#### Macroeconomics 3

Monetary macro

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## Today's lecture

 A sort of introduction to the literature on Monetary macro from an arbitrary perspective

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- Empirics of short run monetary macro
- Small scale New-Keynesian model (3 eq)
- Medium scale New-Keynesian models
- Monetary and fiscal policy interaction

Monetary macro: evidence and DSGE

Christiano, Eichenbaum, and Evans (1999)

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- Clarida, Gali, and Gertler (2000)
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## The impact of monetary shocks

Christiano, Eichenbaum, and Evans (1999)

- The central question of modern monetary macro is the short run effect of monetary policy shocks
- It is widely agreed that in the long run the quantity theory holds

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- In the short run, what's the effect of monetary policy?
- Monetary rule vs monetary shocks?

## Monetary policy rule vs shocks?

- monetary policy is usually modeled as a feedback rule... the CB observes and respond to some macro aggregates
- Rule does not explain fully the instrument
- monetary policy shock
- The rule is usually called Taylor rule and the intuitions for the shock are many
  - changes in the preferences of the central bank... preference of the members of the FOMC
  - may be related to agents expectations about monetary policy (FED's aversion to dissapoint expectations)
  - ► measurement errors in the observed variables by the FED

#### Christiano, Eichenbaum, and Evans (1999)

Estimate the following VAR

$$Z_t = \begin{bmatrix} X_{1t} \\ S_t \\ X_{2t} \end{bmatrix}$$

- Here, the policy instrument  $(S_t)$  is the federal funds rate
- X<sub>1t</sub> includes variables at period t that are part of the information set of the central bank: output, price deflator, index for commodity prices
- X<sub>2t</sub> includes total reserves, nonborrowed reserves plus extended credit, and M1 (or M2), all in logs.
- They have a robustness with  $S_t = NBR$

## Christiano, Eichenbaum, and Evans (1999)

► IRF

- persistent rise in FF and persistent fall in NBR (a monetary policy shock has strong liquidity effect)
- total reserves does not fall initially (borrowed reserves)...
   M1 follows TR

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negative delayed effect on GDP (2 quarters later)

no price effect

#### The model

- We introduce a benchmark model to think about the effects of monetary policy shocks
- Simplified version of Schmitt-Grohé and Uribe (2007)
  - Monopolistic competition
  - Sticky prices (potentially can include sticky wages)
  - Capital accumulation and endogenous labor supply
  - Taylor rule: monetary policy that responds to inflation

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- Continuum of intermediate goods producers
- each of them produces a variety in the interval [0, 1]
- given that each firm produces a differentiated good, they have market power and are demand takers

$$a_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} a_t$$

- variety *i* producer takes demand as given. This demand depends on its price and the average price of the economy
- *P<sub>it</sub>* is a choice variable for the firm that takes demand as given

- Key friction: firms can change prices with *α* probability each period... only *α* firms will not be able to change prices
- what's the average duration?
- Probability of adjusting after one period is  $(1 \alpha)$
- Probability of adjusting after 2 period is  $\alpha(1-\alpha)$
- Probability of adjusting after 3 period is  $\alpha^2(1-\alpha)$
- Average number of periods before readjusting:  $(1 - \alpha) + \alpha(1 - \alpha)2 + \alpha^2(1 - \alpha)3 + ... = \frac{1}{1 - \alpha}...$  why?

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$$(1 - \alpha) + \alpha (1 - \alpha)2 + \alpha^{2} (1 - \alpha)3 + \dots$$
  
=  $(1 - \alpha) [1 + \alpha 2 + \alpha^{2} 3 + \dots]$   
=  $(1 - \alpha) [1 + \alpha + \alpha + \alpha^{2} + \alpha^{2} + \alpha^{2} + \dots]$   
=  $(1 - \alpha) [1 + \alpha + \alpha^{2} + \alpha^{3} + \dots] + (1 - \alpha) [\alpha + \alpha^{2} + \dots] + \dots$   
=  $(1 - \alpha) \frac{1}{1 - \alpha} + (1 - \alpha) \frac{\alpha}{1 - \alpha} + (1 - \alpha) \frac{\alpha^{2}}{1 - \alpha} + \dots$   
=  $1 + \alpha + \alpha^{2} + \dots = \frac{1}{1 - \alpha}$ 

standard: calibrate  $\alpha$  to match the frequency of price adjustment

Frequency of price adjustment

Macro models and micro-data seems to differ a bit

Bils and Klenow (2004): studies price adjustment using micro-data and finds that half of the prices last at most 4.3 months (this is much faster than what standard macro models assume). Also finds that frequency of price changes differ across goods

