Supplementary Online Appendix

Difference between short and long term risk aversion: an optimal asset allocation perspective, by Jesus Gonzalo (Universidad Carlos III de Madrid) and Jose Olmo (University of Southampton)

Further Empirical Resuls for Different Investment Horizons

Tables A.1 to A.3 reveal interesting insights about risk aversion for $K = 12$ to $K = 36$. The results for these investment horizons also support the presence of dynamics in risk aversion driven by the fluctuation of the state variables. The column of the bottom panel reporting the linear dynamics in the risk aversion coefficient provides overwhelming statistical evidence of the significance of the U.S. credit spread and S&P 500 trend for $K = 12$ to $K = 36$. The statistical significance of the state variables in driving the optimal portfolio weights increases for larger investment horizons. Thus, for $K = 24$, the detrended short-term interest rate is also significant, and for $K = 36$ we observe that the four state variables are statistically significant.

The analysis of nonlinearities in risk aversion also reveals interesting findings. The overall risk aversion level clearly increases from the short to the long term but the differences across investment horizons are not large. More importantly, the difference between the short and the long term is characterized by the threshold value κ_0 that is estimated from the data. For consistency across investment horizons we only report in Tables A.1 to A.3 the estimates of θ_0 for $\widehat{\kappa}_N = 7$, however, the optimal values of κ_0 obtained from (17) are 7, 11 and 7 for $K = 12, 24, 36$, respectively. The presence of nonlinearity in risk aversion is tested using the likelihood ratio test comparing the linear model against the model with two regimes. The p-values are equal to 0.12 for $K = 12$ and zero for $K = 24$ and $K = 36$.

[Insert Tables A.1 to A.3 about here]

The results in Tables A.1 to A.3 show clear evidence of the influence of the state variables in driving the dynamics of the optimal allocation to the bond index. In particular, the four state variables are statistically significant under the three different types of risk aversion considered in the paper and across investment horizons. The results show the statistical significance of the detrended short-term interest rate, the U.S. credit spread and the one-month average of the excess bond and stock returns, however, the S&P 500 trend is not statistically significant for any K. These results are common across risk aversion scenarios. It is also worth noting that the magnitude of the parameter estimates Λ increases with K. This finding suggests that the exposure of the optimal portfolio weights to the fluctuations of the state variables is larger as the investment horizon increases.

We illustrate the results for $K = 12, 24, 36$ further by plotting the dynamics of the risk aversion function γ_t in Figures A.1 to A.3. The top panel reports the constant and linear dynamic risk aversion coefficient (2) defined as $\gamma(j) = \exp(\widehat{\gamma}_c)$ and $\gamma_t(j) = \exp(\widehat{\gamma}' Z_{t+j})$, respectively. The bottom panel plots the nonlinear version of the risk aversion function (2). For comparison purposes, we report separately the short-term dynamics $\exp(\widehat{\gamma}' Z_{t+j})$ and the long-term dynamics $\exp((\widehat{\gamma}' + \widehat{\eta}') Z_{t+j})$. The comparison across top panels reveals rises in risk aversion as K increases. The bottom panels illustrate the additional effect of long-term risk aversion to the short-term risk aversion component. The contribution of long-term risk aversion to the overall risk aversion function is very significant during the first half of the decade of 1980 and the 2007-2009 crisis period. We also observe spikes in long-term risk aversion during the second half of the 1990 decade.

[Insert Figures A.1 to A.10 about here]

Figures A.4 to A.7 report the dynamics of the optimal portfolio allocations to the S&P 500 index (α_{st}), the G0Q0 bond index (α_{bt}) and the one-month Treasury bill (α_{ct}) for $K =$

$12, 24, 36, 48$ over the period January 1980 to December 2016. The dashed black line corresponds to the dynamic nonlinear strategy, the dotted red line to the dynamic linear strategy and the solid blue line to the constant risk aversion strategy. The analysis of the optimal allocation to stocks, bonds and cash is very similar across the four investment horizons for this extended sample period. The only remarkable differences are in the magnitude of the weights that increase as the investment horizon K increases. This finding is also noted above when discussing the magnitude of the parameter estimates Λ. For completeness, we also report in Figures A.8 to A.10 the optimal portfolio weights α_t for $K = 12, 24, 36$ for the crisis period 2007-2011. The results are in line with the above discussion. During this period, the differences in the optimal allocation to stocks between the constant risk aversion model and the model exhibiting nonlinear dynamics in risk aversion are more significant as a result of the significant increase in risk aversion in financial markets.

Tables and figures

stock parameters				bond parameters			
	nonlinear	dynamics	constant		nonlinear	dynamics	constant
$\lambda_{s,c}$	0.141 [0.001]	0.142 [0.000]	0.152 [0.000]	$\lambda_{b,c}$	0.429 [0.000]	0.370 [0.002]	0.306 [0.001]
$\lambda_{s,1}$	0.035 [0.190]	0.044 [0.081]	0.027 [0.138]	$\lambda_{b,1}$	−0.091 [0.138]	−0.082 [0.085]	−0.153 [0.138]
$\lambda_{s,2}$	0.038 [0.117]	0.032 [0.141]	0.046 [0.015]	$\lambda_{b,2}$	−0.538 [0.000]	−0.509 [0.000]	−0.424 [0.000]
$\lambda_{s,3}$	0.022 [0.331]	0.000 [0.991]	0.007 [0.674]	$\lambda_{b,3}$	−0.328 [0.001]	−0.315 [0.001]	−0.267 [0.001]
$\lambda_{s,4}$	0.245 [0.000]	0.247 [0.000]	0.268 [0.000]	$\lambda_{b,4}$	−0.268 [0.001]	−0.232 [0.003]	−0.259 [0.000]
short term regime				long term regime			
	nonlinear	linear	constant		nonlinear	linear	constant
γ_c	3.730 [0.000]	3.907 [0.000]	3.849 [0.000]	η_c	0.597 [0.210]	–	–
γ_1	−0.017 [0.865]	−0.078 [0.363]	–	η_1	−0.151 [0.701]	–	–
γ_2	0.312 [0.014]	0.290 [0.001]	–	η_2	0.571 [0.476]	–	–
γ_3	0.178 [0.115]	0.199 [0.003]	–	η_3	0.354 [0.423]	–	–
γ_4	0.253 [0.249]	0.134 [0.490]	–	η_4	−0.205 [0.787]	–	–
κ_o	7						

Table A.1. Parameter estimates of the three different versions of the individual's objective function (3) for $K = 12$ and $\beta = 0.95$. The parameters λ correspond to the portfolio allocations associated to the state variables Z_t: $\lambda_{\cdot,c}$ is for the constant term, $\lambda_{\cdot,1}$ corresponds to the detrended short-term interest rate, $\lambda_{\cdot,2}$ to the U.S. credit spread, $\lambda_{\cdot,3}$ to the S&P 500 trend and $\lambda_{\cdot,4}$ to the one-month average of the excess stock and bond returns. Similarly, the vector γ describes the sensitivities of the risk aversion function (2) with respect to the state variables for the linear segment and η the corresponding sensitivities for the nonlinear segment of the function. κ_0 denotes the estimate of the threshold value corresponding to these parameter estimates. P-values are in squared brackets.

