-run properties implied by each of these models can be characterized using the stochastic properties of the partial sum process for z_t. The standard assumptions considered in the macroeconomics or finance literature assume the existence of a "δ," such that $T^{-1/2+\delta} \sum_{t=1}^{T} z_t \longrightarrow \sigma\ H(.)$, where "$\delta$" is a model-specific constant and H is a model-specific zero-mean Gaussian process with a given covariance kernel $k(r, s)$. Then, it is clear that the process $C_t = \mu + z_t$ is summable (see Berenguer-Rico and Gonzalo, 2014). This is the main reason why Proposition 3 holds for these three persistent processes.

Proposition 3. *Let $C_t = \mu + z_t, t = 1, ..., T$, with z_t any of the following three processes: (i) a fractional or long-memory model, with $1/2 < d < 3/2$; (ii) a near-unit-root AR model; or (iii) a local-level model. Furthermore, $T^{-1/2+\delta} \sum_{t=1}^{T} z_t \longrightarrow \sigma$ $H(.)$, where "δ" is a model-specific constant and H is a model-specific zero-mean Gaussian process with a given covariance kernel $k(r, s)$. Then, in the LS regression*

$$C_t = \alpha + \beta t + u_t,$$

the t-statistic diverges,

$$t_{\beta=0} = O_p(T^{1/2}).$$

Proof in Appendix A.

In summary, Propositions 2 and 3 imply that Definition 4 can be simplified to the following practical definition.

Definition 7. *(Practical definition 2)*: A characteristic C_t of a functional stochastic process X_t contains a trend if in the LS regression,

$$C_t = \alpha + \beta t + u_t,\ t = 1, ..., T, \tag{12}$$

$\beta = 0$ is rejected.

Several remarks are relevant with respect to this definition: (i) regression (12) has to be understood as the linear LS approximation of an unknown trend function

$h(t)$ (see White, 1980); (ii) the parameter β is the plim of $\widehat{\beta}_{ols}$; (iii) if the regression (12) is the true data-generating process, with $u_t \sim I(0)$, then the OLS $\widehat{\beta}$ estimator is asymptotically equivalent to the GLS estimator (see Grenander and Rosenblatt, 1957); and (iv) in practice, in order to test $\beta = 0$, it is recommended to use a robust HAC version of $t_{\beta=0}$ (see Busetti and Harvey, 2008).

For all these reasons, in the empirical applications we implement Definition 7 by estimating regression (12) using OLS and constructing a HAC version of $t_{\beta=0}$ (Newey and West, 1987). Appendix B includes a detailed analysis of the finite-sample performance of this test for several types of the most common deterministic and stochastic trends. Alternative estimation and testing procedures for certain types of polynomial deterministic trends can be found in Canjels and Watson (1997) and in Vogelsang (1998).

5 Local and Global Warming: Time Series and Cross-sectional Data

We begin this section by recalling the type of data structure we analyze in order to answer the first question of any climate change study: which type of GW exists? Following the convenient example (see (2) in Section 3), X is a local or global temperature, T (number of periods) is measured in years, N has a temporal structure (days) or a cross-sectional dimension (stations in both hemispheres), and $C_t = (C_{1t}, C_{2t}, ..., C_{pt})$ is a vector of p distributional characteristics (mean ($mean$), maximum (max), minimum (min), standard deviation (std), interquartile range (iqr), total range ($range$), kurtosis (kur), skewness (skw), and the following quantiles: $q5$, $q10$, $q20$, $q30$, $q40$, $q50$, $q60$, q70, $q80$, $q90$, and $q95$ estimated from N observations.

In this section, we implement our trend test (Propositions 2 and 3) on two types of data: (i) time series data (local Central England temperatures, for N days and T denoting the period 1772–2017); and (ii) cross-sectional data (global Earth temperature, for N stations and T denoting the period 1880–2015).

In the rest of the section, we describe our data and a unit-root analysis, and apply our trend test (TT) to detect the existence of local and/or global warming.

5.1 Time series data: Local warming

The longest temperature record series (thermometer measured) runs from 1659 to the present. These data are measured monthly and annually for England. There are also daily temperature data that have been measured since 1772. However, there are no instrumental data prior to 1659 because the thermometer was only invented a few decades earlier. These data were originally published by Gordon Manley in 1953 in a database called The Central England Temperature (CET), which have provided monthly mean surface air temperatures for the Midlands region of England, measured in degrees Celsius, since 1659 (Manley, 1953, 1974). Parker et al. (1992) built a daily version of the database from 1772 to the present day, which is updated continuously. They evaluate recent urban warming influences and correct the series after 1974.[3] Figure 1 shows the annual, monthly, and daily versions of the data.[4]

The advantages of the CET climate database are its length and its high frequency. In particular, having daily observations for each year (1772–2017) allows us to compute the distributional characteristics of interest and convert them into time series objects. Figure 2 presents the annual densities from 1772 to 2017, and Figure 3 shows the path of these characteristics.

More recently, a European Union research project (IMPROVE) studied past climatic variability using early daily European instrumental sources. This project collected records of temperatures in different European areas, from the Baltic to the Mediterranean and from the Atlantic to Eastern Europe. IMPROVE's general objectives were to assess correction and homogenization protocols for early daily instrumental records of air temperature and air pressure, but the quality and continuity of the series are highly heterogeneous, and only the Swedish series (Stockholm) continued to be updated by Anders Moberg.[5] In addition to Stockholm, we analyze data from Cadiz and Milan to test the robustness of our CET results. The density of the data (Figure 8) and the econometric results for these additional data are shown in Appendix C.

[3]See Parker et al. (1992), Manley (1953, 1974), and Parker and Horton (2005) for further information on these series. The data are available from http://www.metoffice.gov.uk/hadobs/hadcet/.

[4]An analysis of the statistical properties of the annual and monthly averages of these data can be found in Harvey and Mills (2003) and in Proietti and Hillebrand (2016).

[5]They can be obtained from the Bolin Center for Climate Research: http://bolin.su.se/data/stockholm/.

5.1.1 Results

Before testing for the presence of trends in the distributional characteristics of the CET data, we test for the existence of unit roots. To do so, we use the well-known Augmented Dickey-Fuller test (ADF; Dickey and Fuller, 1979), where the number of lags is selected in accordance with the SBIC criterion. The results in Table 1 show that the null hypothesis of a unit root is rejected for all the characteristics considered.

We test for the presence of a trend in temperature characteristics by applying the proposed TT in regression (12). Table 2 reports the OLS trend slopes and a HAC $t_{\beta=0}$ (p-values from a $N(0,1)$). The latter indicates a significant trend in all of the characteristics. The trends are all positive, except those corresponding to the dispersion measures (*std*, *iqr*, *range*), which are negative. The mean has a trend coefficient of 0.0038, which implies an increase of 0.4 degrees Celsius in 100 years. The highest positive trends occur in the lower quantiles. The trend coefficient of the quantiles ranges from 0.0072 in the 5% quantile ($q5$) to 0.0013 in the 80% quantile ($q80$). Note that this test only confirms the existence of a trend, but says nothing about its nature. This would require an additional empirical study that determines the most suitable type of trend. However, this is beyond the scope of this study, and forms part of on-going research.

These results illustrate the usefulness of our proposed methodology in terms of analyzing a wide set of distributional characteristics of the temperature instead of the mean only. To strengthen this idea, we test for co-trending in different sets of characteristics. The results of a Wald test for different co-trending possibilities appear in Table 3. The null hypothesis of co-trending in all quantiles is rejected. Nevertheless, this is not the case if we test the null of equal trends in groups of quantiles, namely, the lower, medium, and upper quantiles. Finally, to complete this study, we test for the existence of a trend in important spacing characteristics, and find that the difference between the lowest quantile ($q5$) and the median shows a decreasing and significant trend. However, the difference between the highest quantile ($q95$) and the median does not show a statistically significant trend. The minimum temperatures approach the median more rapidly than the maximum temperatures do. This is corroborated by a negative trend in $q_{95} - q_5$, and is in line with the IPCC 2014 summary that reports that winters have warmed more than summers have.