The macro model tend to overpredict persistence and underpredict volatility (prices change less often)

What happens to firms that do not change prices?

$$P_{it} = P_{it-1}$$

$$P_{it} = \pi^* P_{it-1}$$

$$P_{it} = \pi_{t-1}^{\chi} P_{it-1}$$

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•  $\chi$  allows for partial indexation to lagged inflation

Hire labor and capital with Cobb-Douglas technology

$$y_{it} = z_t F(h_{it}, k_{it})$$

Serve demand

$$y_{it} \geq a_{it}$$

- Firms maximize present discounted value of profits
- Period's t profits

$$\Phi_{it} = \frac{P_{it}a_{it}}{P_t} - u_t k_{it} - w_t h_{it}$$

$$\mathbb{E}_0 \sum_{t=0}^{\infty} r_{0,t} P_t \Phi_{it}$$

$$\mathbb{E}_0 \sum_{t=0}^{\infty} r_{0,t} P_t \left\{ \frac{P_{it} a_{it}}{P_t} - u_t k_{it} - w_t h_{it} \right\}$$

subject to

$$z_t F(h_{it}, k_{it}) \ge \left(\frac{P_{it}}{P_t}\right)^{-\eta} a_t$$

Then

$$\mathbb{E}_{0}\sum_{t=0}^{\infty}r_{0,t}P_{t}\left\{\frac{P_{it}a_{it}}{P_{t}}-u_{t}k_{it}-w_{t}h_{it}+mc_{it}\left(z_{t}F(h_{it},k_{it})-a_{it}\right)\right\}$$

note we can chop the problem in the static (choose labor and capital) and the dynamic (update prices) relaxing the constraint a little bit (increasing output a little bit) costs its marginal cost... Lagrange multiplier

$$u_t = mc_{it}z_tF_k(h_{it}, k_{it})$$
$$w_t = mc_{it}z_tF_h(h_{it}, k_{it})$$

Then

$$\frac{u_t}{w_t} = \frac{F_k(h_{it}, k_{it})}{F_h(h_{it}, k_{it})} = \frac{F_k\left(\frac{h_{it}}{k_{it}}, 1\right)}{F_h\left(\frac{h_{it}}{k_{it}}, 1\right)}$$

k/h is constant, then the marginal cost is the same across firms

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$$\mathbb{E}_t \sum_{s=t}^{\infty} \alpha^{s-t} r_{t,s} P_s \left\{ \frac{P_{it} a_{is}}{P_s} - u_s k_{is} - w_s h_{is} \right\}$$

subject to

$$z_s F(h_{is}, k_{is}) \ge \left(\frac{P_{it}}{P_s}\right)^{-\eta} a_s$$

Then

$$\mathbb{E}_{t} \sum_{s=t}^{\infty} \alpha^{s-t} r_{t,s} P_{s} \left\{ \left( \frac{P_{it}}{P_{s}} \right)^{1-\eta} a_{s} - u_{s} k_{is} - w_{s} h_{is} + mc_{s} \left( z_{s} F(h_{is}, k_{is}) - \left( \frac{P_{it}}{P_{s}} \right)^{-\eta} a_{s} \right) \right\}$$

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$$\mathbb{E}_t \sum_{s=t}^{\infty} \alpha^{s-t} r_{t,s} P_s \left\{ (1-\eta) \left( \frac{P_{it}}{P_s} \right)^{-\eta} \frac{a_s}{P_s} + \eta m c_s \left( \frac{P_{it}}{P_s} \right)^{-\eta} \frac{a_s P_s}{P_s P_{it}} \right\} = 0$$

reorder

$$\mathbb{E}_t \sum_{s=t}^{\infty} \alpha^{s-t} r_{t,s} P_s a_s \left(\frac{P_{it}}{P_s}\right)^{-\eta} \left\{\frac{(\eta-1)}{\eta} \frac{P_{it}}{P_s} - mc_s\right\} = 0$$