stock parameters				bond parameters			
	nonlinear	linear	constant		nonlinear	linear	constant
$\lambda_{s,c}$	0.188 [0.000]	0.220 [0.000]	0.217 [0.000]	$\lambda_{b,c}$	0.542 [0.000]	0.483 [0.005]	0.417 [0.000]
$\lambda_{s,1}$	0.066 [0.042]	0.074 [0.002]	0.043 [0.002]	$\lambda_{b,1}$	−0.141 [0.011]	−0.178 [0.000]	−0.251 [0.000]
$\lambda_{s,2}$	0.034 [0.145]	0.037 [0.060]	0.070 [0.000]	$\lambda_{b,2}$	−0.637 [0.000]	−0.727 [0.000]	−0.566 [0.000]
$\lambda_{s,3}$	0.021 [0.456]	0.011 [0.558]	0.023 [0.156]	$\lambda_{b,3}$	−0.354 [0.002]	−0.327 [0.000]	−0.281 [0.000]
$\lambda_{s,4}$	0.291 [0.000]	0.352 [0.000]	0.379 [0.000]	$\lambda_{b,4}$	−0.318 [0.000]	−0.323 [0.000]	−0.351 [0.000]
short term regime				long term regime			
	nonlinear	linear	constant		nonlinear	linear	constant
γ_c	3.479 [0.000]	3.632 [0.000]	3.566 [0.000]	η_c	0.969 [0.004]	–	–
γ_1	−0.065 [0.462]	−0.205 [0.000]	–	η_1	−0.348 [0.321]	–	–
γ_2	0.397 [0.000]	0.349 [0.000]	–	η_2	0.678 [0.098]	–	–
γ_3	0.197 [0.135]	0.228 [0.000]	–	η_3	0.661 [0.014]	–	–
γ_4	0.253 [0.333]	0.079 [0.528]	–	η_4	−0.322 [0.546]		–
κ_o	7						

Table A.2. Parameter estimates of the three different versions of the individual's objective function (3) for $K = 24$ and $\beta = 0.95$. The parameters λ correspond to the portfolio allocations associated to the state variables Z_t: $\lambda_{\cdot,c}$ is for the constant term, $\lambda_{\cdot,1}$ corresponds to the detrended short-term interest rate, $\lambda_{\cdot,2}$ to the U.S. credit spread, $\lambda_{\cdot,3}$ to the S&P 500 trend and $\lambda_{\cdot,4}$ to the one-month average of the excess stock and bond returns. Similarly, the vector γ describes the sensitivities of the risk aversion function (2) with respect to the state variables for the linear segment and η the corresponding sensitivities for the nonlinear segment of the function. κ_0 denotes the estimate of the threshold value corresponding to these parameter estimates. P-values are in squared brackets.

stock parameters				bond parameters			
	nonlinear	linear	constant		nonlinear	linear	constant
$\lambda_{s,c}$	0.212 [0.000]	0.239 [0.000]	0.249 [0.000]	$\lambda_{b,c}$	0.592 [0.000]	0.488 [0.000]	0.458 [0.000]
$\lambda_{s,1}$	0.087 [0.015]	0.083 [0.000]	0.053 [0.003]	$\lambda_{b,1}$	−0.167 [0.002]	−0.196 [0.000]	−0.289 [0.000]
$\lambda_{s,2}$	0.022 [0.403]	0.031 [0.043]	0.075 [0.043]	$\lambda_{b,2}$	−0.690 [0.000]	−0.751 [0.000]	−0.641 [0.000]
$\lambda_{s,3}$	0.008 [0.785]	0.007 [0.596]	0.017 [0.259]	$\lambda_{b,3}$	−0.381 [0.005]	−0.330 [0.000]	−0.331 [0.000]
$\lambda_{s,4}$	0.337 [0.000]	0.372 [0.000]	0.440 [0.000]	$\lambda_{b,4}$	−0.320 [0.000]	−0.321 [0.000]	−0.385 [0.000]
short term regime				long term regime			
	nonlinear	linear	constant		nonlinear	linear	constant
γ_c	3.330 [0.000]	3.614 [0.000]	3.442 [0.000]	η_c	1.025 [0.002]	–	–
γ_1	−0.101 [0.259]	−0.205 [0.000]	–	η_1	−0.376 [0.239]	–	–
γ_2	0.426 [0.000]	0.392 [0.000]	–	η_2	0.618 [0.099]	–	–
γ_3	0.206 [0.108]	0.202 [0.000]	–	η_3	0.644 [0.012]	–	–
γ_4	0.274 [0.209]	0.148 [0.041]	–	η_4	−0.348 [0.464]		–
κ_o	7						

Table A.3. Parameter estimates of the three different versions of the individual's objective function (3) for $K = 36$ and $\beta = 0.95$. The parameters λ correspond to the portfolio allocations associated to the state variables Z_t: $\lambda_{\cdot,c}$ is for the constant term, $\lambda_{\cdot,1}$ corresponds to the detrended short-term interest rate, $\lambda_{\cdot,2}$ to the U.S. credit spread, $\lambda_{\cdot,3}$ to the S&P 500 trend and $\lambda_{\cdot,4}$ to the one-month average of the excess stock and bond returns. Similarly, the vector γ describes the sensitivities of the risk aversion function (2) with respect to the state variables for the linear segment and η the corresponding sensitivities for the nonlinear segment of the function. κ_0 denotes the estimate of the threshold value corresponding to these parameter estimates. P-values are in squared brackets.