To close this section, we conduct a parallel study using temperature data for the other cities mentioned previously: Stockholm, Cadiz, and Milan (see Figure 8). The results in Table 11 for the unit roots and the trend analysis in Tables 12, 13, and 14 lead to the same conclusions. Summarizing our findings, we have identified patterns in the distributional characteristics of temperatures that are common for different cities with different geographic positions. This infers that this may be a global phenomenon. The next section investigates this conjecture in further detail.

5.2 Cross-sectional data: GW

The Climate Research Unit (CRU) offers monthly and yearly data of land and sea temperatures in both hemispheres from 1850 to the present, collected from different stations around the world.[6] Each station temperature is converted to an anomaly, taking 1961–1990 as the base period,[7] and each grid-box value, on a five-degree grid, is the mean of all the station anomalies within that grid box.[8] This database (in particular, the annual temperature of the Northern Hemisphere) has become one of the most widely used to illustrate GW from records of thermometer readings. These records form the blade of the well-known "hockey stick" graph, frequently used by academics and other institutions, such as, the IPCC. In this paper, we prefer to base our analysis on raw station data (see density in Figure 4). These data show high variability at the beginning of the period, probably due to the few number of stations in this early stage of the project, as noted by Jones et al. (2012). Following

[6] HadCRUT4 is a global temperature data set, providing gridded temperature anomalies across the world, as well as averages for the hemispheres and for the globe as a whole. CRUTEM4 and HadSST3 are the land and ocean components of this overall data set, respectively. These data sets were developed by the Climatic Research Unit (University of East Anglia) in conjunction with the Hadley Centre (UK Met Office), with the exception of the sea surface temperature (SST) data set, which was developed solely by the Hadley Centre. We use CRUTEM version 4.5.0.0, which can be downloaded from (https://crudata.uea.ac.uk/cru/data/temperature/). A recent revision of the methodology can be found in Jones et al. (2012).

[7] To avoid biases from the different elevations of stations, monthly average temperatures are reduced to anomalies from the period with best coverage (1961–1990). Because many stations do not have complete records for the period 1961–1990, they are estimated using neighboring records or using other sources of data.

[8] Today, many other institutions collect climate and temperature data. These include the National Oceanic and Atmospheric Administration (NOAA), which presents daily and monthly raw temperature data, classified by country and station, from 1961 to the present, and the National Aeronautics and Space Administration (NASA) offers raw monthly data for stations, anomalies for countries, and a method of homogenization of station data since 1880. Furthermore, Berkeley University offers raw monthly data for stations and anomalies for countries for land temperatures since 1750, for land and ocean temperatures since 1850, and experimental daily land data since 1880.

these authors, our study period begins in 1880 and ends in 2015 (2016 and 2017 data are still under revision).[9]

The construction of the characteristics deserves a little attention. Although there are over 10,000 stations on record, the effective number fluctuates each year. It reached a minimum in 1850 and a maximum during the period 1951–2010. Furthermore, the geographic distribution of stations is not homogeneous. Coverage is denser over the more populated parts of the world, particularly in the United States, Southern Canada, Europe, and Japan. In contrast, coverage is sparser over the interior of the South American and African continents and over Antarctica. This provokes a disequilibrium between the Northern Hemisphere (NH) and the Southern Hemisphere (SH). To guarantee the stability of the characteristics over the whole sample, we select only those stations with data for all years in the sample period, which forces us to reduce the sample size. Applying this procedure to the sample period 1880–2015, we have N=290 stations. Figure 5 shows where they are situated on a world map, and Figure 6 shows the distributional characteristics as time series objects. These characteristics are constructed from stations' annual averages, calculated using monthly temperature records. Note that a benefit of using stable raw station data is that we always have perfect knowledge of every observation, and can easily detect the origin of any extreme observations or outliers.[10]

This method of building characteristics has consequences that should be mentioned. The mean calculated from the filtered raw data does not match that reported by the CRU, which is calculated as the weighted average of all non-missing, grid-box anomalies in each hemisphere. The weights used are the cosines of the central latitudes of each grid box, and the global average is a weighted average of those of the NH and the SH. These weights are "two" for the NH and "one" for the SH. Therefore, we carry out an additional study using data grids to show that the key results do not change (results available upon request).

In summary, we analyze raw global data (stations instead of grids) for the period

[9] Raw station data can present other homogenization problems. Therefore, in addition to carrying out data cleansing, we investigate in detail the controls imposed by the CRU in order to identify and correct significant inhomogeneities (Jones et al., 2012).

[10] Figure 6 shows four outliers, which are the result of the way in which annual means are constructed. Two Russian stations around latitude 51 (codes 308790 and 313690) have few observations in the central months of the year in 1919 and 1924. For most of the stations considered, there are observations every, or almost every month. Therefore, the annual average is representative. We have verified the robustness of our results in two ways: eliminating the stations causing the outliers, and interpolating the missing values. The results do not change. Therefore, we use the original raw station data.

1880 to 2015. However, for reasons of homogeneity and stability, we use only data from stations that are represented in the whole sample period.

5.2.1 Results

The ADF test rejects the null hypothesis of a unit root for all of the characteristics (see Table 4), with the exception of $q80$. The unit-root analysis has been completed in two ways. First, we applied the ADF test station by station, and counted the number of rejections. Second, we carried out a battery of panel unit-root tests. The results shown in Table 5 reinforce the conclusion of no unit roots in our temperature data. Similar unit-root test results are obtained from stable grid data.[11]

The results change if we include all existing grids in each year (whether they have observations or not during the whole sample). In this case, we cannot reject the null of a unit root in many of the annual characteristics, including the mean.[12] This is consistent with the widespread belief that the global temperature has a unit root, a result that comes from an analysis of the annual mean temperature in the Northern Hemisphere (see Kaufmann et al., 2006, 2010, 2013). Nevertheless, this result is not maintained either at monthly frequency (Global, NH and SH), or individually grid by grid, (84% of the times the unit root is rejected)(results available upon request). Therefore, it seems that the unit root found in some part of the literature can be a consequence of temporal or spatial aggregation that produces artificial persistence (see Taylor, 2001). Other researchers (see Gay-García et al., 2009; Estrada et al., 2013) attribute the non-stationarity to the presence of structural breaks in the deterministic trend.[13] In both cases, TT is able to detect the existence of a trend (see Propositions 2 and 3.)

Finally, we apply TT to the characteristics calculated using the cross-sectional data. The results, displayed in Table 6, lead to the same conclusions obtained from the distributional characteristics of the time series data.[14] This similarity is evident in Figure 7, which compares the trend slope coefficients estimated from the time-

[11]Following the same logic as for our station data, we consider only those grids that are represented in the whole sample period. This yields a total of 160 grids.

[12]Using the same data and following a pure functional approach, Chang et al. (2015) find some evidence of unit-root behavior in some moments. Nevertheless, a panel unit-root test based on all the grids rejects the unit-root hypothesis (results available upon request).

[13]In this study, we have not considered structural breaks because they are model dependent and our approach is not. We focus only on detecting the existence of a trend, not on the nature of the trend.

[14]This similarity can be extended to the analysis of co-trending, (Table 7), although we reject that upper quantiles have the same trend coefficients.

series and the cross-sectional analyses. This finding endorses the behavior patterns of temperature distribution as a global phenomenon. In summary, we find trends in most of the C_{it} $(i = 1, ..., p)$ considered, and they are stronger for the lower quantiles than they are for the mean and upper quantiles. Dispersion measures such as iqr, std, and $range$ show a negative trend. Therefore, we conclude that GW is not only a phenomenon described by an increase in the average temperature, but also one of a larger increase in the lower quantiles, producing a decreasing dispersion.

6 Conclusion

This study proposes a novel approach to modeling the evolution of certain distributional characteristics of a functional stochastic process (moments, quantiles, etc.). This is possible because these distributional characteristics can be obtained as time series objects and, therefore, we can apply existing tools (modeling, inference, forecasting, etc.) available in the time series literature. We present a simple robust trend test that is able to detect unknown trend components in any of these characteristics.

By defining GW as the existence of an increasing trend in the characteristics measuring the central tendency or position (quantiles) of the temperature distribution, testing for a trend is equivalent to testing for the existence of GW, and even more important, for the type of GW we have.

