Macroeconomics 3

Business Cycles: Facts and models

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Today's lecture

- A sort of introduction to the literature on Real Business Cycle
- A quick overview of empirical motivation and facts, but mainly disagreement and questions on the sources of business cycle fluctuations
- What are the main business cycle facts?
- Long standing discussion: what drives the business cycle? how should we design models that study it?

US business cycle

Cooley and Prescott (1995)

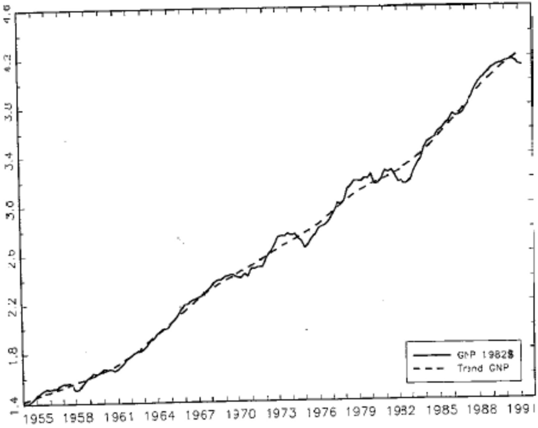


Figure 1.1 Log of Real GNP and Its Growth Component

Trillions of 1982 dollars (log scale).

Note: From Cooley and Prescott (1995)

US business cycle

Cooley and Prescott (1995)

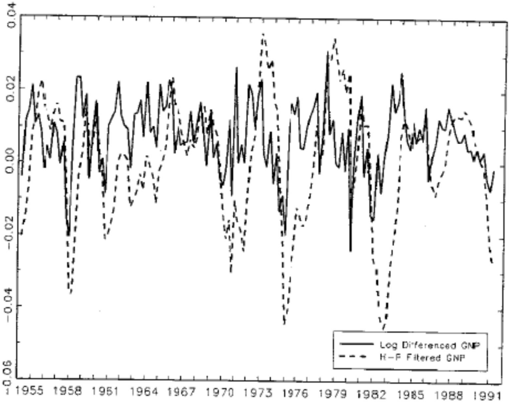


Figure 1.2 H-P–Filtered Cyclical Component and Log-Differenced Real GNP

Note: From Cooley and Prescott (1995)

US business cycle

Cooley and Prescott (1995)

ble 1.1
yclical Behavior of the U.S. Economy: Deviations from Trend of Key Variables, 1954:I–1991:II

Variable	SD%	Cross-Correlation of Output with: $x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
ıtput component												
GNP	1.72	.02	.16	.38	.63	.85	1.0	.85	.63	.38	.16	−.02
ɔnsumption expenditures												
CONS	1.27	.25	.42	.57	.72	.82	.83	.67	.46	.22	−.01	−.20
CNDS	0.86	.22	.40	.55	.68	.78	.77	.64	.47	.27	.06	−.11
CD	4.96	.24	.37	.49	.65	.75	.78	.61	.38	.11	−.13	−.31
vestment												
INV	8.24	.04	.19	.38	.59	.79	.91	.76	.50	.22	−.04	−.24
INVF	5.34	.08	.25	.43	.63	.82	.90	.81	.60	.35	.09	−.12
INVN	5.11	−.26	−.12	.05	.30	.57	.79	.88	.83	.60	.46	.24
INVR	10.7	.42	.55	.65	.72	.74	.63	.39	.11	−.14	−.33	−.43
Ch. INV	17.3	−.03	.07	.22	.38	.53	.67	.51	.27	.04	−.15	−.30
overnment purchases												
GOVT	2.04	.03	−.01	−.03	−.01	−.01	.04	.08	.11	.16	.25	.32
xports and imports												
EXP	5.53	−.48	−.42	−.29	−.10	.15	.37	.50	.54	.54	.52	.44
IMP	4.88	.11	.19	.31	.45	.62	.72	.71	.52	.28	.04	−.18
abor input based on household survey												
HSHOURS	1.59	−.06	.09	.30	.53	.74	.86	.82	.69	.52	.32	.11
HSAVGHRS	0.63	.04	.16	.34	.48	.63	.62	.52	.37	.23	.09	−.05

US business cycle (Cooley and Prescott (1995))

Features of the data

- Output and hours of work have similar volatility in magnitude
- Employment fluctuates as much as output but weekly hours fluctuate less: fluctuation of total hours is driven by entry/exit in the work force
- Consumption of non-durables is smooth
- Investment and durable consumption fluctuate more than output
- Capital stock is less volatile and uncorrelated with output
- Government expenditure is acyclical

US business cycle

More from Uhlig (2003)

	output		hours		labor prod.		wages		TFP	
	HP	Δ	HP	Δ	HP	Δ	HP	Δ	HP	Δ
output	1	1								
hours	0.86	0.70	1	1						
labor prod.	0.54	0.68	0.04	-0.04	1	1				
wages	0.13	0.18	-0.11	-0.24	0.42	0.49	1	1		
TFP	0.81	0.85	0.40	0.23	0.92	0.96	0.32	0.40	1	1

Table 1: *Correlations, using the Hodrick-Prescott (HP)-filter and the first difference filter to detrend the data.*

- Compute labor productivity and TFP
- Output is correlated with hours and labor productivity: key
- Labor productivity and hours are uncorrelated Capital and TFP

Some facts about world business cycle (Uribe and Schmitt-Grohé (2017))