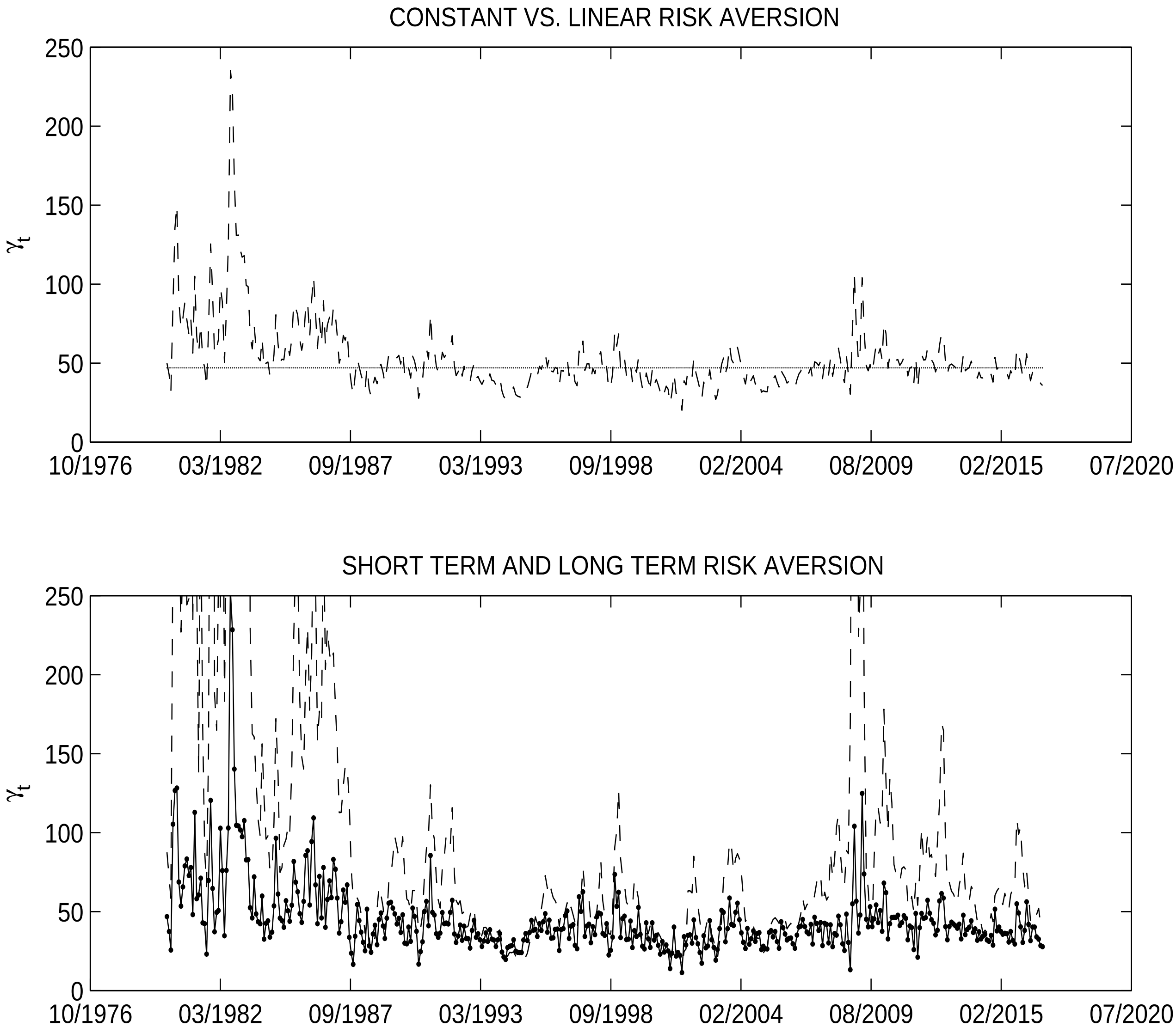

Figure A.1. Dynamics of risk aversion over the period January 1980 to December 2016 for $K = 12$, $\beta = 0.95$. Top panel compares the constant and linear versions of the risk aversion function (2). Flat line for constant risk aversion and dashed line for dynamic risk aversion. Bottom panel compares the two segments defining the nonlinear version of (2). Dotted line for the short-term dynamics of risk aversion (γ parameters in (2)) and dashed for the long-term dynamics (η parameters in (2)).

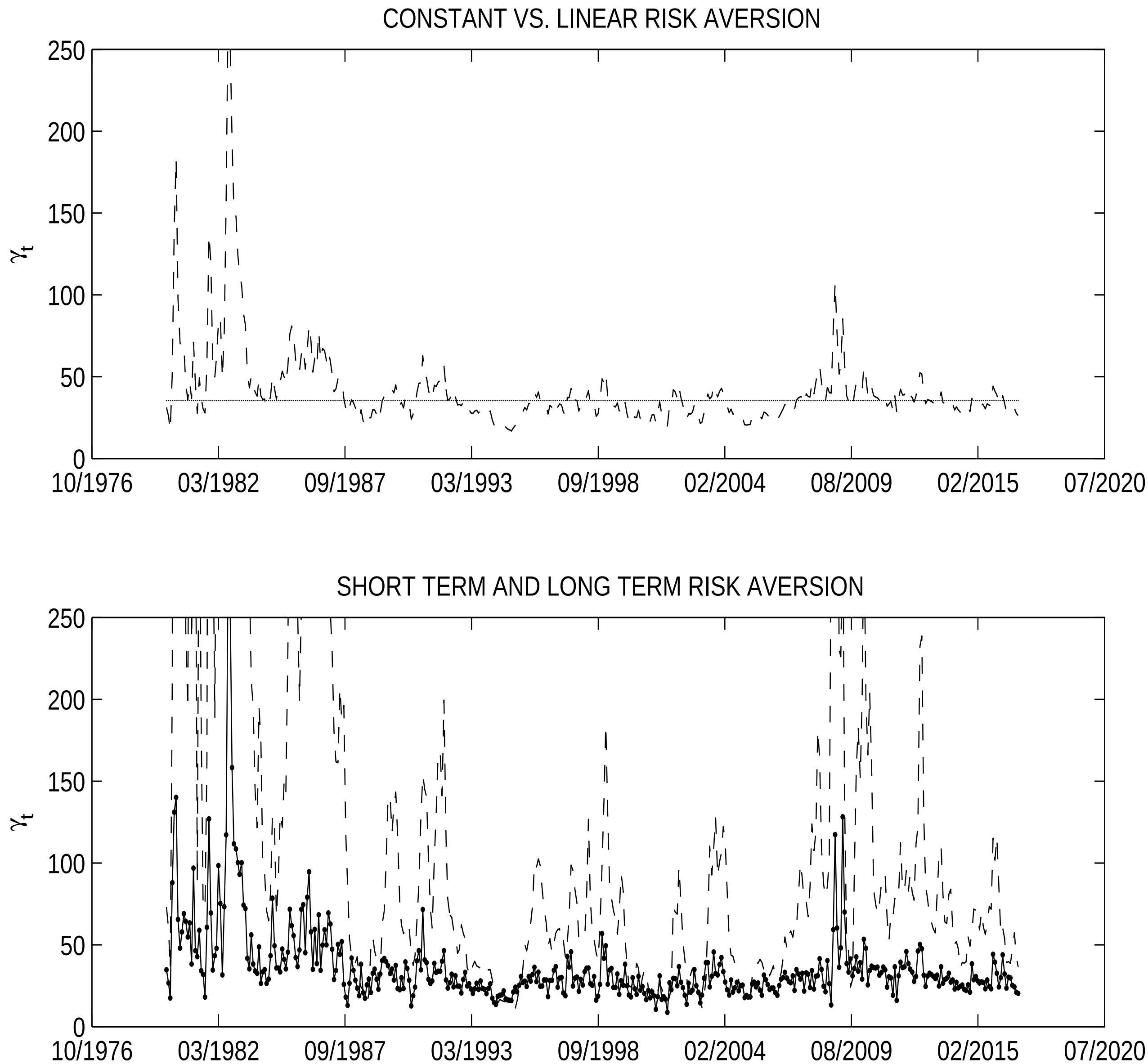

Figure A.2. Dynamics of risk aversion over the period January 1980 to December 2016 for $K = 24$, $\beta = 0.95$. Top panel compares the constant and linear versions of the risk aversion function (2). Flat line for constant risk aversion and dashed line for dynamic risk aversion. Bottom panel compares the two segments defining the nonlinear version of (2). Dotted line for the short-term dynamics of risk aversion (γ parameters in (2)) and dashed for the long-term dynamics (η parameters in (2)).