We apply our methodology to two types of data: (i) time series distributional characteristics, measured in Central England; and (ii) the global Earth temperature, with cross-sectional distributional characteristics. In both cases, we obtain the same conclusions: (i) there is a trend component in all the distributional characteristics of interest, and this trend is stronger in the lower quantiles than it is in the mean, median, and upper quantiles; and (ii) the distributional characteristics that capture the dispersion of the temperature have a negative trend (lower quantiles evolve toward the median faster than the upper quantiles do). Therefore, there is clear evidence of local (CET) and global (Earth surface) warming. This warming is stronger in the lower temperatures than in the rest of the distribution. This result can have very serious consequences, such as an acceleration of the ice melting process. Future international climate agreements should consider this, and not focus only on the mean temperature.

Note that the proposed trend test is able to detect the existence of an unknown trend, but not the nature of the trend component. This provides two directions for future research: (i) modeling the correct trend, and developing methods to forecast this trend component; and (ii) finding the causes of these distributional trends (see Arrhenius, 1896).

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7 Tables

Table 1
ADF unit root test (CET data)

Characteristic	ADF-SBIC	p-value	lags
mean	-8.09	0.000	1
max	-14.55	0.000	0
min	-15.13	0.000	0
std	-16.18	0.000	0
iqr	-16.44	0.000	0
range	-16.40	0.000	0
kur	-16.53	0.000	0
skw	-13.42	0.000	0
q5	-14.28	0.000	0
q10	-14.28	0.000	0
q20	-14.27	0.000	0
q30	-8.89	0.000	1
q40	-8.70	0.000	1
q50	-8.42	0.000	1
q60	-4.94	0.000	3
q70	-5.35	0.000	3
q80	-14.72	0.000	0
q90	-14.47	0.000	0
q95	-15.01	0.000	0

Notes: Annual distributional characteristics of temperature from daily Central England data (1772-2017). Lag-selection according to SBIC criterion.

Table 2
Trend test (CET data)

Characteristic	Coeff	p-value
mean	0.0041	0.0000
max	0.0038	0.0027
min	0.0112	0.0000
std	-0.0020	0.0000
iqr	-0.0042	0.0000
range	-0.0074	0.0000
kur	0.0003	0.0552
skw	0.0003	0.0682
q5	0.0073	0.0000
q10	0.0068	0.0000
q20	0.0063	0.0000
q30	0.0055	0.0000
q40	0.0048	0.0000
q50	0.0039	0.0000
q60	0.0028	0.0009
q70	0.0019	0.0127
q80	0.0016	0.0240
q90	0.0019	0.0346
q95	0.0024	0.0145

Notes: Annual distributional characteristics of temperature from daily Central England data (1772-2017). OLS estimates and HAC $t_{\beta=0}$ from regression: $C_t = \alpha + \beta t + u_t$.

Table 3
Co-trending analysis (CET data)

Characteristic	Wald test	p-value
QUANTILES		
All quantiles (q5, q10,...,q90,q95)	44.909	0.000
Lower quantiles (q5-q30)	1.685	0.640
Medium quantiles (q40-q60)	2.470	0.291
Upper quantiles (q70-q95)	0.239	0.971
SPACING		
	Trend-coeff.	p-value
q50-q5	-0.003	0.005
q95-q50	-0.002	0.094
q95-q5	-0.005	0.000

Notes: Annual distributional characteristics (quantiles) of temperature from daily Central England data (1772-2017). In the top panel, Wald test of the null hypothesis of equality of trend coefficients of a given set of characteristics. P-values calculated from bootstrap critical values. In the bottom panel, the TT is applied to the difference between two representative quantiles.

Table 4
ADF unit root test (CRU station data)

Characteristic	ADF-SBIC	p-value	lags
mean	-8.04	0.000	0
max	-5.84	0.000	3
min	-8.84	0.000	0
std	-5.46	0.000	1
iqr	-6.39	0.000	1
range	-3.53	0.041	3
kur	-4.24	0.005	3
skw	-10.20	0.000	0
q5	-8.80	0.000	0
q10	-8.01	0.000	0
q20	-8.75	0.000	0
q30	-9.14	0.000	0
q40	-9.09	0.000	0
q50	-9.15	0.000	0
q60	-8.99	0.000	0
q70	-8.80	0.000	0
q80	-2.66	0.267	3
q90	-3.37	0.060	3
q95	-3.77	0.021	3

Notes: Annual distributional characteristics calculated from CRU station data (1880-2015). Lag-selection according to SBIC criterion.

Table 5
Additional unit root analysis (CRU station data)

ADF UNIT ROOT TEST BY STATIONS		
% rejections with all stations	88.99	
% rejections with NH stations	89.93	
PANEL UNIT ROOTS TESTS		
Levin, Lin and Chu	-16.50	0.000
Breitung	-8.00	0.000
Im, Pesaran and Shin	-16.13	0.000
Fisher (ADF)	415.70	0.000
Fisher (PP)	708.46	0.000

Notes: In the top panel, percentage of rejections of the ADF test station by station, considering all stations from 1880 that have at least 30 observations. In the bottom panel, panel unit root tests of the 19 distributional characteristics. We use the test of Breitung (2000) and Levin et al. (2002) that assumes common persistence parameters across cross-sections. Fisher-type tests proposed by Maddala and Wu (1999) and Choi (2001) and those suggested by Im et al. (2003) allow different persistence parameters across cross-sections.

Table 6
Trend test (CRU station data)

Characteristic	Coeff	p-value
mean	0.0104	0.0000
max	0.0052	0.0003
min	0.0184	0.0000
std	-0.0027	0.0000
iqr	-0.0003	0.4196
range	-0.0132	0.0006
kur	0.0006	0.1161
skw	-0.0001	0.3120
q5	0.0147	0.0000
q10	0.0146	0.0000
q20	0.0125	0.0000
q30	0.0109	0.0000
q40	0.0109	0.0000
q50	0.0112	0.0000
q60	0.0111	0.0000
q70	0.0117	0.0000
q80	0.0105	0.0000
q90	0.0025	0.0752
q95	0.0005	0.4614

Notes: Annual distributional characteristics of temperature with CRU station data (1880-2015). OLS estimates and HAC $t_{\beta=0}$ from regression: $C_t = \alpha + \beta t + u_t$.

Table 7
Co-trending analysis

Characteristic	Wald test	p-value
Quantiles		
All quantiles (q5, q10,...,q90,q95)	33.888	0.000
Lower quantiles (q5-q30)	4.718	0.194
Medium quantiles (q40-q60)	0.020	0.990
Upper quantiles (q70-q95)	19.478	0.000
Spacing		
	Trend-coeff.	p-value
q50-q5	-0.004	0.007
q95-q50	-0.011	0.035
q95-q5	-0.014	0.011

Notes: Annual distributional characteristics (quantiles) of temperature of CRU station data (1880-2015). In the top panel, Wald test of the null hypothesis of equality of trend coefficients of a given set of characteristics. P-values calculated from bootstrap critical values. In the bottom panel, the TT is applied to the difference between two representative quantiles.

