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Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624 98 75

PRICE CAP REGULATION WITH CAPACITY WITHHOLDING*

Ana B. Lemus[1] and Diego Moreno[2]

Abstract

A monopolist facing an uncertain demand makes ex-ante capacity decisions involving irreversible investments, and then chooses its output up to capacity upon the realization of demand. In equilibrium, capacity is low and underused. Imposing a binding price cap leads to an increase of capacity as well as expected output and total surplus, and to a decrease of expected price. The optimal price cap trades off the incentives for capacity investment and capacity withholding, and is well above the marginal cost. Price cap regulation alone cannot eliminate inefficiencies. When the unit cost of capacity is high the comparative static properties of price caps relative to the price cap than maximizes capacity investment ρ* are analogous to those obtained when the demand is known with certainty, and the optimal price cap is ρ*. When the unit cost of capacity is low, however, the expected output and surplus decrease with the price cap above and around ρ*, and therefore the optimal price cap is below ρ*.

Keywords: Monopoly, Market Power, Price Cap Regulation, Capacity Investment, Capacity Withholding, Demand Uncertainty.

[1]Departamento de Economía, Universidad Carlos III de Madrid, alemus@eco.uc3m.es
[2]Departamento de Economía, Universidad Carlos III de Madrid, dmoreno@eco.uc3m.es

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1 Introduction

Since Littlechild (1983)'s report, when precise information about demand and cost is available, price cap regulation is regarded as an effective instrument to mitigate market power, foster cost minimization and ultimately enhance expected surplus. (In contrast to rate-of-return regulation, used for most of the 20th century to regulate public utilities, which distorts incentives for cost minimization – see, e.g., Joskow (1972) – or cost reduction – see, e.g., Cabral and Riordan (1989).) In an industry where firms have market power, when the demand and cost are known with certainty the introduction of a binding price cap rises firms' marginal revenue near the equilibrium output, and leads to an increase of the equilibrium output and the expected surplus and to a decrease of the market price. Moreover, under broad regularity conditions on the demand and cost functions, for price caps above marginal cost the output and the expected surplus decrease and the market price increases with the price cap. Further, in the most favorable conditions (e.g., when firms produce the good with constant returns to scale), a price cap equal to marginal cost is able to eliminate inefficiencies.

We study the effectiveness of price cap regulation in a monopolistic setting under demand uncertainty. (Demand uncertainty may be interpreted also as variations of demand over time, as is common in electricity markets – see, e.g., Green and Newbery (1992).) In our model, the monopolist makes ex-ante capacity decisions involving irreversible investments, and then decides its output up to capacity upon the realization of demand – i.e., the monopolist may withhold capacity if it finds it beneficial to do so. In this setting, inefficiencies arise both because the monopolist installs a low level of capacity in order to commit to high prices, and because the monopolist withholds capacity for low realizations of the demand in order to avoid prices to fall too low. In electricity markets, generators may declare their capacity to be unavailable to the market – see Baldick and Hogan (2002). Indeed, data for the California electricity market during the time period May 2000-December 2001 show that some generators did not supply to the market all of their uncommitted capacity at the price cap – see Cramton (2003) and Joskow and Kahn (2002).

We show that, analogously to the benchmark case of a deterministic demand, the introduction of a binding price cap raises the firms' marginal return to capacity investment near the equilibrium capacity, and leads to an increase of the equilibrium capacity, the expected output and the expected total surplus, and to a decrease of the expected market price. However, even in the most favorable conditions (specifically, when the unit cost of capacity is constant) a price cap is unable to eliminate inefficiencies. Further, price caps above but near the unit cost of capacity are suboptimal because they reduce the return to capacity investment below its cost and lead the monopolist to install no capacity. The optimal price cap tends to be well above the unit cost of capacity, as it must trade off appropriately the incentives for capacity investment and those for capacity withholding. When the unit cost of capacity is high the first effect is dominant and the optimal price cap is the price cap that maximizes capacity investment. When the unit cost of capacity is low, however, reducing the price cap below the price cap that maximizes capacity investment may increase expected surplus. (Thus, maximizing capacity investment does not warrant maximizing expected surplus.) In either case, a price cap is a poor regulatory instrument as it is unable to provide the appropriate incentives for capacity investment and simultaneously eliminate the inefficiencies arising from capacity withholding.