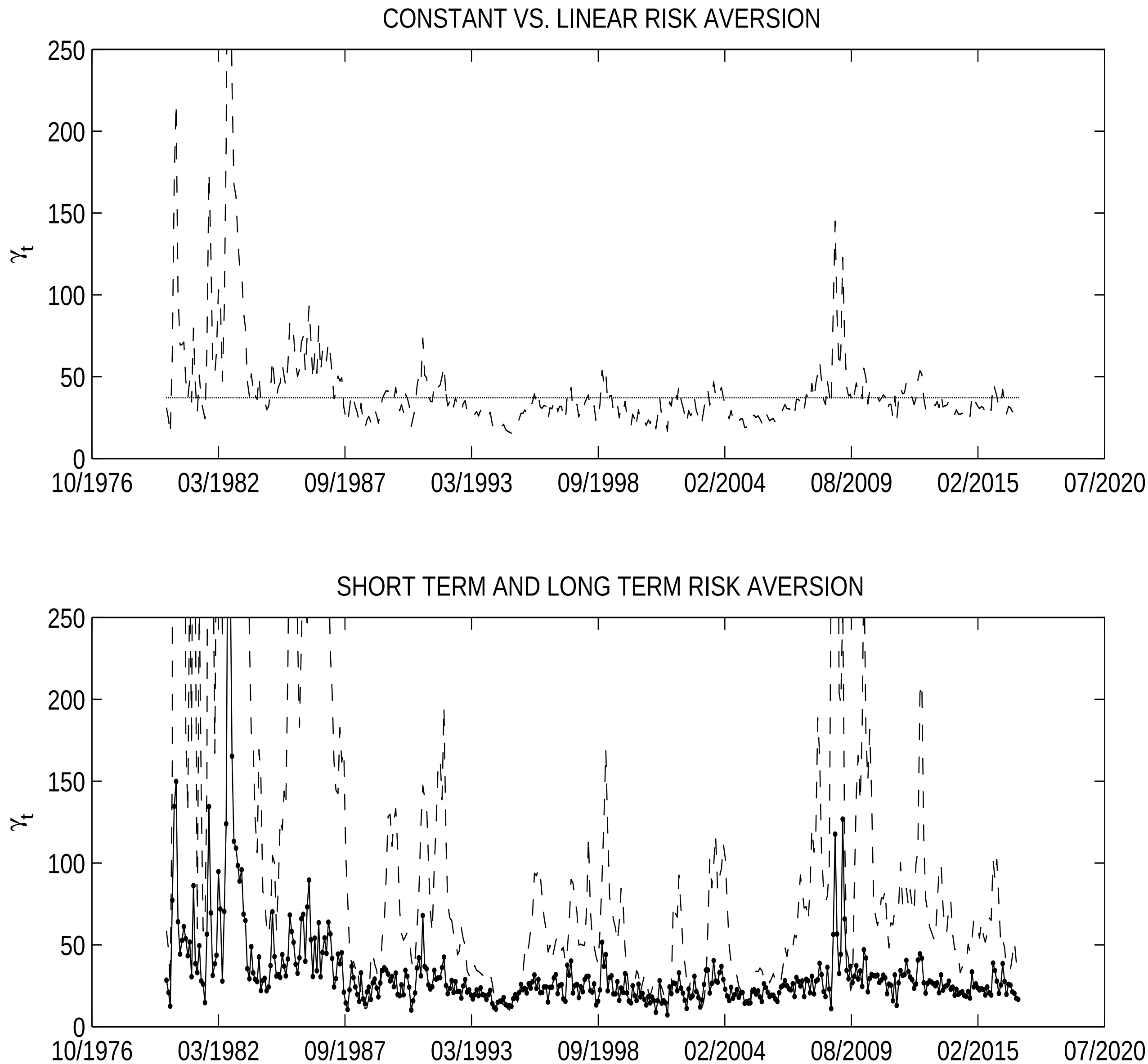

Figure A.3. Dynamics of risk aversion over the period January 1980 to December 2016 for $K = 36$, $\beta = 0.95$. Top panel compares the constant and linear versions of the risk aversion function (2). Flat line for constant risk aversion and dashed line for dynamic risk aversion. Bottom panel compares the two segments defining the nonlinear version of (2). Dotted line for the short-term dynamics of risk aversion (γ parameters in (2)) and dashed for the long-term dynamics (η parameters in (2)).