Table 1.3: HP-Filtered Business Cycles

Statistic	All Countries	Poor Countries	Emerging Countries	Rich Countries
Standard Deviations				
σ_y	3.79	4.12	3.98	2.07
σ_c/σ_y	1.08	1.09	1.23	0.87
σ_g/σ_y	2.29	2.53	2.29	1.23
σ_i/σ_y	3.77	3.80	3.79	3.62
σ_x/σ_y	3.50	3.47	3.67	3.42
σ_m/σ_y	3.65	3.70	3.52	3.63
$\sigma_{tb/y}$	1.79	1.64	2.92	0.89
$\sigma_{ca/y}$	1.78	1.71	2.63	1.02
Correlations with y				
y	1.00	1.00	1.00	1.00
c	0.60	0.53	0.68	0.82
g/y	-0.08	0.02	-0.06	-0.56
i	0.69	0.65	0.71	0.86
x	0.19	0.18	0.13	0.30
m	0.32	0.23	0.46	0.58
tb/y	-0.18	-0.08	-0.34	-0.37
tb	-0.20	-0.11	-0.36	-0.36
ca/y	-0.32	-0.29	-0.39	-0.38
ca	-0.33	-0.29	-0.41	-0.37
Serial Correlations				
y	0.46	0.39	0.60	0.55
c	0.36	0.29	0.44	0.53
g	0.51	0.48	0.52	0.65
i	0.34	0.27	0.45	0.46
x	0.47	0.47	0.44	0.46
m	0.42	0.43	0.44	0.33
tb/y	0.39	0.36	0.42	0.47
ca/y	0.39	0.36	0.39	0.54
Means				
tb/y	-1.3	-1.6	-1.4	-0.0
$(x+m)/y$	36.5	32.5	46.4	40.4

Note. See table 1.1. The variables y, c, g, i, x, and m are HP filtered in logs and expressed in percent deviations from trend, and the variables tb/y and ca/y are HP filtered in levels and expressed in percentage points of output. The variables tb and ca were scaled by the secular component of GDP and then HP-filtered. The parameter λ of the HP filter takes the value 100.

Facts

1. US is much less volatile than the rest of the world
2. $std(c_t)/std(y_t) = 1.02$... the role of durable goods
3. US $corr(g_t, y_t) = -0.32$... government spending is strongly counter-cyclical
4. US is much less open than the rest of the world, $(x + m)/y = 18\%$, while for the world is about twice as much
5. Conditional on income level

Facts overall

1. High global volatility
2. High volatility of government consumption
3. Global rank of volatilities
4. Procyclical aggregate demand components
5. Countercyclical trade balance and the current account
6. Acyclical Government consumption to GDP
7. Persistence

Facts Emerging Economies

1. Excess volatility of emerging and poor countries
2. Less consumption smoothing
3. Countercyclical trade balance increases with income
4. Countercyclical government spending increases with income

Business cycle facts

1. These are some of the most important facts that business cycle models aim to replicate and understand
2. A second question to the design of DSGE models is about the shocks
3. What type of shocks drive the business cycle fluctuations
4. Everything here is open to discussion

Cochrane (1994)

- Cochrane (1994) looks for the main shock that drives business cycles (in USA)
- What drives business cycle? Factor prices (oil), monetary policy, government spending, taxes, technology shocks, bank regulation, international shocks, sectorial shocks (shifters)
- How can we interpret technology shocks (all other!)
- Consumption (demand) shocks... but consumption is endogenous so: news shocks about all other shocks?

Cochrane (1994)

- Why to look for one shock? maybe there are multiple shocks
- maybe different shocks operate for different crisis
- Still, there are regularities: (1) investment and durables fall more than output, (2) hours fall as much as output and (3) consumption fall much less than output.
- usually crisis look pretty much alike (this claims for a unique shock)
- Different shocks induce different co-movement

Cochrane (1994)

Table 1:

	Shock and Horizon											
	1 Qtr.			1 Year			2 Year			3 Year		
Var. of	$M2$	y	p	$M2$	y	p	$M2$	y	p	$M2$	y	p
$M2$	100	0	0	99	1	0	98	0	2	94	1	5
y	1	99	0	32	68	0	70	30	0	82	17	1
p	1	3	96	0	7	92	1	17	83	3	24	73

Variance decomposition from $M2 - y - p$ VAR. Table entries are percent of horizon step ahead forecast error variance of the row variable explained by the column shock. VARs in log-levels with 4 lags, orthogonalized in the given order $(M2, y, p)$. Quarterly data 1959:1–1992:4.

Cochrane (1994)

Table 4:

VAR	Forecast error σ^2 1Q	1Y	2Y	3Y	Var Δy 1Q	1Y
M1 y p	3	16	20	20	11	19
y M1 p	3	1	1	1	9	9
M1 y p; trend	2	8	7	8	10	16
M1 ff c y p	3	2	3	8	8	10
M1 ff c y p, error corr.	2	6	5	4		
M1 ff c y p; e.c.; MS shocks	5	7	5	4		

Percent of output variance explained by M1 shocks.

Cochrane (1994)

Table 12:

Author	Statistic or Observation	Fraction
Prescott	$\frac{\sigma^2(HP\ filtered\ y_{model})}{\sigma^2(HP\ filtered\ y_{data})}$	70%
Eichenbaum	Sampling error	78% +/- 64%
Blanchard-Quah	$\sigma^2(y)$ from perm. shock	$\frac{1Y\ \ 2Y\ \ 3Y}{12\%\,27\%\,37\%}$
Rotemberg-Woodford	$\frac{\sigma^2(E_t(y_{t+k})-y_t)model}{\sigma^2(E_t(y_{t+k})-y_t)data}$	0.002%
Christiano	$\frac{\sigma^2(y-a)}{\sigma^2(y)}, 1 - corr(y,a)$	tiny
Beveridge-Nelson	B-N trend, not HP filter	0.003%
Gordon	corr(shock, output)	0
Burnside, Eichenbaum, Rebelo	Labor hoarding	31%

Summary of calculations of the contribution of technology shocks to output variability. Author gives the inspiration for the calculation: numbers are my calculations, not theirs.