This is optimal pricing for monopolistic competition, marginal costs equal marginal revenue (in present value!) Marginal cost is the same for all firms, so optimal price too! The equilibrium is symmetric

The last expression is not useful, we cannot put it in the computer

$$\underbrace{\mathbb{E}_{t}\sum_{s=t}^{\infty}\alpha^{s-t}r_{t,s}P_{s}a_{s}\left(\frac{P_{it}}{P_{s}}\right)^{-\eta}mc_{s}}_{P_{t}x_{2t}}=\underbrace{\mathbb{E}_{t}\sum_{s=t}^{\infty}\alpha^{s-t}r_{t,s}P_{s}a_{s}\left(\frac{P_{it}}{P_{s}}\right)^{-\eta}\frac{(\eta-1)}{\eta}\frac{P_{it}}{P_{s}}}_{P_{t}x_{1t}}$$

$$P_t x_{2t} = \mathbb{E}_t \sum_{s=t}^{\infty} \alpha^{s-t} r_{t,s} P_s a_s \left(\frac{P_{it}}{P_s}\right)^{-\eta} mc_s$$
$$P_t x_{1t} = \mathbb{E}_t \sum_{s=t}^{\infty} \alpha^{s-t} r_{t,s} P_s a_s \left(\frac{P_{it}}{P_s}\right)^{-\eta} \frac{(\eta-1)}{\eta} \frac{P_{it}}{P_s}$$

still not useful, but we will rewrite them in recursive way for  $x_{1t}$  and  $x_{2t}$ 

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#### Households

$$\max \mathbb{E}_0 \sum_{t=0}^{n} \beta^t u(c_t, h_t)$$
where  $c_t = \left[ \int_0^1 c_{it}^{1-\frac{1}{\eta}} di \right]^{\frac{1}{1-\frac{1}{\eta}}}$ 
Note, again, we have 2 problems. The intertemporal and the intratemporal problem to choose consumption of each variety.
The intratemporal problem can be solved by a cost minimization problem

 $\infty$ 

$$\min P_t c_t = \int_0^1 P_{it} c_{it} di$$

subject to the aggregation technology  $c_t = \begin{bmatrix} \int_0^1 c_{it}^{1-\frac{1}{\eta}} di \end{bmatrix}_{t=1}^{\frac{1}{1-\frac{1}{\eta}}}$ 

#### Households

Solution for the intratemporal problem

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t$$

with

$$P_t = \left(\int_0^1 P_{it}^{1-\eta} di\right)^{\frac{1}{1-\eta}}$$

We assume that households own capital

$$k_{t+1} = (1-\delta)k_t + x_t$$

with  $x_t = \left[\int_0^1 x_{it}^{1-\frac{1}{\eta}} di\right]^{\frac{1}{1-\frac{1}{\eta}}}$ i.e. same substitutability between investment and consumption goods

#### Households

Now we have to solve the intertemporal households problem Budget constraint

$$\mathbb{E}_{t}r_{t,t+1}A_{t+1} + P_{t}c_{t} + P_{t}x_{t} = P_{t}w_{t}h_{t} + P_{t}u_{t}k_{t} + A_{t} + P_{t}\int_{0}^{1}\Phi_{it}di$$

By no arbitrage

$$\frac{1}{R_t} = \mathbb{E}_t r_{t,t+1}$$

$$u'_c(c_t, h_t) = \lambda_t$$

$$\frac{u'_h(c_t, h_t)}{u'_c(c_t, h_t)} = w_t$$

$$\lambda_t = \beta \mathbb{E}_t \left[ (u_{t+1} + 1 - \delta) \lambda_{t+1} \right]$$

$$\lambda_t r_{t,t+1} = \beta \left[ \lambda_{t+1} \frac{P_t}{P_{t+1}} \right]$$



$$y_{it} = c_{it} + x_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t + \left(\frac{P_{it}}{P_t}\right)^{-\eta} x_t$$
$$z_t F(h_{it}, k_{it}) = \left(\frac{P_{it}}{P_t}\right)^{-\eta} (c_t + x_t)$$

Integrate over all varieties

$$z_t F(h_t, k_t) = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di \left(c_t + x_t\right)$$

The first integral does not drop!