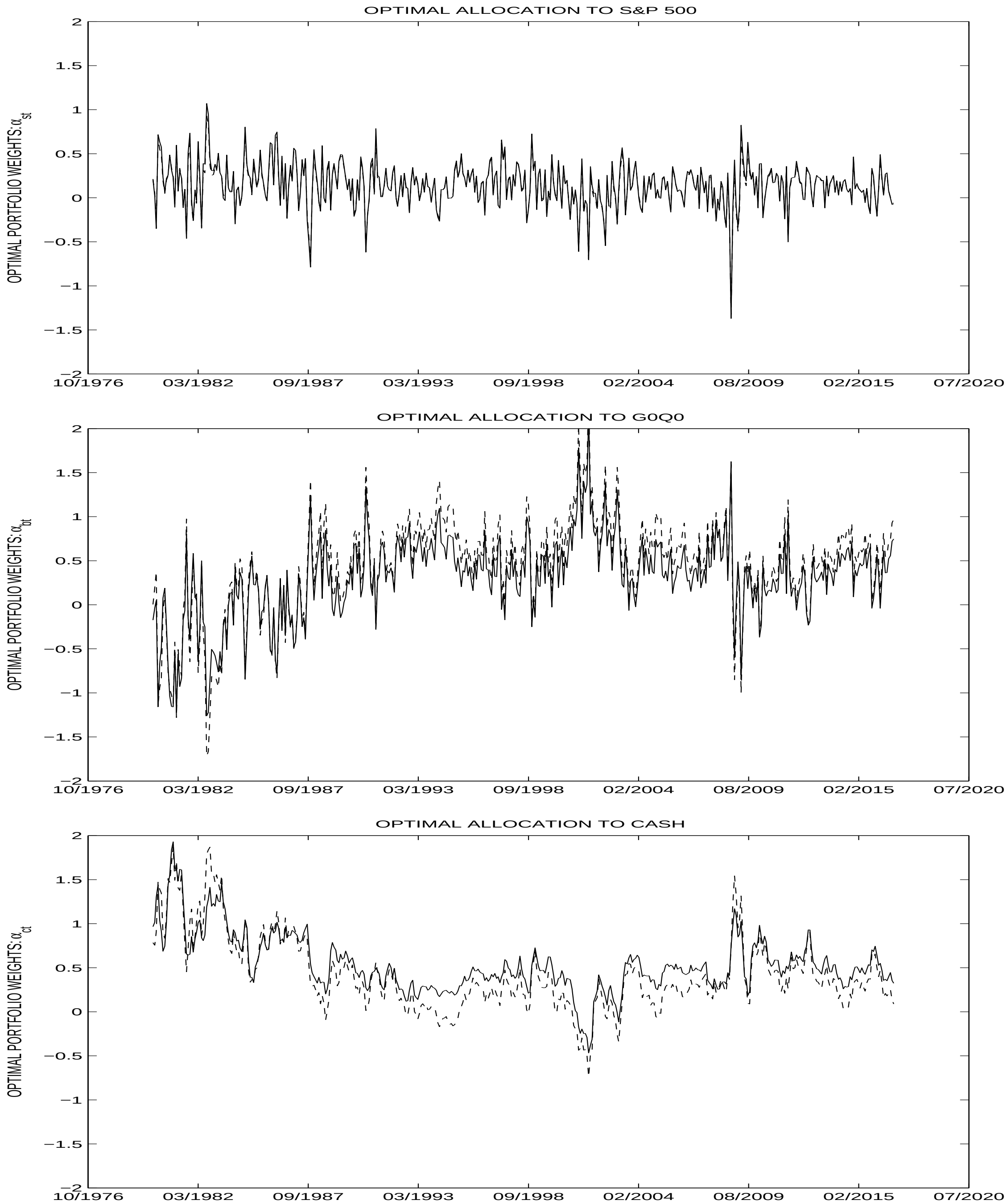

Figure A.4. Dynamics of the optimal portfolio allocation to stocks, bonds and cash over the period January 1980 to December 2016 for $K = 12$, $\beta = 0.95$. Dashed black line for the dynamic nonlinear strategy and solid blue line for the constant risk aversion strategy.

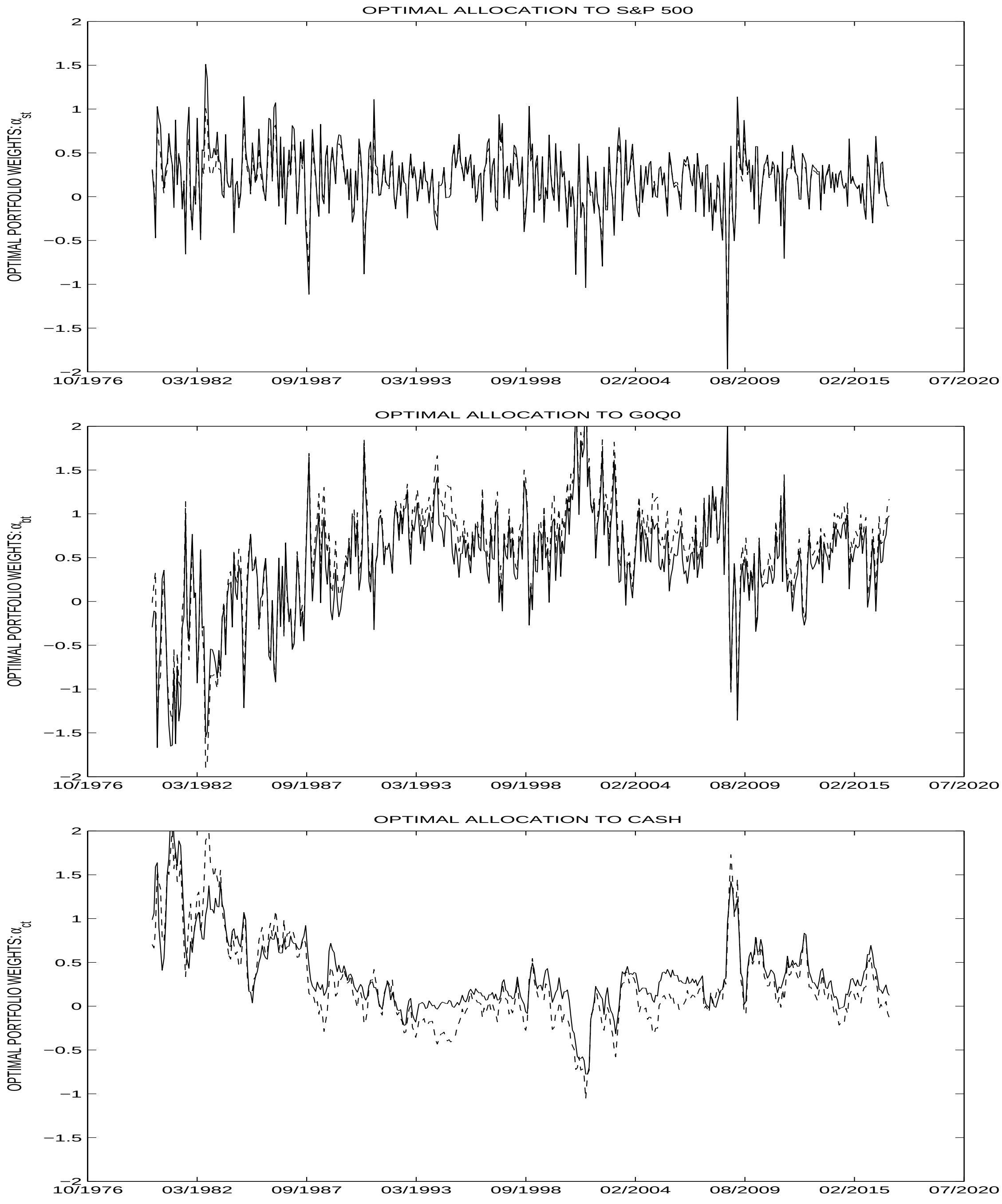

Figure A.5. Dynamics of the optimal portfolio allocation to stocks, bonds and cash over the period January 1980 to December 2016 for $K = 24$, $\beta = 0.95$. Dashed black line for the dynamic nonlinear strategy and solid blue line for the constant risk aversion strategy.