Cochrane (1994): A few findings: skipping the details

- Analyzing which shock matters the most is a highly theoretical question (even if the approach seems pure empirical)
- Data management matters: ▸ HP Filter ▸ Beveridge Nelson ▸ Linear/Quadratic ▸ Growth rates
- Objective to find exogenous innovations but "exogenous shocks are rare": governments usually have reaction functions, big part of the literature is endogenizing technology shocks

Cochrane (1994) A few findings: skipping the details

- If we find a shock that matters, it matters through a transmission channel. So looking for a shock and a channel is not independent
- Data and many studies seem to suggest that technology shocks and monetary shocks can account for no more than 20% of output variability
- There have been many recent studies that pick up from this, let's move on
- There are many ways of looking at the data and different approaches will give different facts and answers to what model should we write

Detour: VAR

Reduced form VAR

$$y_t = \nu + A_1 y_{t-1} + ... + A_p y_{t-p} + u_t$$

with u_t mean zero, white noise. $\mathbb{E}(u_t u_t') = \Sigma_u$

Notation: define matrix of polynomial in the lag operator

$A(L) = I_K - A_1 L - ... - A_p L^p$

$$A(L) y_t = u_t$$

Stable if $\det(A(z)) = \det(I_K - A_1 z - ... - A_p z^p) \ = 0$ has all roots outside the complex unit circle

Detour: VAR

Rewrite the system for $Y_t = (y_t', y_{t-1}', ..., y_{t-p+1}')'$

$$Y_t = \boldsymbol{\nu} + \mathbf{A} Y_{t-1} + U_t$$

$$\boldsymbol{\nu} = \begin{bmatrix} \nu \\ 0 \\ \vdots \\ 0 \end{bmatrix} ; \mathbf{A} = \begin{bmatrix} A_1 & A_2 & \cdots & A_p \\ I_K & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \cdots \\ 0 & 0 & I_K & 0 \end{bmatrix} ; U_t = \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} ;$$

If stable VAR, the polynomial can be inverted

This can be estimated with standard tools

Detour: VAR

Forget about the constant. What we really want is the structural VAR

$$B_0 y_t = B_1 y_{t-1} + ... + B_p y_{t-p} + w_t$$

the 2 of them are related

$$y_t = \underbrace{B_0^{-1} B_1}_{A_1} y_{t-1} + ... + \underbrace{B_0^{-1} B_p}_{A_p} y_{t-p} + \underbrace{B_0^{-1} w_t}_{u_t}$$

To move between representation you need to know B_0

Structural, deep, interpretation of w_t shocks

WTG, set $\Sigma_w = I_K$. Then $\Sigma_u = B_0^{-1} B_0^{-1\prime}$

Detour: VAR

Note that $u_t = B_0^{-1} w_t$

$\Sigma_u = B_0^{-1} B_0^{-1\prime}$. Can we use information in Σ_u to recover B_0?

Yes, but won't be enough to pin down all the elements, as Σ_u is a Variance-covariance matrix and hence, symmetric.

We have only $K(K+1)/2$ independent equations.

This is only a necessary condition for identification

Detour: VAR

Recursive (Cholesky) identification

Define a lower triangular matrix P such that $\Sigma_u = PP'$. This P matrix is the lower triangular Cholesky decomposition of Σ_u

Then, $P = B_0^{-1}$, then B_0 is also lower triangular... we just made identifying assumptions saying that variables ordered first do not respond contemporaneously to those ordered at the bottom of the variable vector

There is a different P for different ordering of variables

This is a famous strategy in monetary VARs

Blanchard and Quah (1989)

- What drives the BC? Demand or supply shocks?
- Decompose GNP and unemployment into the effects of a permanent and transitory shocks (supply and demand, respectively)
- Introduce the Long Run identification for VARs.
- no shock has a long run effect on unemployment, but supply has a long run effect on output while the demand shock does not
- The disturbance with long run effect on output is associated to productivity shocks, demand induced shocks instead has a effect that is supposed to vanish

Blanchard and Quah (1989): Procedure

- First step: Estimate the reduced form equation

$$A(L)z_t = u_t, \quad \text{where} \;\; A(0) = I \;\; \text{and} \;\; \mathbb{E}(u_t u_t') = \Omega$$

- with $z = [\Delta Y \;\; U]'$ and $A(L)$ is a lag-polynomial operator of order p (stationary) and $u_t = [u_Y \;\; u_U]'$ are reduced form coefficients, serially uncorrelated

$$B_0 A(L) z_t = B_0 u_t$$

- produce the structural VAR, i.e.

$$B(L) z_t = w_t$$

- B(L)=B$_0 - B_1 L - ... - B_p L^p = B_0 A(L)$

Blanchard and Quah (1989): Procedure

- v is a reduced form vector of shocks, combination of the structural shocks, w_t
- $w_t = [w_t^{as} w_t^{ad}]'$
- Neither of the shocks have long-run effects on ΔY and U. Still may have long-run effect on output, as output in levels is not constrained by the assumptions of the estimation strategy
- as usual, $\Sigma_u = B_0^{-1} B_0^{-1\prime}$

Blanchard and Quah (1989): Procedure

- we recover the effect of structural shocks on the variables by rewriting the expresion in its MA representation

$$z_t = B(L)^{-1} w_t = \Theta(L) w_t$$

- The long run cumulative effect of demand shocks on output growth if $\sum_{i=0}^{\infty} \Theta_i$
- The long run identification strategy is that this sum is 0

$$\Theta(1) = \begin{bmatrix} \theta_{11}(1) & 0 \\ \theta_{21}(1) & \theta_{22}(1) \end{bmatrix}$$

Blanchard and Quah (1989) findings

- Demand shocks have hump shaped effect on output and unemployment with a peak in 1 year and last up to 3 years
- Positive supply shocks increase output but also unemployment initially
- Shutting down supply shocks, the generated data has peaks and troughs that coincide with the NBER business cycle. Pusshing the idea of demand driven BC
- However, in the Variance decomposition exercise there is substantial uncertainty of the results

Gali (1999)

- Decompose productivity and hours into technology and non-technology components
- Conditional correlations between technology shock and productivity and hours is negative
- Hours fall after technology shocks
- Hard to reconcile the RBC framework here, push for the NK framework

Gali (1999)

- Estimate structural VAR with the identifying restriction that only technology shocks may have permanent effects on the level of productivity
- Design the VAR $[\Delta x_t \quad \Delta n_t]$ where x_t is log-productivity and n_t log-hours, are generated by 2 shocks (technology and non-technology, orthogonal to each other)

$$\begin{bmatrix} \Delta x_t \\ \Delta n_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \epsilon_t^z \\ \epsilon_t^m \end{bmatrix}$$

- ϵ_t^z denote the technology shocks and ϵ_t^m are the non-technology ones
- identifying restriction is $C^{12} = 0$, productivity comes only from technology shocks

Gali (1999)