Under standard regularity assumptions on the demand distribution it is possible to identify the comparative static properties of price caps: Capacity investment is maximal for a binding price cap ρ^* which is above the unit cost of capacity, and increases (decreases) with the price cap below (above) ρ^*. The expected price unambiguously increases with the price cap. Signing the effects of changes in the price cap on expected output and total surplus is subtler as they depend on the magnitude of the effects on capacity investment and capacity withholding, which have opposite signs. Interestingly, when the unit cost of capacity is small the expected output and total surplus decrease with the price cap above and around ρ^*, and thus the optimal price cap is below ρ^*. When the unit cost of capacity is larger, however, the signs of the effects of changes in the price cap on the expected output and total surplus coincide with that of the effect on capacity investment.

There has been a recent interest in studying the effect of price cap regulation when the demand is uncertain. Earle et al. (2007) and Grimm and Zoettl (2010) study an oligopolistic model in which firms make output decisions before the demand is realized and supply their output inelastically and unconditionally. (In this setting, Reynolds and Rietzke (2012) study entry, and Zoettl (2011) studies firms' technological choice.) Earle et al. (2007) show that even when firms produce the good with constant returns to scale the comparative static properties of price caps found when demand is deterministic, which were described above, fail for a "generic demand schedule," and conclude that the standard arguments supporting price cap regulation break down in the presence of demand uncertainty. The proof of this result (Theorem 4) shows that for any demand distribution such that output decreases with the price cap at a given binding price cap $\bar{p}$, it is possible to perturb the demand distribution in such a way that with the new demand distribution output increases with the price cap near $\bar{p}$. (The demand distribution is perturbed on an arbitrarily small interval around $\bar{p}$ by shifting the probability on the interval to the endpoints, thus creating two atoms.)

Of course, Earle et al. (2007)'s result is not at all surprising, as there is no hope that any particular property be preserved on a *large* subset (e.g., a dense subset according to standard topologies) of the set of all probability distributions with bounded support on the real line. However, Grimm and Zoettl (2010) show that under certain regularity conditions on the distribution of demand, prices cap have comparative static properties (relative to the price cap that maximizes capacity) analogous to those arising with a deterministic demand.

The arguably more interesting setting studied in the present paper is strategically very different to that studied by these authors: in the present setting, capacity investment decisions are made ex-ante, whereas output decisions are made at an interim stage upon observing the realization of demand. If we consider an oligopolistic industry, there is a question as to what may the appropriate mode of competition to consider at this interim stage, and there are well known difficulties therein to guarantee existence, uniqueness and symmetry of equilibrium – see, e.g., Reynolds and Wilson (2000), Gabszewicz and Poddar (1997). In the present paper we focus on the monopolistic case in order to avoid these potential conundrums, which are distractions from the issue under scrutiny – the impact of price cap regulation. Nevertheless, Early et al. (2007)'s Theorem 6 claims that their Theorem 4 extends to the case where firms can freely dispose of their output (i.e., upon observing the demand, firms choose how much of their output to supply). Indeed, the monopoly case studied in the present paper may be amenable to such a reduced form analysis. However, we find that a crucial step in their proof fails when the monopolist can withhold capacity – see Appendix A. In fact, in our setting when the cost of capacity is low the perturbation of the demand distribution used in the proof of Earle et al. (2007)'s Theorem 4 has an effect akin to that it would have a flat spot of a deterministic demand: changes in the price cap on this flat spot have no impact on the level of output.

Grimm and Zoettl (2010) also obtain some results for the alternative model where firms can dispose freely of their output. (Again, the caveats about the appropriate strategic analysis of this setting identified above apply here.) For example, they show that a binding price cap increases capacity investment, that inefficiencies cannot be eliminated with a price cap, that capacity investment is zero for price caps near the unit cost of capacity. These results would seemingly apply to our setting. However, they do not study the comparative static properties of price caps in this setting, nor they study the effects of price caps on the expected surplus. Apparently, a mistake in their calculations leads them to conclude that the results they obtain for the model without capacity withholding apply as well to the model with capacity withholding. (Specifically, their calculation of the marginal revenue in their formula (5) is incorrect in region A – see Section 3.) In particular, this mistake leads then to conclude that maximizing the expected surplus amounts to maximizing capacity. However, we show that when the cost of capacity is small maximizing surplus entails a lower price cap than the price cap that maximizes capacity.