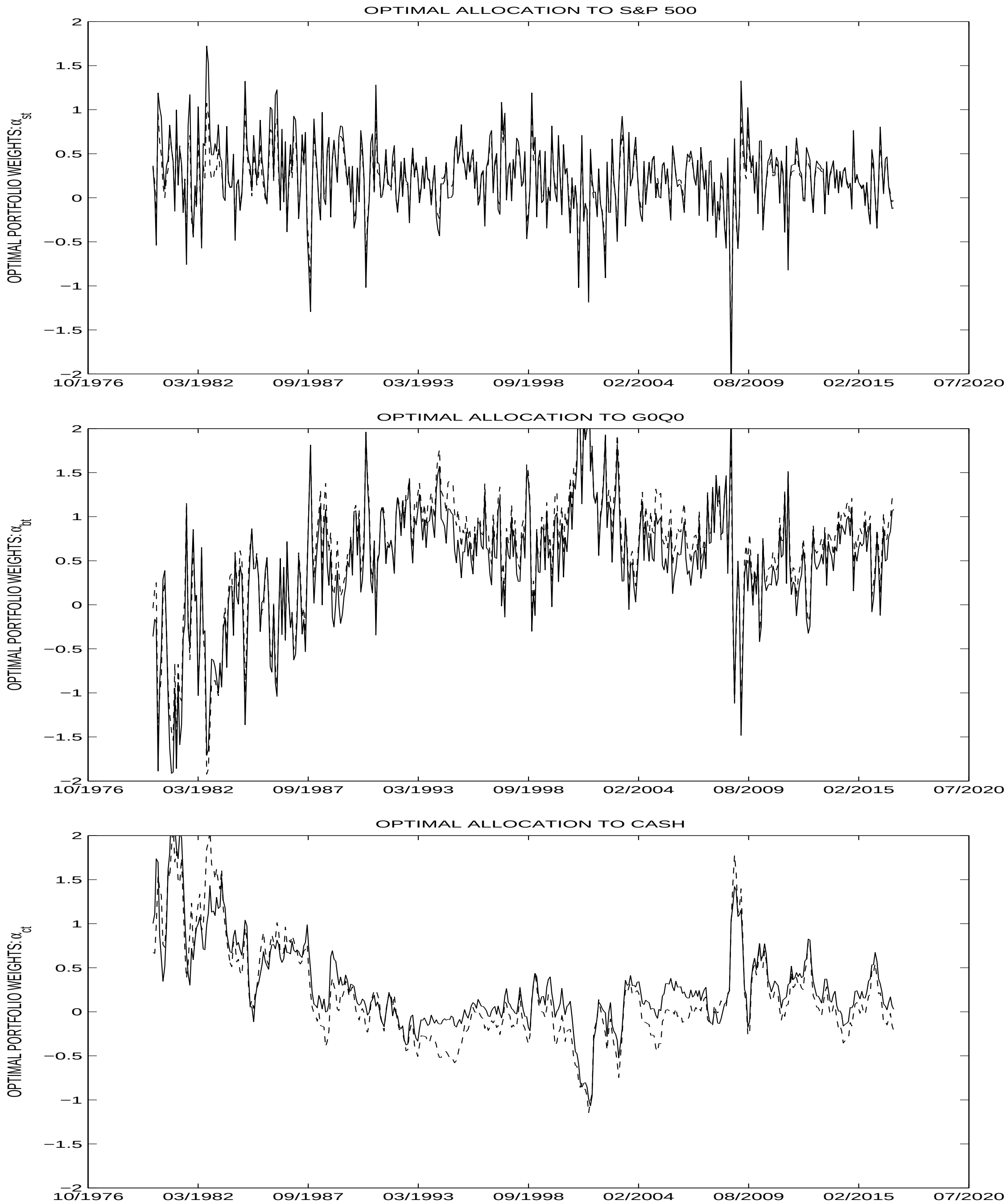

Figure A.6. Dynamics of the optimal portfolio allocation to stocks, bonds and cash over the period January 1980 to December 2016 for $K = 36$, $\beta = 0.95$. Dashed black line for the dynamic nonlinear strategy and solid blue line for the constant risk aversion strategy.