- Using this bi-variate model with US data (1948:1-1994:4)

TABLE 1—CORRELATION ESTIMATES: BIVARIATE MODEL

	Unconditional	Conditional: Technology	Conditional: Nontechnology
Panel A: First-differenced labor			
Hours	−0.26**	−0.82**	0.26**
	(0.08)	(0.12)	(0.12)
Employment	−0.02	−0.84**	0.64**
	(0.07)	(0.26)	(0.13)
Panel B: Detrended labor			
Hours	−0.26**	−0.81**	0.35*
	(0.08)	(0.11)	(0.20)
Employment	−0.02	−0.35	0.38
	(0.07)	(0.49)	(0.56)

Notes: Table 1 reports estimates of unconditional and conditional correlations between the growth rates of productivity and labor input (hours or employment) in the United States, using quarterly data for the period 1948:1–1994:4. Standard errors are shown in parentheses. Significance is indicated by one asterisk (10-percent level) or two asterisks (5-percent level). Conditional correlation estimates are computed using the procedure outlined in the text, and on the basis of an estimated bivariate VAR for productivity growth and labor-input growth (Panel A) or productivity growth and detrended labor input (Panel B). Data sources and definitions can be found in the text.

Note: From Gali (1999)

- Conditional correlations between technology shock and productivity and hours is negative
- Hours fall after technology shocks

Angeletos (2018)

- Identify a single shock that accounts for big share of business-cycle volatility
- Look at its properties and see if they have information about a parsimonious model of the business cycle
- Drawback, it is not really a fundamental shock what they identify...

Angeletos (2018): objective and results

- Provide a summarized description of the data (patterns of co-movement)
- The patterns do not seem to fit the TFP story, or the news, or demand shocks in the context of a NK model... instead seems to be in line with some sort of non-inflationary demand shock
- Some related studies: Angeletos and La'O (2010, 2013), Bai, Rios-Rull, and Storesletten (2017), Beaudry and Portier (2018), Beaudry, Galizia, and Portier (2018), Benhabib, Wang, and Wen (2015), Huo and Takayama (2015), and Ilut and Saijo (2018).

Angeletos (2018): more results

- Identify the shock with max contribution to GDP, Unemployment, total hours and investment at BC frequencies (6-32 quarters)
- The maximal identified shock looks alike for all those (idea of a “main driver of BC fluctuations”)
- This shock is disconnected from the long run behavior of these variables (as suggested in BQ(1989) and Gali (1999))
- It is disconnected from TFP, inflation and Phillips Curves

Angeletos (2018): The method

- Estimate a VAR
- Identify the linear combination of VAR residuals with max contribution to GDP, Unemployment, total hours and investment at BC frequencies (6-32 quarters)
- Compare to other shocks: this makes a collection of one dimensional cuts of the data
- Data: GDP, investment, consumption, hours and labor productivity in the non-farm business sector, unemployment, the labor share, inflation (GDP deflator), and the federal funds rate
- 1960 - 2007: stop in 2007 due to the ZLB

Angeletos (2018): The method

- Estimate a VAR using a few variables (X_t)

$$A(L)X_t = u_t$$

- $A(L)$ is a lagged polynomial matrix and $\mathbb{E}u_t u_t' = \Sigma$. $A(0) = A_0 = I$, inverting the polynomial $B(L) = A(L)^{-1}$

$$X_t = B(L)u_t$$

- Suppose there are some independent shocks ϵ_t such that $u_t = S_t \epsilon_t$ and assume $\mathbb{E}\epsilon_t \epsilon_t' = I$ with $SS' = \Sigma$
- This S is not unique without further restrictions as it could be rewritten by $S = \tilde{S}Q$ for an orthonormal Q

Angeletos (2018): The method

- Let's get $\tilde{S}$ by Cholesky (i.e. recursive recovery of the $_t$)
- Then for X_t you can have a VMA(∞) representation

$$X_t = C(L)Q\epsilon_t = \sum_{\tau=0}^{\infty} C_\tau Q\epsilon_{t-\tau}$$

- Then a column j of C_τ, $C_{\tau,j}$ gives the effect of the j element in ϵ on the VAR variables at horizon τ
- If you take a column q of the Q matrix and define the shock as the linear combination $q'\epsilon_t$, you get the impact of this combination of shocks

Angeletos (2018): The method

- How to choose q? The authors choose q to maximize the contribution of linear combination of $q'\epsilon_t$ to some variable over a particular frequency
- Note, the linear combination of shocks has no structural interpretation as there is no identifying restrictions applied to choose the q
- Findings

Angeletos (2018): Findings

Targeting max FEVD Unemployment

Table 1: Variance Contributions of Unemployment Shock

	u	Y	h	I	C	π	R	r	TFP	Y/h	w	wh/Y
Short Run	68.15	59.93	55.99	65.02	20.67	10.70	27.03	15.73	6.02	12.15	5.11	29.96
Long Run	11.85	4.17	8.83	4.84	3.96	12.48	21.09	16.40	4.11	4.05	5.32	5.63

Note: The shock is constructed by targeting unemployment. The first row (Short Run) reports the contribution of this shock to the volatility of the various variables between 6 and 32 quarters, the second row (Long Run) between 80 quarters and ∞. The variables are denoted as follows: $Y =$GDP, $h =$hours in the non-farm business sector, $I =$investment, $C =$consumption, $u =$unemployment rate, $\pi =$inflation rate (GDP deflator), $R =$nominal interest rate (Federal Funds rate), $r =$real interest rate (with expected inflation constructed from the VAR), TFP$=$ utilization-adjusted Total Factor Productivity, $Y/h =$ labor productivity, $w =$real wage, $wh/Y =$labor share.

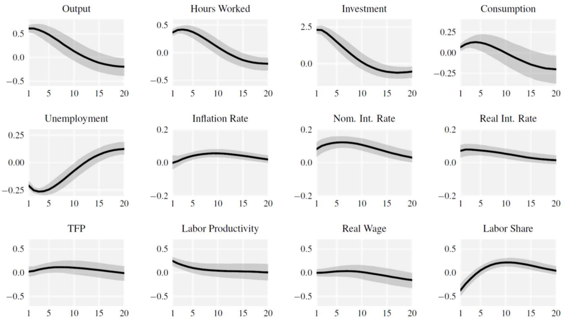
Note: From Angeletos (2018)

- Explains big share of many variables
- Does not explain long run
- Affects little TFP, inflation and labor productivity

Angeletos (2018): Findings

Targeting max FEVD Unemployment

Figure 1: Impulse Response Functions to the Unemployment Shock



Shaded area : 68% HPDI.