Other authors have study price cap regulation in the presence of exogenous technological progress – in our setting the unit cost of capacity and production are constant over the regulatory period. Biglaiser and Riordan (2000), for example, study the incentive properties of price cap to produce optimal capacity investment and replacement. In their setting, they find that price caps provide better incentives than rate-of-return regulation, although in their setting (as in ours) optimal price caps must deal with a trade off involving the incentives for capacity investment and replacement.

In an oligopolistic industry, Roques and Savva (2009) study the effect of price caps on the timing of investments when demand is uncertain, and find that as in our setting a low price cap may be suboptimal as it may disincentivize investment.

Dobbs (2004) studies the effect intertemporal price cap regulation when a monopolist facing demand uncertainty has to decide the size and timing of its investments, and shows that optimal price caps lead to under investment and quantity rationing. Dixit (1991) studies a competitive market in which demand is uncertain and firms make ex-ante irreversible investments, and shows that introducing price ceilings lead to delay investments and higher prices over time.

The paper is organized as follows. We describe the monopoly in Section 2. In Section 3 we derive the monopoly equilibrium when a regulator imposes a price cap. We study the comparative static properties of price caps in Section 4. In Section 5 we study optimal price caps. We discuss an example in Section 6, and we conclude in Section 7. Appendix A contains an exercise showing that Earle et al. (2007)'s Theorem 4 fails in our setting. Appendix B studies a version of our model assuming full capacity utilization, and discusses the differing results obtained in that setting.

2 A Monopoly with Demand Uncertainty

Consider a monopoly that produces a good whose demand is uncertain and must decide how much capacity to install before the demand is realized. (As noted above, demand uncertainty may be interpreted also as variations of demand over time – see, e.g., Green and Newbery (1992).) For simplicity we assume that the market demand is $D(X,p) = \max\{X-p, 0\}$, where X is a random variable with support on a bounded interval of $\mathbb{R}_+$ and *p.d.f.* f. Once capacity is installed the good can be produced with constant returns to scale up to capacity. We assume without loss of generality that the production cost is zero. Also we assume that the unit cost of installing capacity is a positive constant c.

In this setting, when the demand is deterministic, i.e., when the support of X is a single point $x \in (c, \infty)$, the monopoly equilibrium output is $q^* = (x-c)/2$ and the market price is $p^* = (x+c)/2$. Introducing a price cap $\rho \in [c, (x+c)/2)$ increases of the monopolist's marginal revenue around q^*, and leads to an increase of the monopolist's output to $q(\rho) = x - \rho$ and a decrease of the market price to $p(\rho) = \rho$ – see Figure 1. (A price cap $\rho > (x+c)/2$ is *non-binding,* and therefore has no effect on the monopoly equilibrium.)

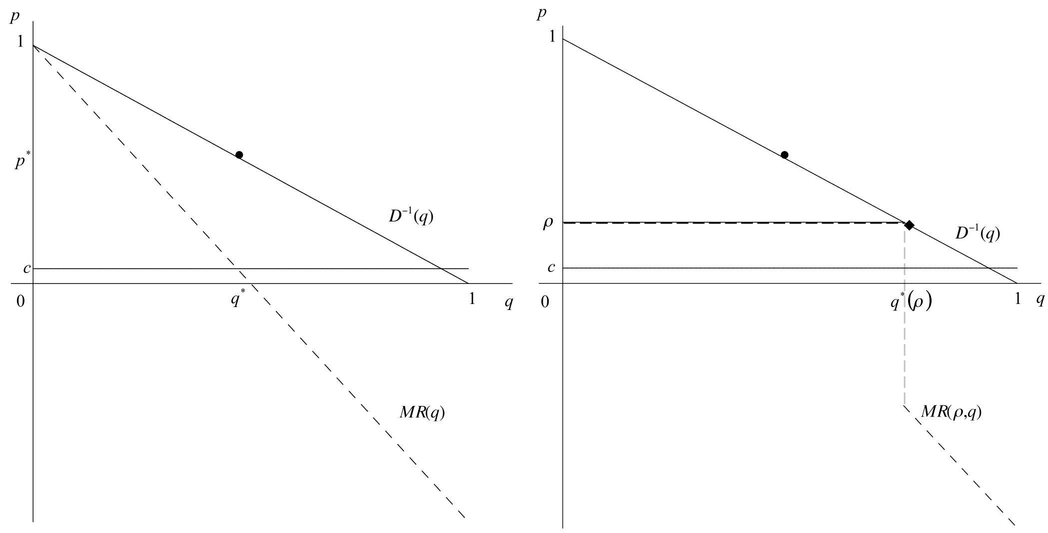

Figure 1: Price Caps when Demand is Known with Certainty

Moreover, for $\rho \in (c, (x+c)/2)$ a decrease of the price cap increases the output, decreases the price, and increases the expected surplus as well the consumer surplus. Thus, the optimal price cap (i.e., the price cap that maximizes the expected surplus) is $\rho = c$. These properties extend to symmetric oligopolistic markets – see, e.g., Theorem 1 in Earle et al. (2007).