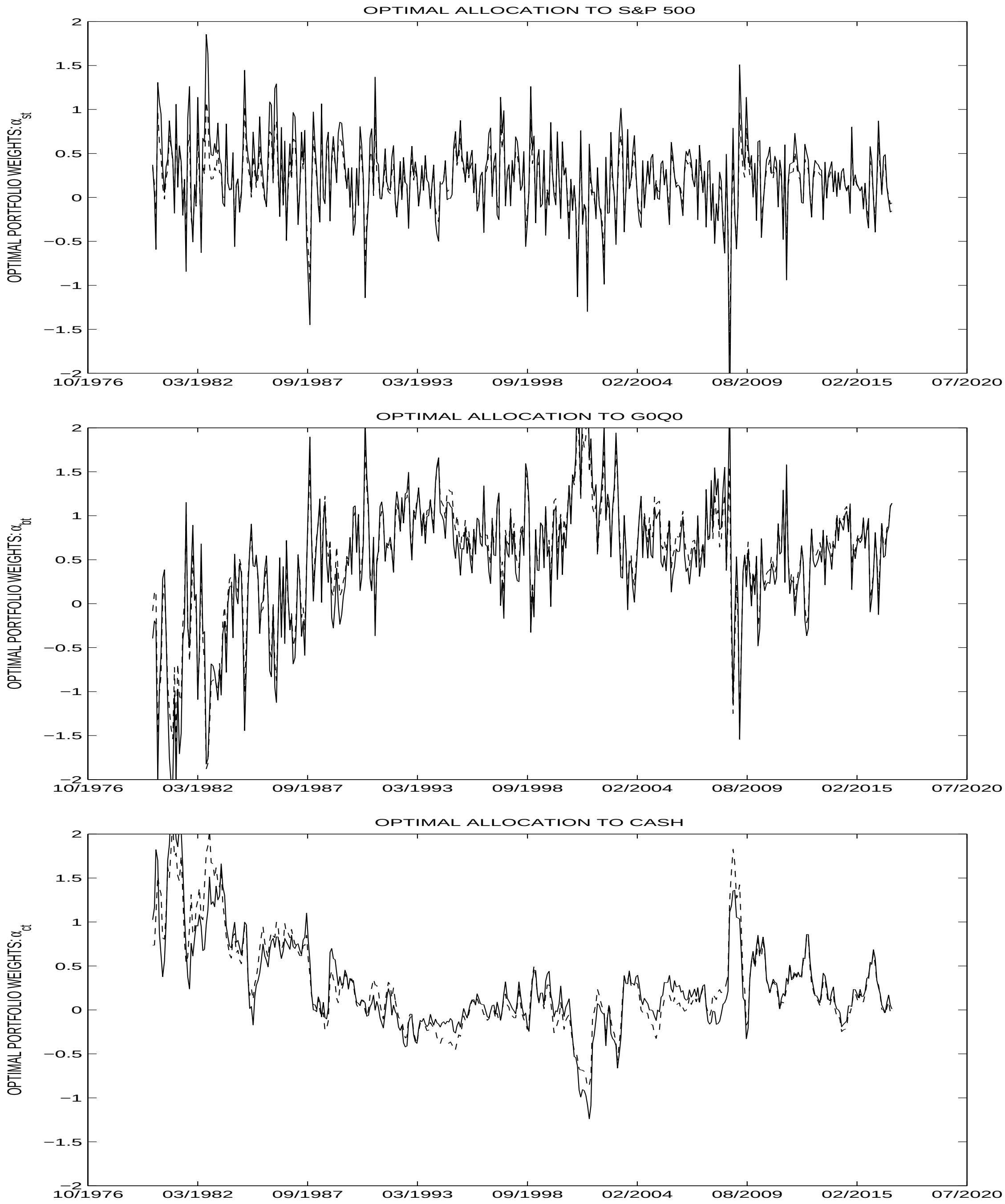

Figure A.7. Dynamics of the optimal portfolio allocation to stocks, bonds and cash over the period January 1980 to December 2016 for $K = 48$, $\beta = 0.95$. Dashed black line for the dynamic nonlinear strategy and solid blue line for the constant risk aversion strategy.

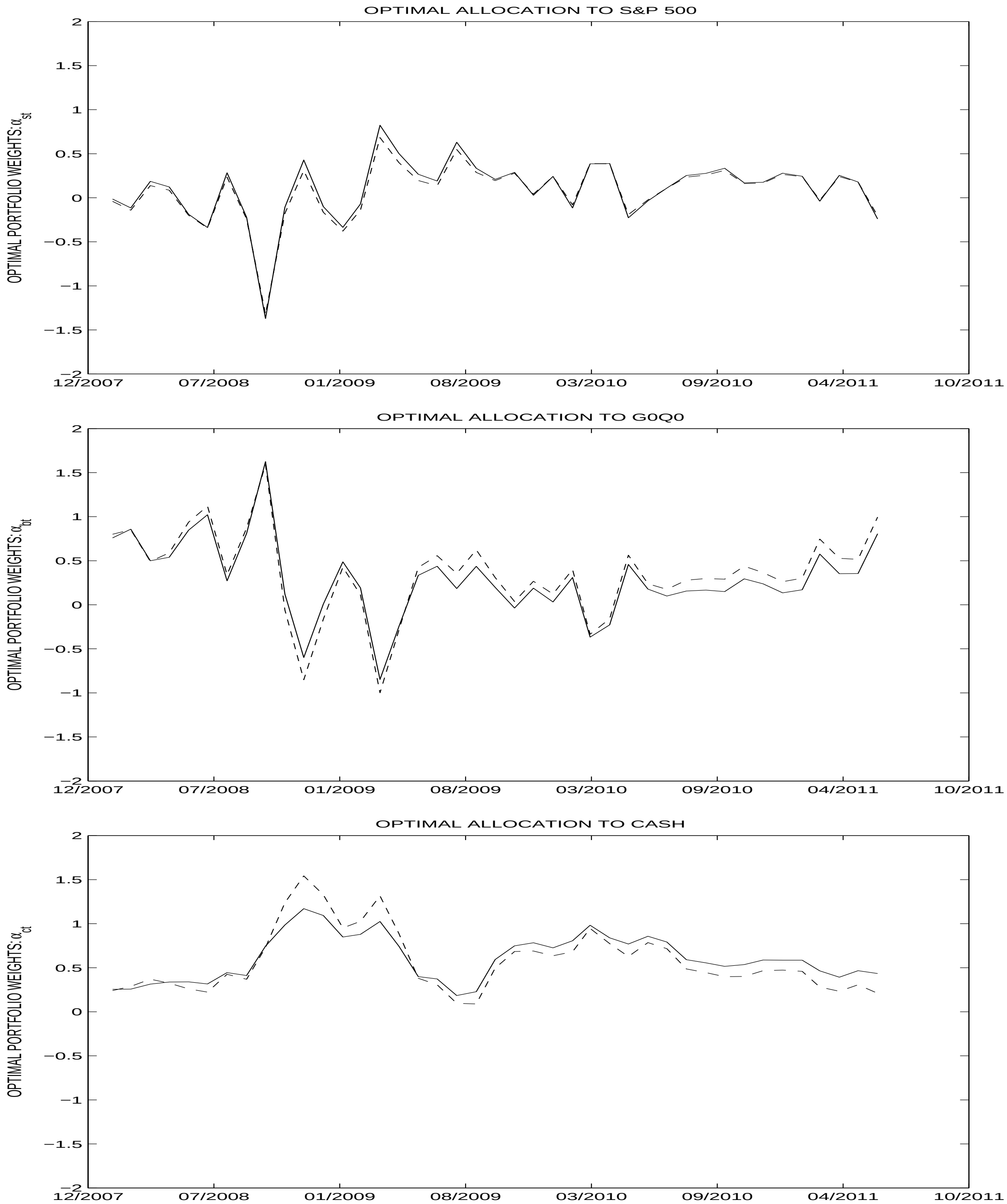

Figure A.8. Dynamics of the optimal portfolio allocation to stocks, bonds and cash over the period 2007-2011 for $K = 12$, $\beta = 0.95$. Dashed black line for the dynamic nonlinear strategy and solid blue line for the constant risk aversion strategy.