Note: From Angeletos (2018)

- Generates realistic business cycles: right co-movement

Angeletos (2018): Findings

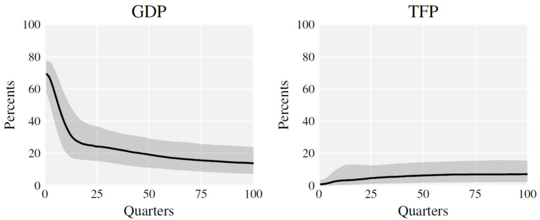
Targeting max FEVD Unemployment

- The shock fades out at long horizons (in line with BQ89), drivers of BC are different from the long run
- Procyclical movement in inflation. In the context of a NK, this shock should look like a demand shock, but it is fairly week link
- weak delayed response of real wages and a countercyclical response of labor share
- procyclical movement in the real interest rate

Angeletos (2018): Findings

TFP?

Figure 5: Variance Contributions to GDP and TFP



Shaded area: 68% HPDI

Note: From Angeletos (2018)

- The shock does not contribute much to the variance of TFP (measured as utilization adjusted as Fernald (2004))
- All the evidence so far does not seem to support RBC type of models (basic) that relate BC to productivity shocks

Angeletos (2018): Findings

What about inflation?

- The NK model may be supported by these evidence depending on the behavior of inflation
- Study the shock that max the FEVD of inflation at the BC frequency

Angeletos (2018): Findings

Targeting max FEVD Unemployment

Table 5: Inflation vs Unemployment Shock

Target	u	Y	h	I	C	π
Inflation	8.86	8.93	10.01	5.84	19.06	80.78
Unemployment	68.15	59.93	55.99	65.02	20.67	10.70

Note: From Angeletos (2018)

- Detached from unemployment
- Does not explain long run
- Affects little TFP, inflation and labor productivity

Other references

- Chari, Kehoe, and McGrattan (2007)
- Chari, Kehoe, and McGrattan (2008) and a reply by Christiano and Davis (2005)

RBC model

- Studying data and facts served as a motivation
- We want to inquire whether a simple stochastic model can explain aggregate dynamics of the economy
- Pre-Keynes already studied the business cycle and by the 1930s we had a few facts about the long run growth ▸ Kaldor
- This last point highlighted a major task: develop a theory of growth (NGM)
- Modern business cycle theory starts here: are the drivers of growth different from those of the cycle? can we use the same model to think about both?

RBC model

- RBC model is an extension of the NGM with two main additional ingredients:
  - Shocks
  - Endogenous leisure/labor choice
- Using this model, it was shown that sources of growth are different from sources of the business cycle

Table: Sources of variability Cooley and Prescott (1995)

Δ in output per worker	Secular Growth	Business cycle
Due to ΔK	1/3	0
Due to ΔL	0	2/3
Due to ΔA	2/3	1/3

RBC model

- We design a model that is stable along a balance growth path... there will be a pattern of steady state growth
- This imposes restrictions on preferences
- and restrictions on production

IES in consumption is invariant to the scale of consumption

Why?

In Steady State, the marginal product of capital is constant (and equal to the interest rate)
Also, since consumption grows at a constant rate, the ratio of discounted marginal utilities is also equal to $(1+r)$... then IES must be constant an independent from consumption

RBC model

Restrictions on preferences II

Income and substitution effect associated with sustained growth in labor productivity must not alter labor supply

Why?

This comes because hours worked cannot grow in steady state. To reconcile it with a growing MPL (induce by the permanent labor augmenting technological change) income and substitution effect should cancel out

Variation in hours are associated to intertemporal substitution induced by capital accumulation (without capital accumulation hours wouldn't move), see King, Plosser, and Rebelo (1988)

RBC model

Restrictions on production

Permanent technical change has to be labor augmenting

Why?

A type of $Y_t = A_t K_t^{1-\alpha}(X_t L_t)^{\alpha}$ and a standard law of motion of capital imply that steady state growth for output, consumption, capital and investment are the same and equal to the permanent technology growth rate

In that way we will be able to identify a steady state around the growing economy

RBC model

- I follow the notation in Sargent and Ljungqvist, ch12 and Jesus Fernandez-Villaverde notes
- s_t denotes the realization of a stochastic event
- s^t denotes the history of realizations of stochastic event up to period t
- $\pi(s^t)$ denotes a probability measure and conditional probabilities are given by $\pi(s^\tau | s^t)$

RBC model

Households' problem

Representative agent maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(c_t(s^t)\right) + \psi \log\left(1 - l_t(s^t)\right) \right\}$$

subject to

$$c_t(s^t) + x_t(s^t) = w_t(s^t) l_t(s^t) + r_t(s^t) k_t(s^{t-1}), \forall t \geq 0$$

We assume complete markets (there is a complete set of Arrow securities)

RBC model

Firms' problem

Representative firm maximizes profits, static problem

$$y_t(s^t) = e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t)\right)^{1-\alpha}$$

Labor augmenting technological change (permanent)

In the static problem, the firm hires capital... does not own it (under complete markets it doesn't matter)

$$r_t(s^t) = \alpha e^{z_t} k_t(s^{t-1})^{\alpha-1} \left(X_t l_t(s^t)\right)^{1-\alpha}$$

$$w_t(s^t) = (1-\alpha) e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t)\right)^{-\alpha}$$

RBC model

Investment Firms' problem

Capital evolves accordingly to

$$k_{t+1}(s^t) = (1-\delta)k_t(s^{t-1}) + x_t(s^t)$$

RBC model

Technology

We can have permanent change to be stochastic (Aguiar and Gopinath (2007))

or not: $X_t = (1+\mu)^t$, linear constant trend

TFP stationary shock follows AR(1)

$$z_t = \rho z_{t-1} + \sigma \epsilon_t$$

with ϵ_t standard Normal

RBC model

3 concepts of Equilibrium

Arrow-Debreu (all trading at period 0): requires introducing prices at period 0 to value all future claims in all states of nature