8 Figures

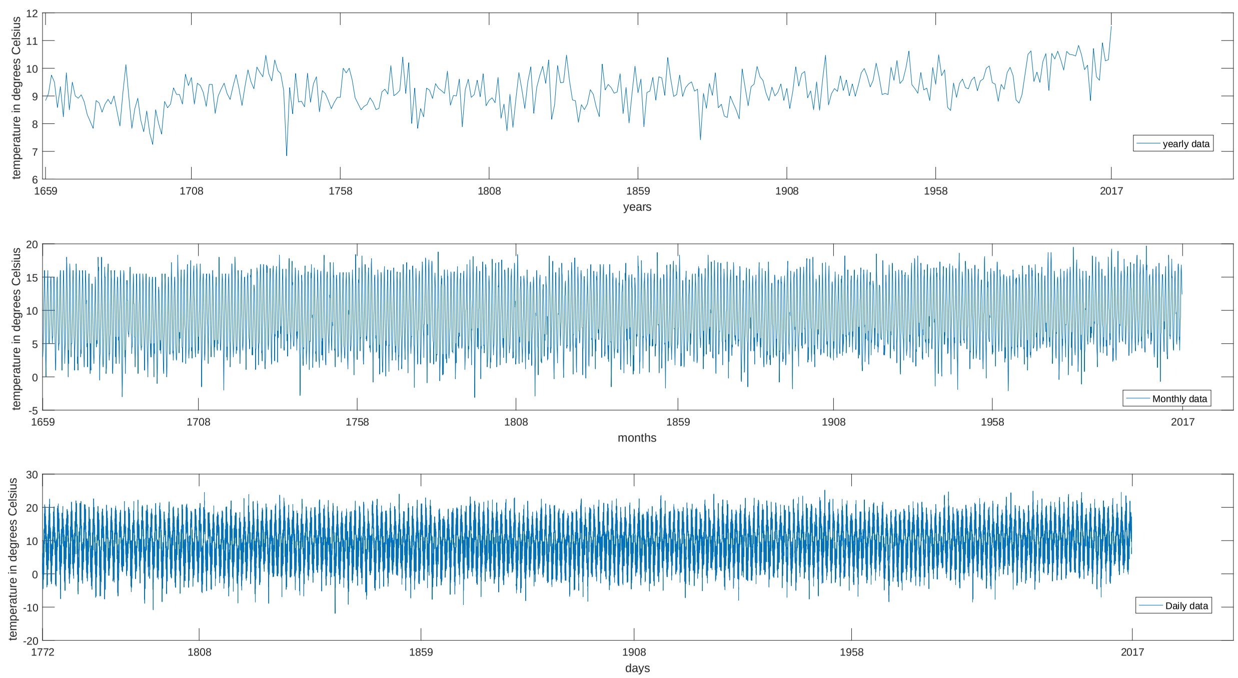


Figure 1
Recorded temperature data from Central England at different frequencies

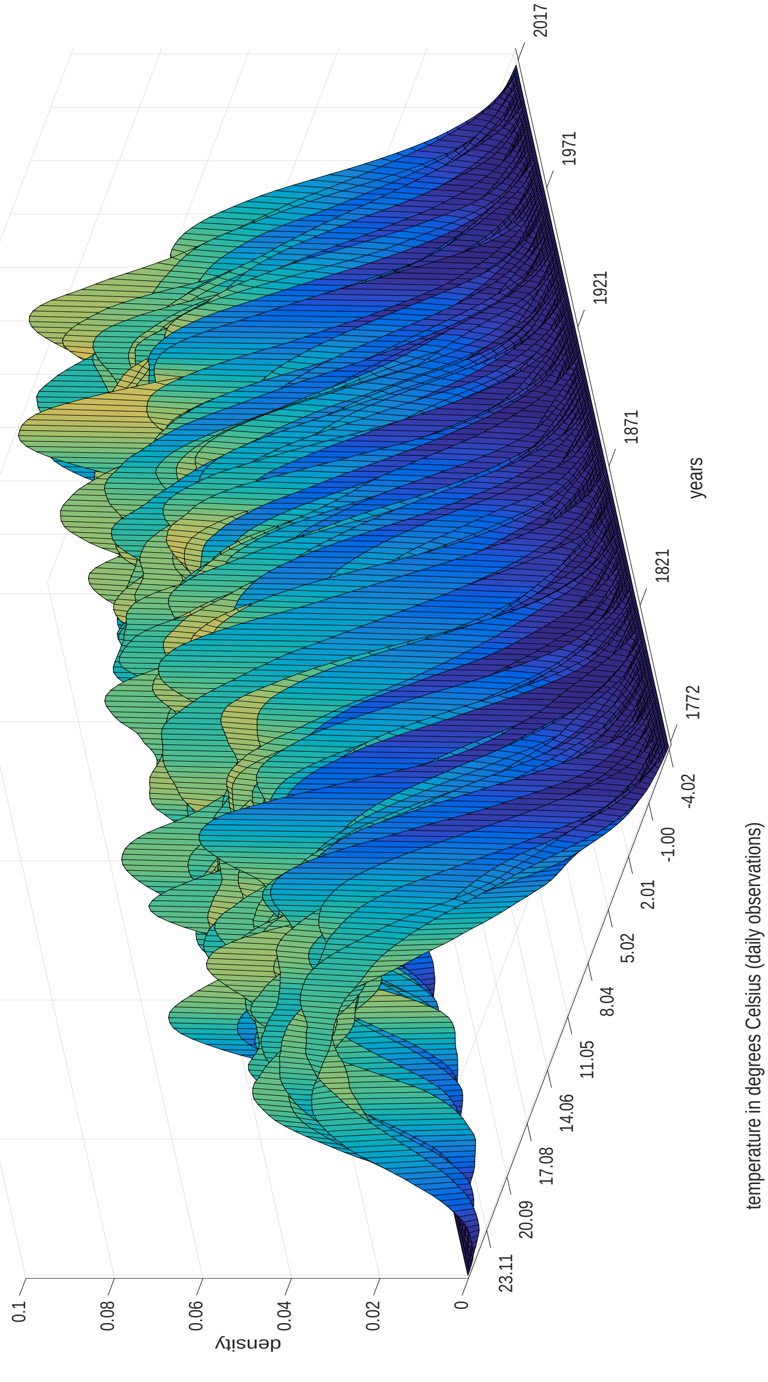


Figure 2
Central England temperature densities calculated with daily data

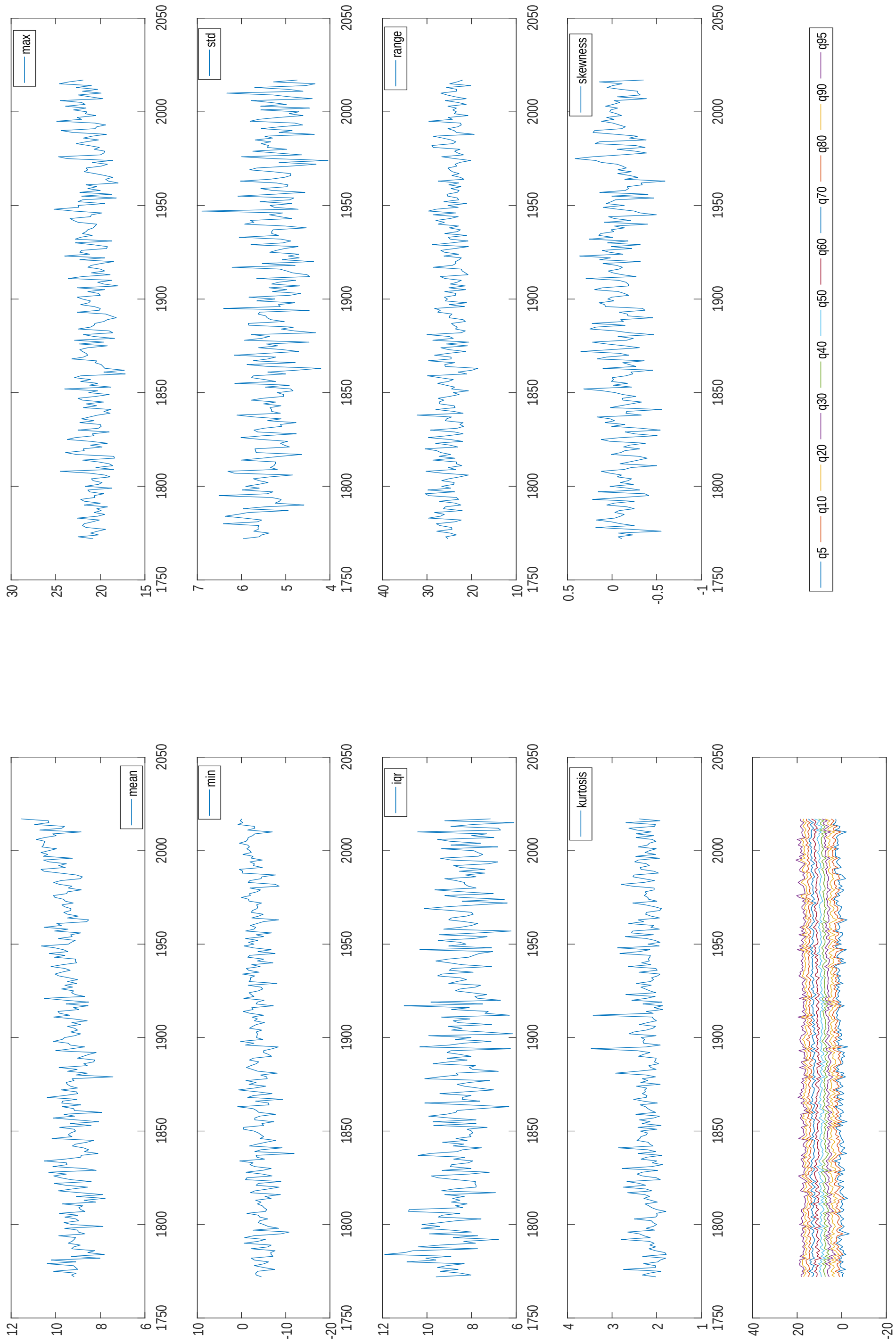


Figure 3
Annual temporal distributional characteristics of Central England temperature (degrees Celsius) calculated with daily data