Our purpose is to examine the impact of price caps when demand is uncertain. We assume that once the demand parameter X is realized, it is observed by the monopolist, who then decides how much to produce, and may withhold capacity if doing so is beneficial. (Alternatively, one may interpret this setting as if the monopolist decides its output before demand is realized, but once demand is realized the monopolist decides how much to supply, and may supply less than its total output.) Since the cost of capacity is sunk and the cost of production up to capacity is zero, then for each realization of the demand parameter X the monopolist's output maximizes revenue on $[0, k]$, where k is the monopolist's installed capacity.

In order to reduce notation and facilitate the presentation and the interpretation of our results, we assume that the support of X is the interval $[0, 1]$. This assumption entails a small loss of generality because the cost of production given capacity and the lower bound on the values of unit costs of capacity c coincide with the lower bound of the support of X. We denote by F the $c.d.f$ of f. Also we rule out the trivial case where the monopolist installs no capacity by assuming that $c < E(X)$. (Note that $E(X)$ is the expected maximum willingness to pay for the good.)

3 Monopoly Equilibrium with a Price Cap

Assume that a regulatory agency imposes a price cap $\rho \in [0,1]$. Since the cost of capacity is sunk, at this stage the monopolist maximizes revenue. Using the results for the case of a deterministic demand described above it is easy to see that if the monopolist had an unlimited capacity, then the equilibrium output for each demand realization $x \in [0,1]$ would be $q = x - \rho \leq 1 - \rho$ if $\rho < 1/2$, and $q = x/2 \leq 1/2$ if $\rho \geq 1/2$ (i.e., if ρ is non-binding for any demand realization) – recall that cost of production is zero. Hence levels of capacity $k > \max\{1-\rho, 1/2\}$ are suboptimal since the monopolist would always have idling capacity, and therefore may increase its profit by installing less capacity – recall that the unit cost of capacity is $c > 0$. Thus, we study the monopolist's problem for price cap-capacity pairs $(\rho, k) \in [0,1]^2$ such that $k \leq \max\{1-\rho, 1/2\}$.

Figure 2 describes a partition of this set of price cap-capacity pairs into three regions, $A = \{(\rho, k) \in [0,1]^2 \mid \rho \leq k \leq 1-\rho\}$, $B = \{(\rho, k) \in [0,1]^2 \mid k < \min\{1-\rho, \rho\}\}$, and $C = \{(\rho, k) \in [0,1]^2 \mid 1-\rho \leq k \leq 1/2\}$.

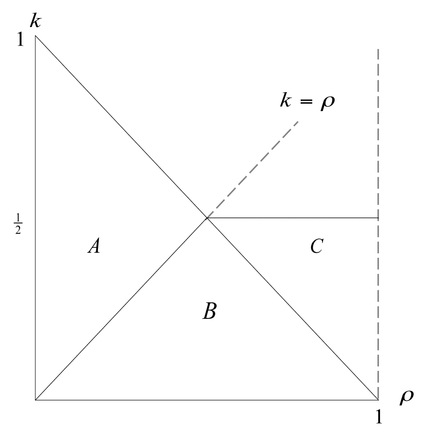

Figure 2: Relevant Price Cap-Capacity Pairs

We calculate the equilibrium price $P(\rho, k)$ and output $Q(\rho, k)$ in these regions for each realization of the demand parameter X. Table 1A describes the prices and output for $(\rho, k) \in A$.

X	$[0, 2\rho)$	$[2\rho, \rho + k)$	$[\rho + k, 1]$
$P(\rho, k)$	$x/2$	ρ	ρ
$Q(\rho, k)$	$x/2$	$x - \rho$	k

Table 1A: Equilibrium output and price for $(\rho, k) \in A$.

In region A, for low demand realizations $x < 2\rho$ the monopolist withholds capacity and the price cap is non-binding. For intermediate demand realizations $2\rho \leq x < \rho+k$ the price cap is binding because the monopolist continuous withholding capacity, serving the demand at the price cap. For high demand realizations $x \geq \rho + k$, the monopolist supplies its entire capacity, the price cap remains binding and the demand is rationed. In this region the market price $P(\rho, k)$ does not depend on the level of installed capacity k.