Sequential trading: requires making explicit trading in Arrow securities

Recursive competitive: requires writing the problem in recursive way and aggregate states

RBC model

Arrow-Debreu

An Arrow-Debreu equilibrium is a set of prices $\{\hat{p}_t(s^t), \hat{w}_t(s^t), \hat{r}_t(s^t)\}_{t=0, s^t \in S^t}^{\infty}$ and allocations $\{\hat{c}_t(s^t), \hat{l}_t(s^t), \hat{k}_t(s^t)\}_{t=0, s^t \in S^t}^{\infty}$ such that

- Given prices, $\{\hat{c}_t(s^t), \hat{l}_t(s^t), \hat{k}_t(s^t)\}_{t=0}^{\infty}$ solves

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(c_t(s^t)\right) + \psi \log\left(1 - l_t(s^t)\right) \right\}$$

st

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \hat{p}_t(s^t) \left(c_t(s^t) + k_{t+1}(s^t)\right) \leq$$

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \hat{p}_t(s^t) \left(\hat{w}_t(s^t) l_t(s^t) + (\hat{r}_t(s^t) + 1 - \delta) k_t(s^{t-1})\right)$$

RBC model

Arrow-Debreu

- Given prices, $\{\hat{l}_t(s^t), \hat{k}_t(s^t)\}_{t=0}^{\infty}$ are chosen to minimize costs

$$\hat{r}_t(s^t) = \alpha e^{z_t} \hat{k}_t(s^{t-1})^{\alpha-1} \left(X_t \hat{l}_t(s^t) \right)^{1-\alpha}$$

$$\hat{w}_t(s^t) = (1-\alpha) e^{z_t} \hat{k}_t(s^{t-1})^{\alpha} \left(X_t \hat{l}_t(s^t) \right)^{-\alpha} X_t$$

- Markets clear

$$c_t(s^t) + k_{t+1}(s^t) - (1-\delta) k_t(s^{t-1}) = e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t) \right)^{1-\alpha}$$

RBC model

Sequential markets

Representative agent maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(c_t(s^t)\right) + \psi \log\left(1 - l_t(s^t)\right) \right\}$$

subject to

$$c_t(s^t) + x_t(s^t) + \sum_{s_{t+1}|s^t} Q_t(s^t, s_{t+1}) a_{t+1}(s^t, s_{t+1}) =$$

$$w_t(s^t) l_t(s^t) + r_t(s^t) k_t(s^{t-1}) + a_t(s^t), \forall t \geq 0$$

$$a_{t+1}(s^t, s_{t+1}) \geq -A_{t+1}(s^{t+1})$$

RBC model

Sequential Market equilibrium

An SME is a set of prices $\{\hat{Q}_t(s^t, s_{t+1})\}_{t=0, s^t \in S^t, s_{t+1} \in S}^{\infty}$ for Arrow securities, $\{\hat{w}_t(s^t), \hat{r}_t(s^t)\}_{t=0, s^t \in S^t}^{\infty}$ and allocations $\{\hat{c}_t(s^t), \hat{l}_t(s^t), \hat{k}_t(s^t), \{\hat{a}_{t+1}(s^t, s_{t+1})\}_{s_{t+1} \in S}\}_{t=0, s^t \in S^t}^{\infty}$ such that

- Given prices, $\{\hat{c}_t(s^t), \hat{l}_t(s^t), \hat{k}_t(s^t), \{\hat{a}_{t+1}(s^t, s_{t+1})\}_{s_{t+1} \in S}\}_{t=0}^{\infty}$ solves the households problem
- Given prices, $\{\hat{l}_t(s^t), \hat{k}_t(s^t)\}_{t=0}^{\infty}$ are chosen to min costs

$$\hat{r}_t(s^t) = \alpha e^{z_t} \hat{k}_t(s^{t-1})^{\alpha-1} \left(X_t \hat{l}_t(s^t) \right)^{1-\alpha}$$

$$\hat{w}_t(s^t) = (1-\alpha) e^{z_t} \hat{k}_t(s^{t-1})^{\alpha} \left(X_t \hat{l}_t(s^t) \right)^{-\alpha} X_t$$

- Markets clear
  $c_t(s^t) + k_{t+1}(s^t) - (1-\delta) k_t(s^{t-1}) = e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t) \right)^{1-\alpha}$

RBC model

Recursive competitive equilibrium

Define the equilibrium as a set of functions that depend on state variables of the model (pay-off relevant states)

The household problem in recursive written as

$$v(k, K, z) = \max_{c,x,l} \left\{ u(c,l) + \beta \mathbb{E}\left[v(k', K', z') | z \right] \right\}$$

subject to

$$c + x = r(K,z)k + w(K,z)l$$

$$k' = (1-\delta)k + x$$

$$K' = (1-\delta)K + X(K,z)$$

$$z' = \rho z + \sigma \epsilon'$$

RBC model

Recursive competitive equilibrium

A RCE for this economy is a value function $v(k, K, z)$, households policy functions, $c(k, K, z)$, $l(k, K, z)$ and $x(k, K, z)$, aggregate policy functions $C(K, z)$, $L(K, z)$ and $X(K, z)$ and price functions $r(K, z)$ and $w(K, z)$ such that

- Given price functions, household policy functions solve the recursive problem of the households
- Firms behavior satisfy

$$r(K,z) = \alpha e^z K(K,z)^{\alpha-1} \left(X_t L(K,z)\right)^{1-\alpha}$$

$$w(K,z) = (1-\alpha) e^z K(K,z)^{\alpha} \left(X_t L(K,z)\right)^{-\alpha} X_t$$

RBC model

Recursive competitive equilibrium

- Aggregate resource constraint is satisfied

$$C(K,z) + X(K,z) = e^{z}K(K,z)^{\alpha}\left(X_t L(K,z)\right)^{1-\alpha}$$

- individual choices are consistent with the aggregate: $c(k,K,z) = C(K,z)$, $l(k,K,z) = L(K,z)$ and $x(k,K,z) = X(K,z)$