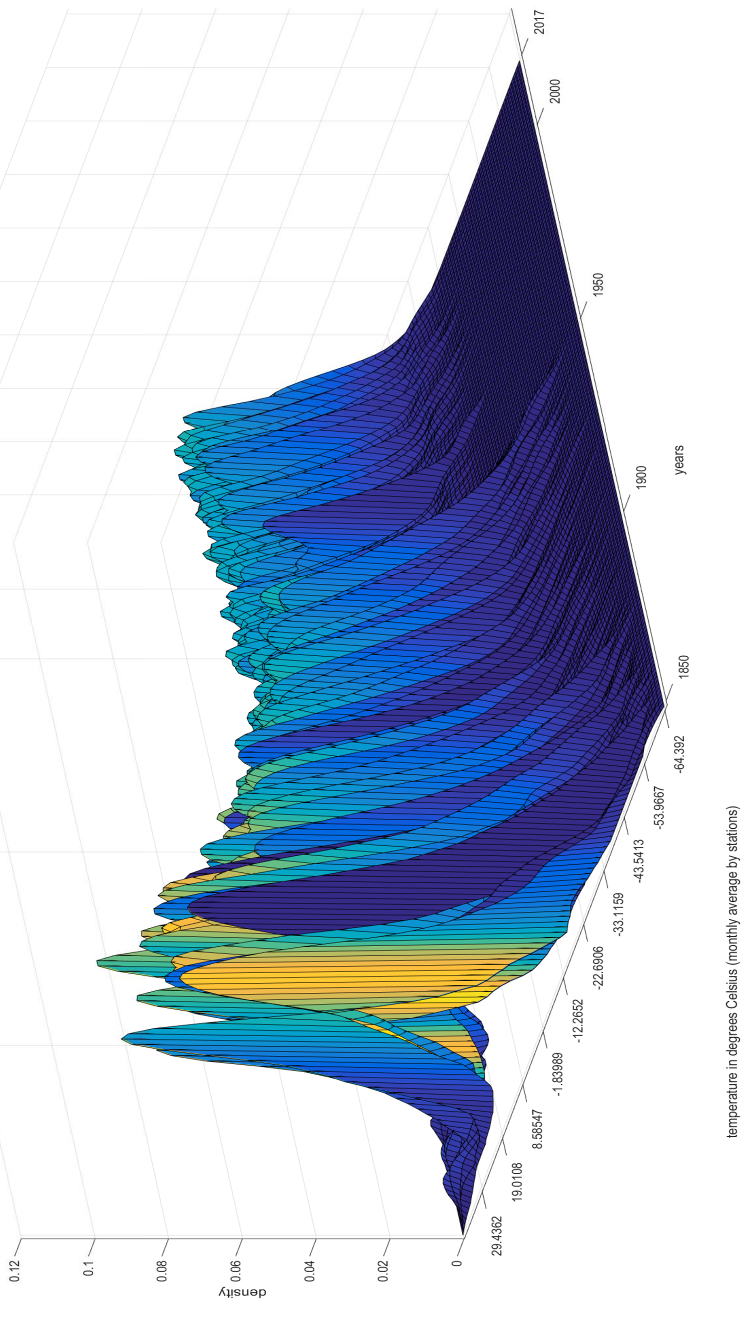


Figure 4
Global temperature densities calculated with CRU station data

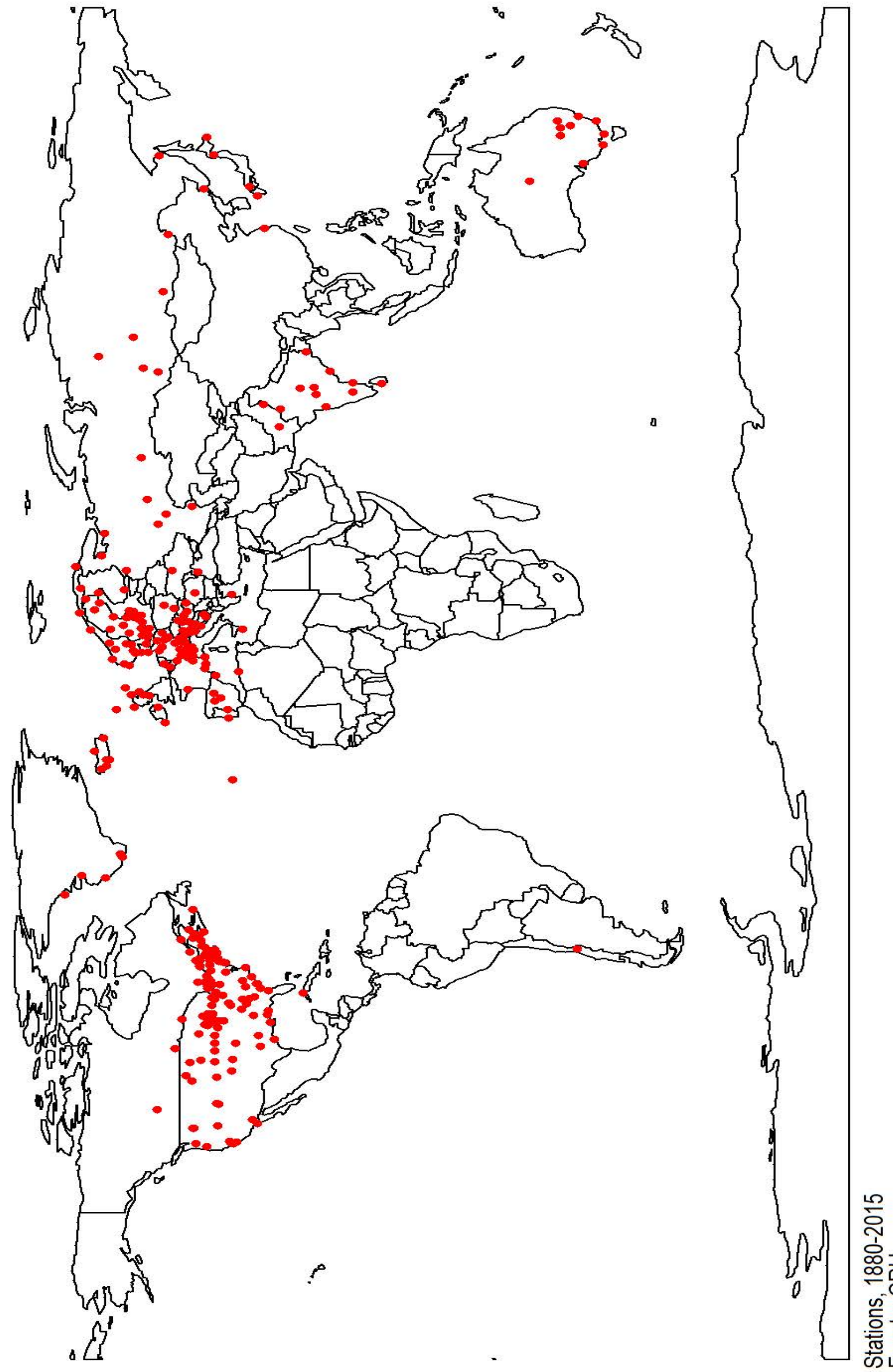


Figure 5
Geographical distribution of selected stations

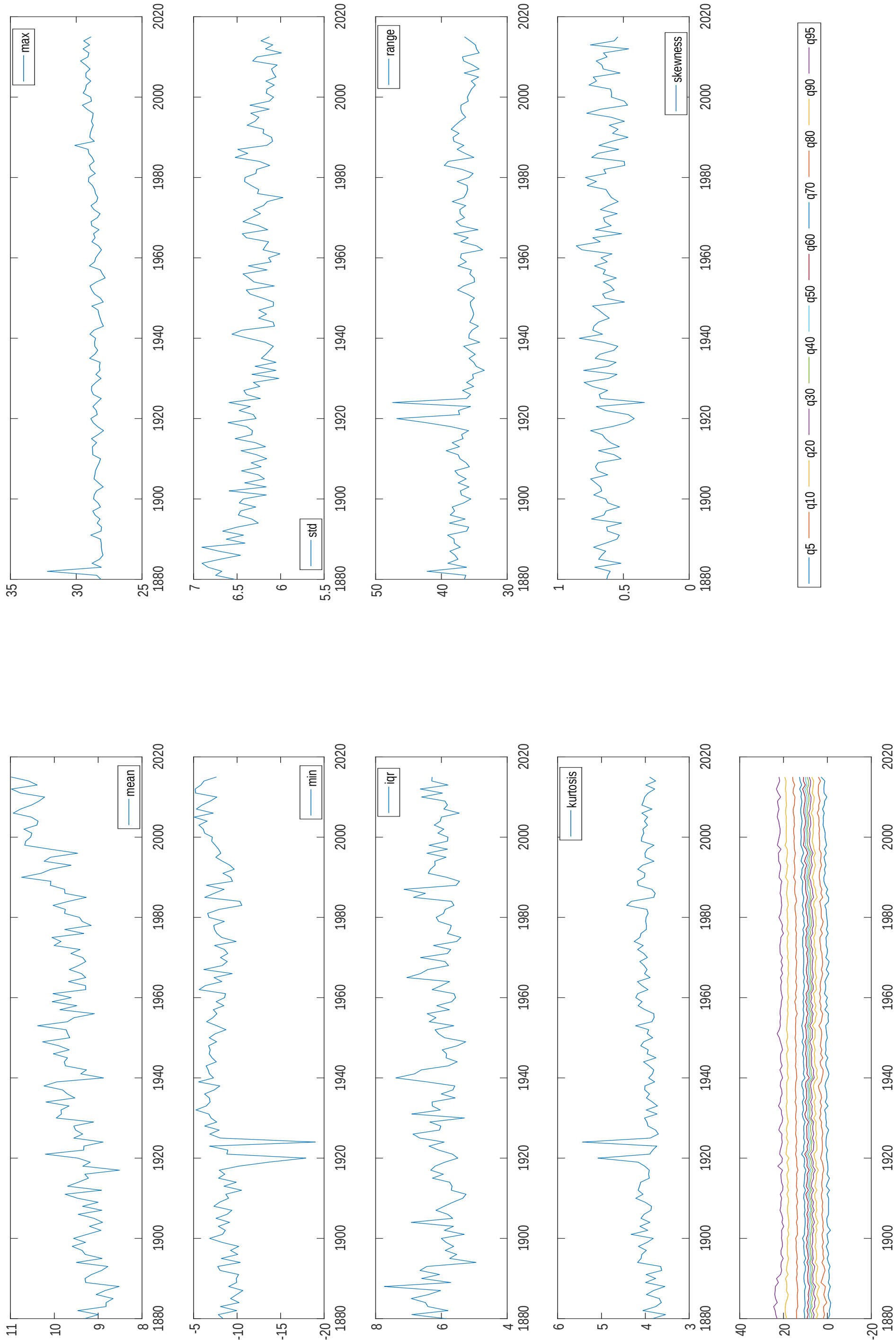


Figure 6
Annual temporal distributional characteristics of Global temperature (degrees Celsius) calculated with CRU station data