Table 1B describes the prices and output for $(\rho, k) \in B$.

X	$[0, 2k)$	$[2k, \rho + k)$	$[\rho + k, 1]$
$P(\rho, k)$	$x/2$	$x - k$	ρ
$Q(\rho, k)$	$x/2$	k	k

Table 1B: Equilibrium output and price for $(\rho, k) \in B$.

In region B, for low demand realizations $x \leq 2k$ the monopolist withholds capacity and the price cap is not binding. For intermediate demand realizations $2k \leq x < \rho+k$ the monopolist supplies its full capacity and the price cap is non-binding. For high demand realizations $x > \rho+k$ the monopolist continues supplying its entire capacity, but the price cap is binding and the demand is rationed. In this region the market price $P(\rho, k)$ depends on the level of capacity.

Table 1C describes the prices and output for $(\rho, k) \in C$.

X	$[0, 2k)$	$[2k, 1]$
$P(\rho, k)$	$x/2$	$x - k$
$Q(\rho, k)$	$x/2$	k

Table 1C: Equilibrium output and price for $(\rho, k) \in C$.

In region C, the price cap is never binding. The monopolist withholds capacity only for low demand realizations $x < 2k$, and supply its entire capacity otherwise. Demand is never rationed. The market price $P(\rho, k)$ depends on the level of capacity.

Note an important feature of equilibrium that stands in contrast to the case of deterministic demand: whether the price cap is binding or not equilibrium involves demand rationing. Demand rationing arises since capacity decisions are made ex-ante, and capacity cannot be built instantaneously.

In both A and B, if the demand at the price cap is so low that the monopolist's expected marginal revenue is positive, then price cap is effectively non-binding. If the demand at the price cap is above capacity, then monopolist supplies its full capacity. For intermediate demand realizations, however, the equilibrium differs in these two regions.

If region A, since capacity is large relative to the price cap, i.e., $k > \rho$, then for intermediate demand realizations $2\rho \leq x < \rho + k$ the expected marginal revenue is negative at $q = D(x, \rho)$, and therefore the monopolist serves the demand at the price cap, thus maintaining idling capacity, and the price cap is binding – see Figure 3A. Hence a marginal decrease of the price cap leads to an increase of output and a decrease of the market price for all demand realization on this range, much as in the standard case of a monopolist with an unlimited capacity facing a deterministic demand.

If region B, since capacity is small relative to the price cap, i.e., $k < \rho$, then for intermediate demand realizations $2k \leq x < \rho + k$ the expected marginal revenue at $q = D(x, \rho)$ is positive, and therefore the monopolist supplies its full capacity and the price cap remains non-binding – see Figure 3B. In this case a marginal decrease of the price cap has no effect on the level of output.

The monopolist's revenue is

$$R(\rho, k) = P(k, \rho)Q(\rho, k),$$

and its expected profit is

$$\bar{\Pi}(\rho, k) = E\left(R(\rho, k) - ck\right) = E\left(R(\rho, k)\right) - ck,$$

Clearly $\bar{\Pi}$ is continuous on $A \cup B \cup C$.