$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di$$

is the cost of price dispersion

## Aggregation

those that update prices choose the same one  $\tilde{P}_t$  while the other ones keep the previous period prices

$$s_{t} = (1 - \alpha) \left(\frac{\tilde{P}_{t}}{P_{t}}\right)^{-\eta} + \int_{\alpha} \left(\frac{P_{it-1}}{P_{t}}\right)^{-\eta} di$$
$$s_{t} = (1 - \alpha) \left(\frac{\tilde{P}_{t}}{P_{t}}\right)^{-\eta} + \left(\frac{P_{t-1}}{P_{t}}\right)^{-\eta} \int_{\alpha} \left(\frac{P_{it-1}}{P_{t-1}}\right)^{-\eta} di$$
$$s_{t} = (1 - \alpha) \left(\frac{\tilde{P}_{t}}{P_{t}}\right)^{-\eta} + \left(\frac{P_{t-1}}{P_{t}}\right)^{-\eta} s_{t-1}$$

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## Aggregation

Now, the cost of price dispersion is always larger than 1. Let

$$\nu = \left(\frac{P_{it}}{P_t}\right)^{1-\eta}$$

From the price definition we know that

$$1 = \int_0^1 \nu_{it} di = \left(\int_0^1 \nu_{it} di\right)^{\frac{\eta}{\eta-1}}$$

But, by Jensen's inequality

$$1 = \left(\int_0^1 \nu_{it} di\right)^{\frac{\eta}{\eta-1}} \le \int_0^1 \left(\nu_{it}^{\frac{\eta}{\eta-1}} di\right) = s_t$$

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Price level evolves as

$$P_t^{1-\eta} = \int_0^1 P_{it}^{1-\eta}$$

From the price definition we know that

$$P_t^{1-\eta} = (1-\alpha)\tilde{P}_t^{1-\eta} + \int_{\alpha} P_{it-1}^{1-\eta}$$
$$1 = (1-\alpha)\tilde{p}_t^{1-\eta} + \alpha \left(\frac{P_{t-1}}{P_t}\right)^{1-\eta}$$

The last step comes from the fact that price change is random + have continuum of firms  $\rightarrow$  the integral over a subset of the unit interval is proportional to the integral over the whole interval

#### Optimality conditions

Use  $\tilde{p}_t = \frac{P_t}{P_t}$  and  $\Pi_t = \frac{P_t}{P_{t-1}}$ . Simplify the system abstracting from capital. Collect Optimality conditions

$$s_{t} = (1 - \alpha)\tilde{p}_{t}^{-\eta} + alpha\Pi_{t}^{\eta}s_{t-1}$$

$$z_{t}h_{t} = s_{t}c_{t}$$

$$1 = (1 - \alpha)\tilde{p}_{t}^{1-\eta} + \alpha\pi_{t}^{\eta-1}$$

$$x_{t}^{1} = \tilde{p}_{t}^{1-\eta}a_{t}\frac{\eta - 1}{\eta} + \alpha\mathbb{E}_{t}r_{t,t+1}\left(\frac{\tilde{p}_{t}}{\tilde{p}_{t+1}}\right)^{1-\eta}\Pi_{t+1}^{\eta}x_{t+1}^{1}$$

$$x_{t}^{2} = \tilde{p}_{t}^{-\eta}a_{t}mc_{t} + \alpha\mathbb{E}_{t}r_{t,t+1}\left(\frac{\tilde{p}_{t}}{\tilde{p}_{t+1}}\right)^{-\eta}\Pi_{t+1}^{\eta+1}x_{t+1}^{2}$$