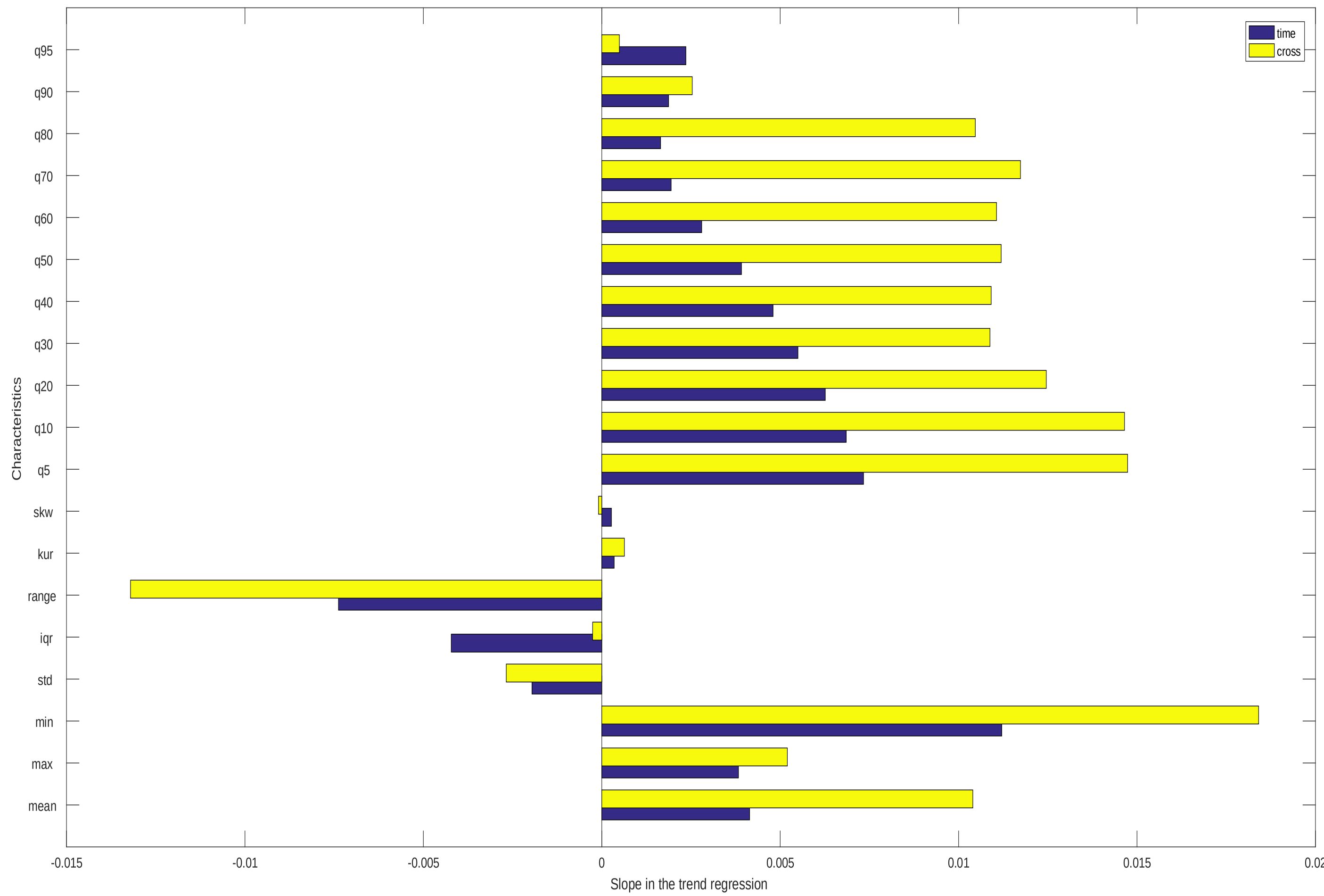


Figure 7

Trend comparison in distributional characteristics with time series (CET) and cross-sectional (CRU) data

Note. This figure reports the $\widehat{\beta}$ estimated from the regression $C_t = \alpha + \beta t + u_t$, applied to the temporal distributional characteristics with time series (CET) and cross-sectional (CRU) data.