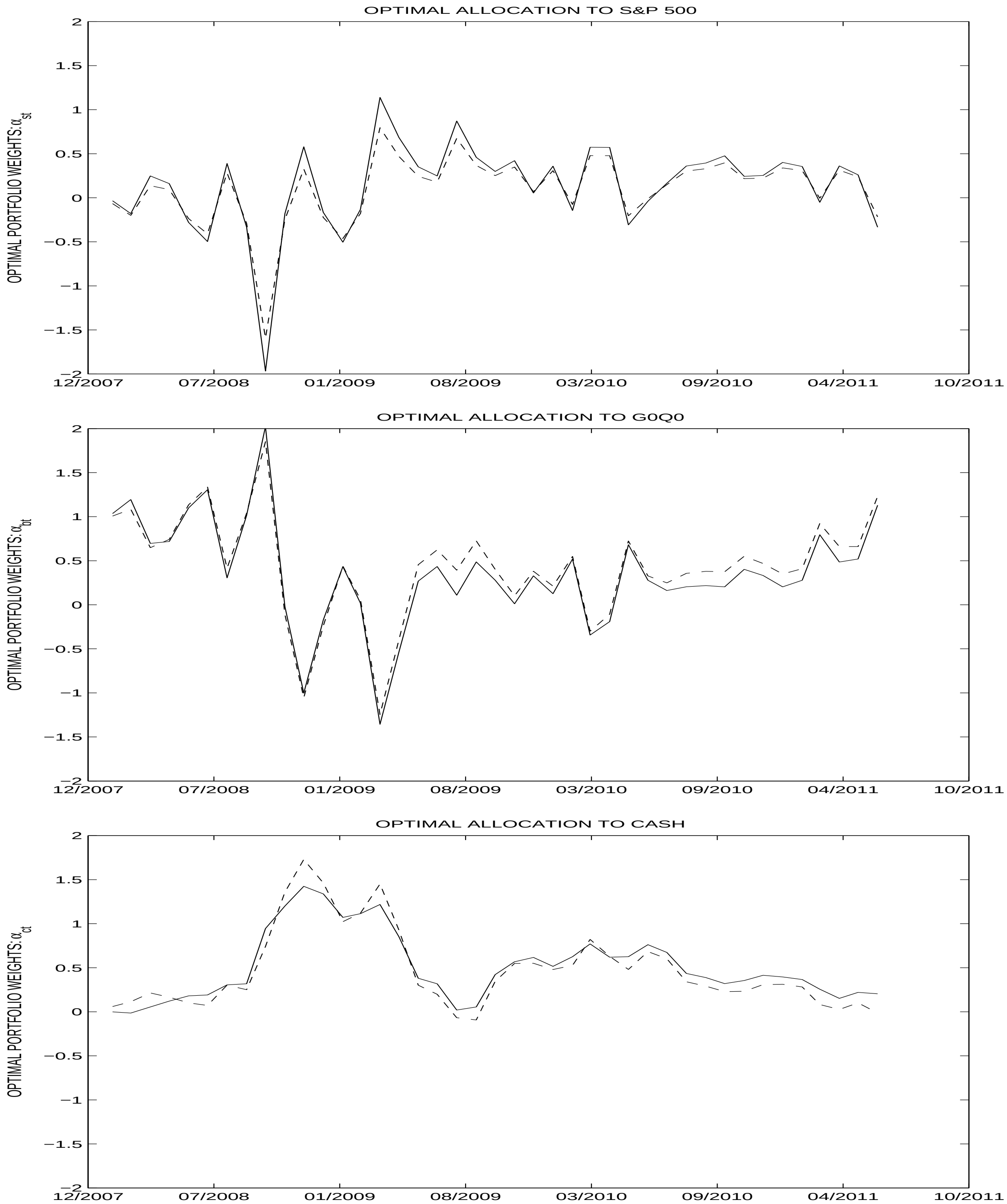

Figure A.9. Dynamics of the optimal portfolio allocation to stocks, bonds and cash over the period 2007-2011 for $K = 24$, $\beta = 0.95$. Dashed black line for the dynamic nonlinear strategy and solid blue line for the constant risk aversion strategy.

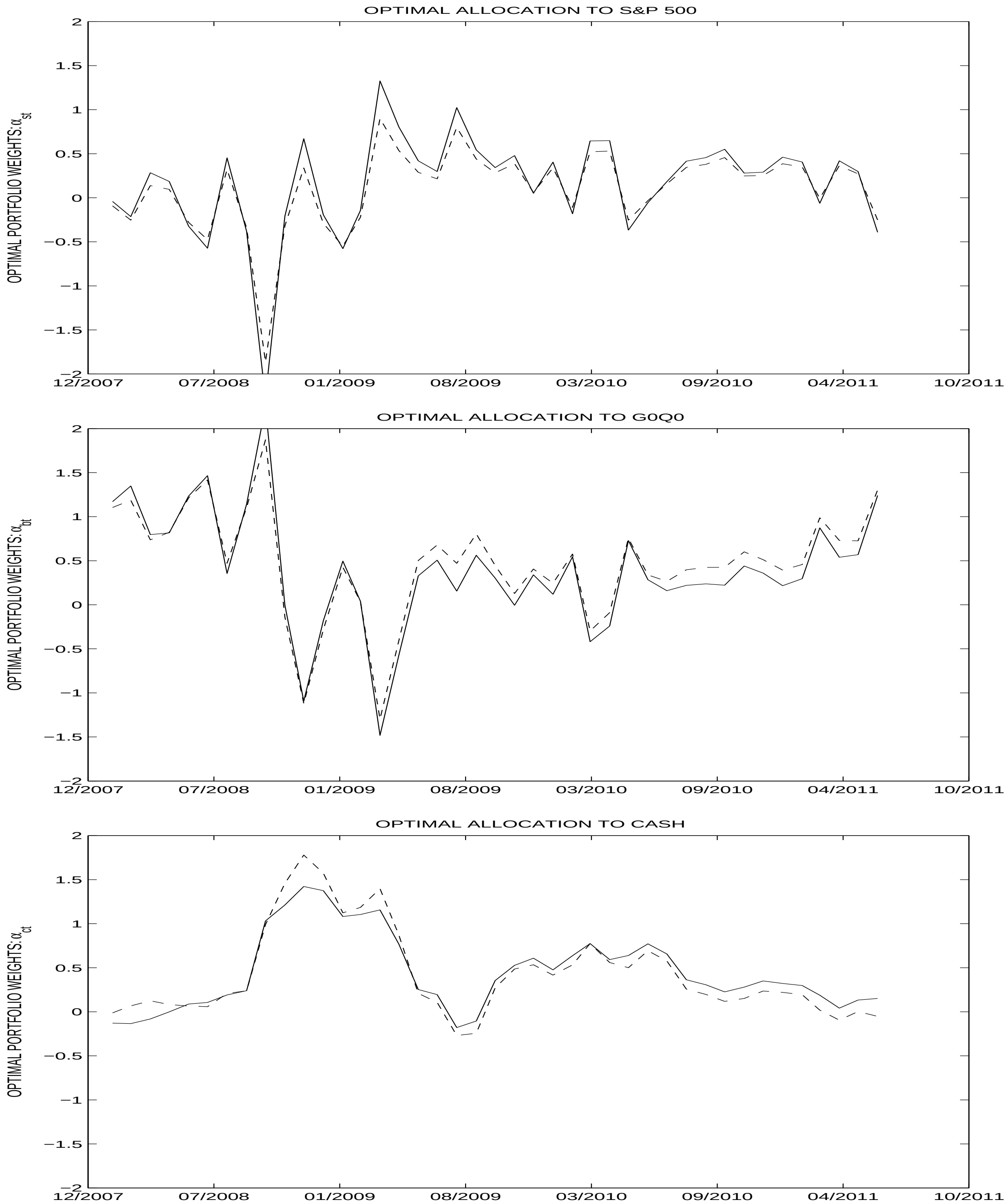

Figure A.10. Dynamics of the optimal portfolio allocation to stocks, bonds and cash over the period 2007-2011 for $K = 36$, $\beta = 0.95$. Dashed black line for the dynamic nonlinear strategy and solid blue line for the constant risk aversion strategy.