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# Optimality conditions

$$\begin{aligned} x_t^2 &= x_t^1 \\ w_t &= mc_t z_t F_h(h_t) \\ \frac{1}{R_t} &= \mathbb{E}_t r_{t,t+1} \\ u_c'(c_t,h_t) &= \lambda_t \\ \frac{u_h'(c_t,h_t)}{u_c'(c_t,h_t)} &= w_t \\ \lambda_t r_{t,t+1} &= \beta \left[ \lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \end{aligned}$$

## Optimality conditions

Unknowns are:  $\{c_t, h_t, \lambda_t, w_t, mc_t, s_t, \tilde{p}_t, \pi_t, r_{t,t+1}, x_t^1, x_t^2\}$  given a TFP shock and a policy rule for  $R_t$ Homework 2: solve this model and show how well it does to replicate standard monetary facts.

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Lets simplify the framework a bit more

The very basic setting already implies a short run effect of monetary shocks/policy in the real economy, in line with IS/LM

One key difference, intertemporal (dynamic) content... current dynamics depend on expected dynamics and are consistent with individual optimization

Clarida, Gali, and Gertler (1999) presents the 3 equation version of this model

Intertemporal IS equation

$$x_t = -\varphi \left[ i_t - \mathbb{E}_t \pi_{t+1} \right] + \mathbb{E}_t x_{t+1} + g_t$$

Phillips curve

$$\pi_t = \lambda x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$

Here,  $x_t$  denote output gap (difference between output and its natural, i.e. flexible price, level)

The dynamic IS equation comes from log-linearizing the consumption Euler equation and impose the feasibility condition  $y_t = c_t + g_t$ 

$$x_t = -\varphi \left[ i_t - \mathbb{E}_t \pi_{t+1} \right] + \mathbb{E}_t x_{t+1} + g_t$$

Current output depends on expected future output... higher expected future output rises todays output... Demand driven fluctuations, people expect to be richer in the future, the wealth effect push to higher consumption today that pushes output up

The negative effect with respect to the real interest rate reflects intertemporal substitution effect

Iterating forward

$$x_t = \mathbb{E}_t \sum_{j=0}^{\infty} \left\{ -\varphi \left[ i_{t+j} - \mathbb{E}_t \pi_{t+1+j} \right] + g_{t+j} \right\}$$

Phillips curve

$$\pi_t = \lambda x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$$

 $\lambda$  is a reduced form of many structural parameters

It is a log-linear approximation of the aggregation of pricing decision by firms

 $u_t$  is a cost-push shock: it is basically anything that can affect expected marginal costs (allows for non-demand driven inflation)

Iterating forward

$$\pi_t = \mathbb{E}_t \sum_{j=0}^{\infty} \left\{ \beta^j \left[ \lambda x_{t+j} + u_{t+j} \right] \right\}$$

Close the model assuming that  $i_t$ , the nominal interest rate, is the monetary instrument

This is what is usually called the 3 equation model and contains the main ingredients and intuition of the medium scale models

Now with this very simple setting we focus on optimal policy

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# Optimal policy

$$\max -\frac{1}{2}\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \gamma x_{t+j}^2 + \pi_{t+j}^2 \right] \right\}$$

 $\gamma$  weights the relative importance of the output gap versus inflation

The expression implicitly assumes that target inflation is zero This is a sort of pragmatic approach to optimal policy... takes a given loss function

Rotemberg and Woodford (1999) provide a formal justification to this (maybe obtained from a quadratic approximation of the utility based welfare function)

# Optimal policy

Rules versus discretion

Problem: choose the path for output gap and inflation that maximize the objective function

Now: values for these variables depend on current policies but also expectations about future policies

This is why rules and discretion may lead to different dynamics

Credibility about future policies become an important issue

CB under discretion chooses interest rate re-optimizing every period

CB under a rule chooses a plan at the beginning and sticks to it

## Optimal policy without commitmment

- Each period the CB chooses  $\{x_t, \pi_t, i_t\}$  to maximize the
- objective function subject to the Phillips curve. The IS equation is used to pin down the interest rate
- It cannot manipulate expectation due to the credibility issue... then it has to take expectations as given
- Note there are no state variables, so the problem reduces to a sequence of static problems

$$-\frac{1}{2}\left(\gamma x_t^2+\pi_t^2\right)+F_t$$

subject to

$$\pi_t = \lambda x_t + f_t$$

where  $F_t$  and  $f_t$  are the remainder part of the function that the government takes as given