Appendix

Appendix A: Proofs

Proof of Proposition 1

See Sections 16.1 and 16.2 in Hamilton (1994).

Proof of Proposition 2

Part 1: Asymptotic behavior of OLS $\widehat{\beta}$:

$$\widehat{\beta} = \frac{\sum_{t=1}^{T}(C_t - \overline{C})(t - \overline{t})}{\sum_{t=1}^{T}(t - \overline{t})^2} = \frac{\sum_{t=1}^{T} C_t t - \overline{C}\sum_{t=1}^{T} t - \overline{t}\sum_{t=1}^{T} C_t + \overline{C}\overline{t}}{\sum_{t=1}^{T}(t - \overline{t})^2} \tag{13}$$

Taking into account that

$$\begin{aligned}
&\sum_{t=1}^{T} C_t t = O_p(T^{2+\delta}),\\
&\overline{C}\sum_{t=1}^{T} t = O_p(T^{2+\delta}),\\
&\overline{t}\sum_{t=1}^{T} C_t = O_p(T^{2+\delta}),\\
&\overline{C}\overline{t} = O_p(T^{1+\delta})
\end{aligned}$$

and

$$\sum_{t=1}^{T}(t - \overline{t})^2 = O(T^3),$$

we obtain that $\widehat{\beta} = O_p(T^{\delta-1})$.

Part 2: Asymptotic behaviour of $t_{\beta=0}$:

$$t_{\beta=0} = \frac{\widehat{\beta} - 0}{\sqrt{\widehat{\sigma}_u^2 / \sum_{t=1}^{T}(t - \overline{t})^2}} = \frac{\sum_{t=1}^{T}(C_t - \overline{C})(t - \overline{t})}{\sqrt{\widehat{\sigma}_u^2 \sum_{t=1}^{T}(t - \overline{t})^2}} \tag{14}$$

From Part 1 the numerator is $O_p(T^{2+\delta})$. It is easy to obtain that

$$\widehat{\sigma}_u^2 = \frac{\sum_{t=1}^{T}(C_t - \widehat{\alpha} - \widehat{\beta}t)^2}{T} = \begin{cases} O_p(T^{(1+2\delta-\gamma)}) \text{ for } 0 \leq \gamma \leq 1 \\ O_p(T^{2\delta}) \text{ for } 1 \leq \gamma \leq 1+\delta \end{cases} \quad (15)$$

Taking into account that $\sum_{t=1}^{T}(t-\bar{t})^2) = O(T^3)$, the result follows.

Proof of Proposition 3

For the fractional case, $1/2 < d < 3/2$, see Marmol and Velasco (2002).
For the near unit root as well as for the local level model, the proof follows straightforward from the proof in Durlauf and Phillips (1988) for the pure I(1) case.

Appendix B: Finite-sample performance

In this appendix, the finite-sample performance of our proposed trend test (TT) is analyzed via a Monte Carlo experiment. Sample sizes are T = 200, 500, and 1000. Number of replications is equal to 10,000. In all cases, the significance level is 5% (critical values for a N(0,1)) and a HAC $t_{\beta=0}$ is used. In general, the parameters of a given model have been estimated or selected by fitting that model to the average annual Central England temperature (1772-2015). However, in some cases (super-exponential trends, Gompertz curves and logistic trends), when the fitting is very unstable we use other typical economic series such as the UK nominal GDP per-capita (1800-2010) (from Madisson, 2013) and others (Population, IPI and Wholesale Prices) from Davis (1941).

SIZE

The empirical size is investigated by generating several non-trending models.

- Case 1: A white noise model (WN) from a Normal (0, 1).
- Case 2: $sin(u * t)$, $t = 1, ..., T$, u$\sim U(0,1)$, where u is used to reduce the frequency of the sin function.
- Case 3: An AR(2) process whose parameter values are obtained from fitting an AR(2) to the average annual Central England temperature.

- Case 4: An AR(2) with complex roots. The first parameter is selected from a $U(0,1)$ and the second one from a $U(-1,0)$

Table 8
Size of TT

	T=200	T=500	T=1000
WN	0.0693	0.0583	0.0579
sin(u*t), u∼ $U(0,1)$	0.0582	0.0377	0.0388
AR(2) with estimated parameters	0.1391	0.1150	0.0994
complex roots	0.0499	0.0445	0.0389

POWER

Deterministic trends (See Proposition 2): The power of our TT is investigated by generating data from the main deterministic trends used in the literature plus a N(0,1) white noise term:
(I) Polynomial Trends

$$x(t) = a_0 + a_1 t + a_2 t^2 + ... + a_p t^k \tag{16}$$

with $k = 1$, $k = 2$, k chosen by a SBIC. We also analyze the case of $k = \theta$ with $\theta < 1$. In all these cases the parameters have been estimated or selected by fitting the corresponding polynomial trend to the average annual Central England temperature.
(II) Exponential trends

$$x(t) = a_0 + a_1 e^{\lambda t} \tag{17}$$

- Sub-exponential:

$$x(t) = a_0 + a_1 e^{a_2 t^{\lambda}} \tag{18}$$

  with $\lambda < 1$.

- Super-exponential:

$$x(t) = a_0 + a_1 e^{e^{\lambda t}} \tag{19}$$

  The Gompertz curve can be included within this sub-case:

$$x(t) = e^{a_0 - a_1 e^{-\lambda t}} \tag{20}$$

(III) Logistic Trends

$$x(t) = \frac{a_1}{1 + a_2 e^{-\lambda t}} \tag{21}$$

(IV) Segmented Trends

$$x(t) = a_0 + b_0 d_{1t} + a_1 t + b_2 d_{1t} t \tag{22}$$

with d_{1t} being a dummy variable that takes the value 1 in regime A and 0 in regime B.

(V) Logistic Smooth Transition Trends

$$x(t) = a_0 + a_1 t + (b_0 + b_2 t) S_t(\theta, \tau) \tag{23}$$

with $S_t(\theta, \tau) = (1 + exp(-\theta(t - \tau T)))^{-1}$.

Table 9
Power (deterministic trends) of TT

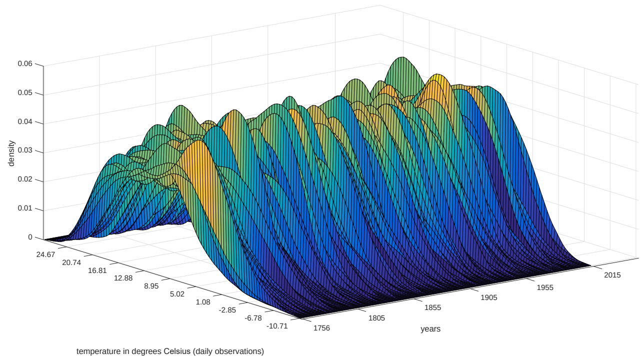
	T=200	T=500	T=1000
Polynomial trend k=1	0.9998	1.0000	1.0000
Polynomial trend k=2	0.9119	1.0000	1.0000
Polynomial trend k=sbic	0.9083	1.0000	1.0000
Polynomial trend $k = \theta < 1$	0.9937	1.0000	1.0000
Exponential	0.8782	1.0000	1.0000
Exponential (sub)	0.8718	1.0000	1.0000
Exponential (super, UK GDP)	1.0000	- (*)	- (*)
Exponential (Gompertz curve, UK GDP)	1.0000	- (*)	- (*)
Logistic (Population)	1.0000	1.0000	1.0000
Logistic (Industrial Production Index)	1.0000	1.0000	1.0000
Logistic (Wholesale Prices)	1.0000	1.0000	1.0000
Segmented trends	1.0000	1.0000	1.0000
Logistic smooth transition (UK GDP)	1.0000	1.0000	1.0000

Stochastic trends (see Proposition 3) Following Müeller and Watson (2008) we consider the three most common long-run models generating stochastic trends: fractional models ($1/2 < d < 3/2$), near unit root models and local level models.