RBC model

Equilibrium conditions

$$\frac{1}{c_t(s^t)} = \beta \mathbb{E}_t \left[\frac{1}{c_{t+1}(s^{t+1})} \left(r_{t+1}(s^{t+1}) + 1 - \delta \right) \right]$$

$$\psi \frac{c_t(s^t)}{1 - l_t(s^t)} = w_t(s^t)$$

$$r_t(s^t) = \alpha e^{z_t} k_t(s^{t-1})^{\alpha-1} \left(X_t l_t(s^t) \right)^{1-\alpha}$$

$$w_t(s^t) = (1-\alpha) e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t) \right)^{-\alpha} X_t$$

$$c_t(s^t) + k_{t+1}(s^t) - (1-\delta) k_t(s^{t-1}) = e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t) \right)^{1-\alpha}$$

$$z_t = \rho z_{t-1} + \sigma \epsilon_t$$

RBC model

Cycle vs growth

This economy grows at the rate of technology growth, $\mu_t = \frac{X_t}{X_{t-1}}$

We will analyze the economy along the equilibrium path, to do so, we rescale the economy by the level of technology

This is implementing a detrending strategy in the model

Rewrite: $\hat{g}_t = \frac{g_t}{X_{t-1}}$, for all non-stationary variables g

RBC model

Stationarized equilibrium conditions

$$\frac{1}{c_t(s^t)} = \beta \mathbb{E}_t \left[\frac{1}{c_{t+1}(s^{t+1})} \left(r_{t+1}(s^{t+1}) + 1 - \delta \right) \right]$$

$$\psi \frac{c_t(s^t)}{1 - l_t(s^t)} = w_t(s^t)$$

$$r_t(s^t) = \alpha e^{z_t} k_t(s^{t-1})^{\alpha-1} \left(X_t l_t(s^t) \right)^{1-\alpha}$$

$$w_t(s^t) = (1-\alpha) e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t) \right)^{-\alpha} X_t$$

$$c_t(s^t) + k_{t+1}(s^t) - (1-\delta) k_t(s^{t-1}) = e^{z_t} k_t(s^{t-1})^{\alpha} \left(X_t l_t(s^t) \right)^{1-\alpha}$$

$$z_t = \rho z_{t-1} + \sigma \epsilon_t$$

RBC model

Parameterizing the model

Assigning values to the parameters is always controversial. Many strategies.

Calibration: to micro-evidence, steady state and long-run averages

Matching moments: volatilities, and cross-correlations

Estimation: Bayesian, ML

Solution method... we already talked a lot about this

Calibration: to micro-evidence, steady state and long-run averages

DISCUSS

RBC model

How does the base model do in replicating facts?

Table 1.2
Cyclical Behavior of the Artificial Economy: Deviations from Trend of Key Variables, 150 Observations

		Cross-Correlation of Output with:										
Variable	*SD%*	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Output	1.351 (0.148)	−.049	.071	.232	.441	.698	1.0	.6983	.441	.232	.071	−.049
Consumption	0.329 (0.041)	.232	.340	.460	.592	.725	.843	.502	.229	.022	−.128	−.234
Investment	5.954 (0.646)	−.112	−.007	.171	.389	.664	.992	.713	.470	.270	.115	−.003
Hours	0.769 (0.083)	−.130	.012	.152	.373	.652	.986	.715	.478	.281	.127	.010
Productivity	0.606 (0.068)	.055	.175	.325	.512	.732	.978	.649	.376	.160	−.002	−.122

Note: Values in parentheses are standard deviations across simulations.

The TFP generates output variability but does not explain it all

Too much consumption smoothing

C, I, N, TFP, are strongly procyclical (a bit too much)

Hours and productivity move together, but not in the data

RBC model

Policy implications

In term of policy implications the RBC model provides a clear insight: macroeconomic crisis are an optimal response to TFP shocks

As optimal responses: it is not possible to implement a policy that improves welfare

In a frictionless world, there is no space for policy interventions

RBC model

A prime on macro-finance

This setting has a direct implication for financial macro

$$\mathcal{M}_{t,t+1} = \beta \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)}$$

With the stochastic discount factor of the households, we can price any asset
i.e. the price of the Arrow security

$$Q(s^t, s_{t+1}) = \pi(s_{t+1}|s^t)\mathcal{M}_{t,t+1}$$

RBC model

A prime on macro-finance

The price of the risk free asset

$$Q^{rf}(s^t, s_{t+1}) = \mathbb{E}_t\left(\mathcal{M}_{t,t+1}\right)$$

The return of a risk-less bond that costs 1 unit of consumption today

$$1 = \mathbb{E}_t\left(\mathcal{M}_{t,t+1} R_t^b(s^t)\right)$$

In the RBC setting this generates the Equity Premium puzzle... Mehra and Prescott (1985)

Relative risk aversion coefficient $-c \times u''(c)/u'(c) = \gamma$, is the reciprocal of IES. Agent that wants to smooth consumption over realizations of state of nature, also wants to do it over time

RBC model

A prime on macro-finance (Mehra and Prescott (1985))

Table 1

Time periods	% growth rate of per capita real consumption: Mean	% growth rate of per capita real consumption: Standard deviation	% real return on a relatively riskless security: Mean	% real return on a relatively riskless security: Standard deviation	% risk premium: Mean	% risk premium: Standard deviation	% real return on S&P 500: Mean	% real return on S&P 500: Standard deviation
1889–1978	1.83 (Std error = 0.38)	3.57	0.80 (Std error = 0.60)	5.67	6.18 (Std error = 1.76)	16.67	6.98 (Std error = 1.74)	16.54
1889–1898	2.30	4.90	5.80	3.23	1.78	11.57	7.58	10.02
1899–1908	2.55	5.31	2.62	2.59	5.08	16.86	7.71	17.21
1909–1918	0.44	3.07	−1.63	9.02	1.49	9.18	−0.14	12.81
1919–1928	3.00	3.97	4.30	6.61	14.64	15.94	18.94	16.18
1929–1938	−0.25	5.28	2.39	6.50	0.18	31.63	2.56	27.90
1939–1948	2.19	2.52	−5.82	4.05	8.89	14.23	3.07	14.67
1949–1958	1.48	1.00	−0.81	1.89	18.30	13.20	17.49	13.08
1959–1968	2.37	1.00	1.07	0.64	4.50	10.17	5.58	10.59
1969–1978	2.41	1.40	−0.72	2.06	0.75	11.64	0.03	13.11