### Optimal policy without commitment

$$x_t = -rac{\lambda}{\gamma}\pi_t$$

The optimal policy implies that the CB leans against the wind, when inflation is high, depress the aggregate demand.

$$x_t = -\lambda q u_t$$
; and  $\pi_t = \gamma q u_t$ 

with  $q = 1/(\lambda^2 + \alpha(1 - \rho\beta))$ . The optimal feedback rule implied by the problem is

$$i_t = \gamma_\pi \mathbb{E}_t \pi_{t+1} + \frac{1}{\varphi} g_t$$

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with  $\gamma_{\pi} > 1$ 

# Optimal policy without commitment A few implications I

Trade off between inflation and output stabilization (only in the presence of cost-push shocks)

- if  $\gamma \to 0$ , then  $\sigma_{\pi} \to 0$
- if  $\gamma \to \infty$ , then  $\sigma_x \to 0$

This trade off disappears without cost push shock... all disturbances comes from aggregate demand so if you stabilize it, you stabilize inflation

# Optimal policy without commitment A few implications II

Optimal policy incorporates inflation targeting If expected inflation rises, interest rates increase to induce a higher real interest rates

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## Optimal policy without commitment A few implications III

The optimal policy has to offset demand shocks  $(g_t)$ :

$$i_t = \gamma_\pi \mathbb{E}_t \pi_{t+1} + \frac{1}{\varphi} g_t$$

The reason is that demand shocks do not force the trade off between objectives, demand shocks increases output gap and increases inflation

# Optimal policy with commitment

Gains from commitment

Suppose the objective function is

$$\max - \frac{1}{2} \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \gamma (x_{t+j} - k)^2 + \pi_{t+j}^2 \right] \right\}$$

here *k* indicates that the optimal output level is larger than the natural output level(maybe there are distortionary taxes and frictions that make the flexible prices output level inefficiently low)

Additional assumption: price setters do not discount the future (simplify the algebra)

Optimality condition is

$$x_t^k = -\frac{\lambda}{\gamma}\pi_t^k + k$$

Optimal policy with commitment Gains from commitment

Plugging in the IS and Phillips curve

$$x_t^k = x_t$$

$$\pi_t^k = \pi_t + \frac{\gamma}{\lambda}k$$

Output gap is not different from the k = 0 case, but you have inflation bias

The private sector discount your tendency to inflate the economy by acting discretionary Inflation rises to the point where the CB is not tempted anymore to increase output

## Optimal policy with commitment Gains from commitment

Optimal rule under commitment in the class of linear rules

$$x_t^c = -\omega u_t$$

#### combining with Phillips curve, implies

$$\pi_t^c = \lambda x_t^c + \beta \mathbb{E}_t \pi_{t+1}^c + u_t = \frac{1 - \lambda \omega}{1 - \beta \rho} u_t$$

or

$$\pi_t^c = \frac{\lambda}{1 - \beta \rho} x_t^c + \frac{1}{1 - \beta \rho} u_t$$

choose optimal value for  $\omega$ 

# Optimal policy with commitment

Gains from commitment

$$\max -\frac{1}{2} (\gamma(x_t^c)^2 + (\pi_t^c)^2) \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( \frac{u_{t+j}}{u_t} \right)^2 \right\}$$

optimality condition is

$$x_t^c = -rac{\lambda}{\gamma^c}\pi_t^c$$

$$\gamma_t^c = \gamma (1 - \beta \rho) < \gamma$$

with

$$x_t = -\lambda q^c u_t;$$
 and  $\pi_t = \gamma^c q^c u_t$ 

with  $q = 1/(\lambda^2 + \alpha^c(1 - \rho\beta))$ .

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# Optimal policy with commitment Gains from commitment

It can be checked that welfare improves with this rule

Plus it is obvious, the planner could have chosen the same values as in the case of discretion but it didn't

However, these (as in discretion) is not really implementable rule, it depends on the observability of cost-push shocks

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