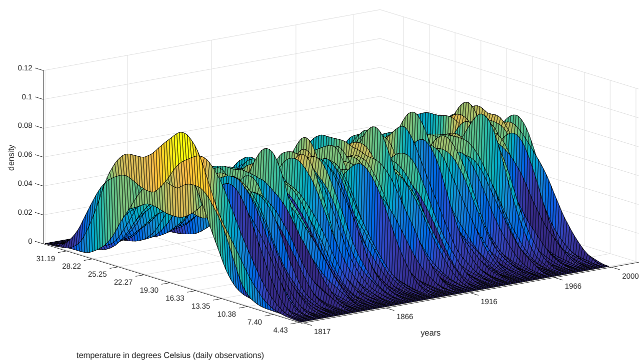
Table 10
Power (stochastic trends) of TT

Fractional model						
d	0.5	0.7	0.9	1	1.2	1.5
T=50	0.4334	0.5512	0.6600	0.7089	0.7868	0.8715
T=100	0.4776	0.6132	0.7112	0.7562	0.8240	0.8887
T=200	0.5326	0.6582	0.7613	0.8058	0.8642	0.9096
T=300	0.5722	0.7103	0.7943	0.8314	0.8829	0.9285
T=50	0.6102	0.7442	0.8253	0.8566	0.8990	0.9423
T=1000	0.6712	0.7913	0.8711	0.8928	0.9253	0.9536
Near Unit root						
	c=30	c=10	c=5	c=0		
T=50	0.1521	0.3537	0.4897	0.7180		
T=100	0.2163	0.4350	0.5572	0.7649		
T=200	0.2879	0.5197	0.6262	0.8060		
T=300	0.3633	0.5860	0.6850	0.8317		
T=50	0.4320	0.6387	0.7324	0.8573		
T=1000	0.5378	0.7193	0.7862	0.8989		
Local level model						
q	0	0.1	0.5	1	5	10
T=50	0.7180	0.7193	0.7151	0.7079	0.5312	0.3347
T=100	0.7649	0.7638	0.7635	0.7609	0.6807	0.5369
T=200	0.8060	0.8058	0.8057	0.8052	0.7722	0.6967
T=300	0.8317	0.8317	0.8312	0.8306	0.8137	0.7686
T=50	0.8573	0.8572	0.8567	0.8573	0.8467	0.8218
T=1000	0.8989	0.8987	0.8986	0.8984	0.8967	0.8862

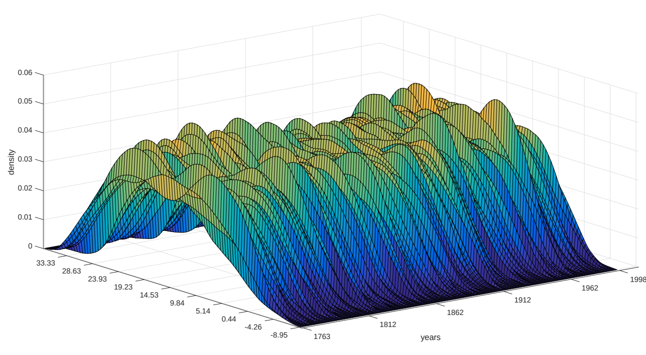
Appendix C: Additional time series data of temperature



(a) Stockolm



(b) Cadiz



(c) Milan

Figure 8

Local temperature densities calculated with daily data (IMPROVE)

Table 11
Unit root tests (IMPROVE data)

Characteristic	Stockholm	Cadiz	Milan
mean	-13.34	-5.94	-12.77
	(0.000)	(0.000)	(0.000)
max	-16.96	-8.45	-13.20
	(0.000)	(0.000)	(0.000)
min	-14.86	-8.97	-13.88
	(0.000)	(0.000)	(0.000)
std	-13.99	-8.46	-13.94
	(0.000)	(0.000)	(0.000)
iqr	-14.14	-9.65	-15.07
	(0.000)	(0.000)	(0.000)
range	-15.88	-11.77	-13.50
	(0.000)	(0.000)	(0.000)
kur	-17.07	-14.37	-13.81
	(0.000)	(0.000)	(0.000)
skw	-14.18	-12.57	-13.62
	(0.000)	(0.000)	(0.000)
q5	-13.51	-6.28	-14.35
	(0.000)	(0.000)	(0.000)
q10	-13.58	-8.38	-14.31
	(0.000)	(0.000)	(0.000)
q20	-13.93	-8.13	-13.71
	(0.000)	(0.000)	(0.000)
q30	-13.82	-7.79	-14.55
	(0.000)	(0.000)	(0.000)
q40	-13.57	-6.76	-13.85
	(0.000)	(0.000)	(0.000)
q50	-13.37	-6.91	-13.69
	(0.000)	(0.000)	(0.000)
q60	-3.26	-10.61	-8.87
	(0.076)	(0.000)	(0.000)
q70	-13.41	-5.82	-13.03
	(0.000)	(0.000)	(0.000)
q80	-13.66	-3.41	-5.77
	(0.000)	(0.053)	(0.000)
q90	-14.91	-4.11	-6.18
	(0.000)	(0.008)	(0.000)
q95	-15.85	-12.06	-13.33
	(0.000)	(0.000)	(0.000)

Notes: Annual distributional characteristics of temperature from IMPROVE daily data. Data from Stockholm (1756-2012), Cadiz (1817-2000) and Milan (1763-1998) from Camuffo and Jones (2002). Stockholm temperatures have been updated to 2015 by Bolin Center Database. P-values in brackets. Lag-selection according to SBIC criterion.

Table 12
Trend test (IMPROVE data, Stockholm)

Characteristic	Coeff	p-value
mean	0.0042	0.0001
max	0.0012	0.2198
min	0.0221	0.0000
std	-0.0038	0.0000
iqr	-0.0045	0.0002
range	-0.0210	0.0000
kur	-0.0006	0.0041
skw	0.0005	0.0094
q5	0.0126	0.0000
q10	0.0099	0.0000
q20	0.0060	0.0001
q30	0.0046	0.0000
q40	0.0050	0.0001
q50	0.0045	0.0005
q60	0.0031	0.0116
q70	0.0016	0.0858
q80	0.0000	0.4865
q90	-0.0006	0.3398
q95	-0.0001	0.4660

Notes: Annual distributional characteristics of temperature from daily Stockholm data (1756-2012). OLS estimates and HAC $t_{\beta=0}$ from regression: $C_t = \alpha + \beta t + u_t$.

Table 13
Trend test (IMPROVE data, Cadiz)

Characteristic	Coeff	p-value
mean	0.0047	0.0000
max	0.0070	0.0042
min	0.0070	0.0489
std	-0.0006	0.2648
iqr	-0.0000	0.4999
range	-0.0037	0.0119
kur	0.0009	0.0011
skw	0.0002	0.2565
q5	0.0050	0.0132
q10	0.0057	0.0009
q20	0.0059	0.0001
q30	0.0060	0.0001
q40	0.0053	0.0009
q50	0.0042	0.0020
q60	0.0044	0.0007
q70	0.0040	0.0027
q80	0.0018	0.0798
q90	0.0036	0.0065
q95	0.0059	0.0002

Notes: Annual distributional characteristics of temperature from daily Cadiz data (1812-2000). OLS estimates and HAC $t_{\beta=0}$ from regression: $C_t = \alpha + \beta t + u_t$.

Table 14
Trend test (IMPROVE data, Milan)

Characteristic	Coeff	p-value
mean	0.0027	0.0002
max	0.0023	0.0679
min	0.0120	0.0000
std	-0.0025	0.0000
iqr	-0.0058	0.0000
range	-0.0097	0.0001
kur	0.0002	0.0492
skw	0.0004	0.0015
q5	0.0067	0.0000
q10	0.0068	0.0000
q20	0.0061	0.0000
q30	0.0047	0.0000
q40	0.0033	0.0034
q50	0.0006	0.3130
q60	-0.0008	0.2091
q70	-0.0005	0.3253
q80	-0.0001	0.4591
q90	0.0007	0.2802
q95	0.0021	0.0437

Notes: Annual distributional characteristics of temperature from daily Milan data (1763-1998). OLS estimates and HAC $t_{\beta=0}$ from regression: $C_t = \alpha + \beta t + u_t$.