From the Lucas (1978) + asset return of $R^e_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$

$$\mathbb{E}_t\left[R^e_{t+1}\mathcal{M}_{t,t+1}\right] = 1$$

RBC model

A prime on macro-finance

Intuition

$$\mathbb{E}_t\left[R^e_{t+1}\mathcal{M}_{t,t+1}\right] = \mathbb{E}_t\left[R^e_{t+1}\right]\mathbb{E}_t[\mathcal{M}_{t,t+1}] + cov\left(R^e_{t+1}\mathcal{M}_{t,t+1}\right)$$

$$\frac{\mathbb{E}_t[R^e_{t+1}] - R^{rf}}{R^{rf}} = -cov\left(R^e_{t+1}\mathcal{M}_{t,t+1}\right)$$

$$\frac{\mathbb{E}_t[R^e_{t+1}] - R^{rf}}{R^{rf}} = -cov\left(R^e_{t+1}, \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right)$$

For a high return you will ask that it pays a lot when consumption is high (negative cov between return and sdf)

Need bout $\gamma = 35$ for standard calibrations... the higher the risk aversion, the higher return you will ask

RBC model

A prime on macro-finance

Habit formation solves the equity premium puzzle in model with exogenous consumption process

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma} \right)$$

$$S_t = (C_t - H_t)/C_t$$

For this specification

$$\frac{-C_t u_{cc}}{u_c} = \frac{\gamma}{S_t}$$

The degree of risk aversion (local curvature of the utility function) is a function of S_t

During bad times, when consumption is too close to the stock of habits, $S_t \to 0$ implies households are extremely risk averse

Will penalize assets that pay low in those scenarios (i.e. positive correlation with consumption)... ask a large return

You can specify the process for H as a function of consumption, or the process of S

RBC model

A prime on macro-finance

Defines the “surplus consumption ratio”. Cochrane and Campbell specifies the log of s_{t+1}

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - \mathbb{E}(\Delta c_{t+1}))$$

$\lambda(s_t)$ defines the sensitivity of surplus to consumption shocks
Define consumption growth as

$$\Delta c_{t+1} = g + v_{t+1}, \qquad v_{t+1} \sim i.i.d.\mathcal{N}(0, \sigma^2)$$

RBC model

A prime on macro-finance

Plugging these expressions, the SDF becomes

$$
\begin{aligned}
\mathcal{M}_{t,t+1} &= \beta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\gamma} \\
&= \beta \exp\left(-\gamma [(1-\phi)(\bar{s} - s_t) + (1 + \lambda(s))_{t+1} + \lambda(s_t) g] \right)
\end{aligned}
$$

$\lambda(s_t)$ is set to match a constant risk-free rate

They generate a procyclical variation of stock prices, a countercyclical stock market volatility, and long-horizon predictability of excess stock returns.

RBC model

A prime on macro-finance

In general equilibrium is not so straight: given the existence of habits consumption just becomes too smooth

EZ preferences: Recursive preferences

$$V_t = \left((1-\beta)(c_t^{\nu}(1-l_t)^{1-\nu})^{\frac{1-\gamma}{\theta}} + \beta \mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right)^{\frac{\theta}{1-\gamma}}$$

with $\theta = \frac{1-\gamma}{1-1/\psi}$... γ is the risk aversion coefficient and ψ is the IES

Rare disaster: Rietz (1988), Barro (2006)

Long-run risks: Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2010)

Construct capital and TFP

$$Y_t = \Gamma_t K_{t-1}^{\alpha} N_t^{1-\alpha}$$

$$K_t = (1-\delta)K_{t-1} + X_t$$

Divide by output

$$\frac{K_t}{Y_t} = (1-\delta)\frac{Y_{t-1}K_{t-1}}{Y_{t-1}Y_t} + \frac{X_t}{Y_t}$$

Evaluate in SS

$$\frac{K}{Y} = \frac{1}{1-\frac{1-\delta}{g_Y}}\frac{X}{Y}$$

Average K to output ratio. Take output, in the first period of your sample and you have an initial condition for capital

Recover TFP from the production function

Detrending

Hodrick-Prescott Filter (1987)

- Here, the trend and cyclical components are identified by solving a minimization problem

$$min_{\{y_t^c, y_t^s\}_{t=1}^T} \left\{ \sum_{t=1}^{T} (y_t^c)^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^s - y_t^s) - (y_t^s - y_{t-1}^s)]^2 \right\}$$

- Subject to $y_t = y_t^c + y_t^s$

Detrending

Hodrick-Prescott Filter (1987)

- Trade-off between minimizing the variance of the cyclical component and keeping the growth rate of the trend constant
- λ regulates this trade-off. The higher λ, the more we penalize changes in the growth rate of the trend component. If λ is infinite the resulting trend is linear
- Ravn and Uhlig (2001)

Detrending

Beveridge Nelson

- If Δy_t has a Wold representation

$$\Delta y_t = \delta + \psi^*(L)\epsilon_t$$

then BN shows that the trend can be written as

$$BN_t = BN_{t-1} + \psi^*(1) \sum_{j=1}^{t} \epsilon_t$$

- i.e., follows a RW without drift

Detrending

Linear/Quadratic

- y_t denotes the log of real output (per capita)
- Assume: $y_t = a + bt + ct^2 + \epsilon_t$
- Set $y_t^c = \epsilon_t$
- $y_t^s = a + bt + ct^2$

Detrending

Growth rates

Kaldor facts

- real output grows at a fairly constant rate
- the stock of capital grows faster than labor input
- the growth rate of capital and output tend to be about the same
- the return on capital is constant (no trend)
- output per-capita growth differs across countries
- economies with high share of profits in total income tend to have higher investment to output ratios

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