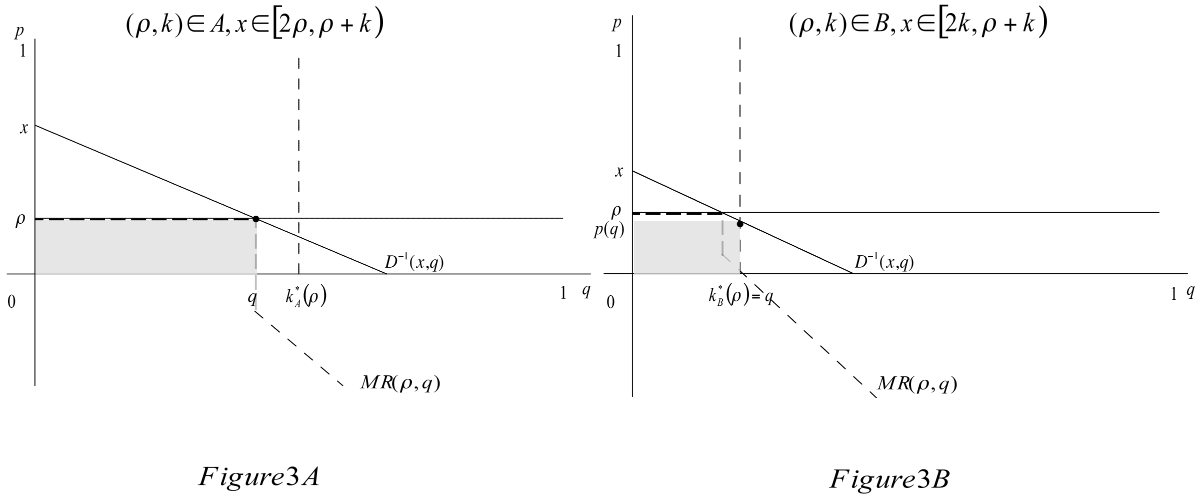

Figure 3: Equilibrium for Intermediate Demand Realizations.

In equilibrium, the monopolist's capacity maximizes $\bar{\Pi}(\rho,\cdot)$. Thus, in an interior equilibrium the capacity k^* is such that the monopolist's expected marginal revenue from installing an additional infinitesimal unit of capacity $\overline{MR}(\rho,k)$, where

$$\overline{MR}(\rho,k) := \frac{\partial E\left(R(\rho,k)\right)}{\partial k},$$

is equal to the marginal cost of capacity c; i.e., k^* solves

$$\overline{MR}(\rho,k) = c. \tag{1}$$

In addition, the second order condition

$$\frac{\partial \overline{MR}(\rho,k)}{\partial k} < 0 \tag{2}$$

holds at k^*.

Using the results described in tables 1A, 1B and 1C we readily calculate the monopolist's expected revenue as a function of its capacity and the price cap. For $(\rho,k)\in A$, we have

$$E\left(R(\rho,k)\right) = \int_0^{2\rho}\left(\frac{x}{2}\right)^2 f(x)dx + \int_{2\rho}^{\rho+k}\rho\left(x-\rho\right)f(x)dx + \int_{\rho+k}^1 \rho k f(x)dx.$$

For $(\rho,k)\in B$, we have

$$E\left(R(\rho,k)\right) = \int_0^{2k}\left(\frac{x}{2}\right)^2 f(x)dx + \int_{2k}^{\rho+k}\left(x-k\right)k f(x)dx + \int_{\rho+k}^1 \rho k f(x)dx.$$

And for $(\rho, k) \in C$, we have

$$E\left(R(\rho, k)\right) = \int_0^{2k} \left(\frac{x}{2}\right)^2 f(x)dx + \int_{2k}^1 (x-k)kf(x)dx.$$

Hence, the expected marginal revenue is

$$\overline{MR}(\rho, k) = \int_{\rho+k}^1 \rho f(x)dx \tag{3}$$

for $(\rho, k) \in A$, it is

$$\overline{MR}(\rho, k) = \int_{2k}^{\rho+k} (x-2k)\, f(x)dx + \int_{\rho+k}^1 \rho f(x)dx \tag{4}$$

for $(\rho, k) \in B$, and it is

$$\overline{MR}(\rho, k) = \int_{2k}^1 (x-2k)f(x)dx \tag{5}$$

for $(\rho, k) \in C$. Since (3) and (4) coincide for $k = \rho$, and (4) and (5) coincide for $\rho > 1/2$ and $k = 1-\rho$, then $\overline{MR}$ in continuous on $A \cup B \cup C$.

In region A, increasing marginally capacity affects the revenue only for high demand realizations $x > \rho + k$ for which the monopolist supplies its entire capacity. For these demand realizations the price cap ρ is binding. Thus, the expected revenue increases by ρ times the probability that the additional marginal unit of capacity is supplied, i.e.,

$$\overline{MR}(\rho, k) = \rho[1 - F(\rho + k)],$$

which is version of equation (3). In region B, a marginal increase of capacity has an additional effect on revenue: for intermediate demand realizations $2k < x < \rho + k$ the price cap is non-binding and the monopolists supplies its full capacity; therefore the marginal revenue is independent of the price cap, and is simply the derivative of $P(\rho, k)Q(\rho, k)$ with respect to k. In region C the price cap is never binding, and therefore this second is the only effect of a marginal increase of capacity.

Differentiating $\overline{MR}$ we have

$$\frac{\partial \overline{MR}(\rho, k)}{\partial k} = -\rho f(\rho + k) < 0 \tag{6}$$

for $(\rho, k) \in A$,

$$\frac{\partial \overline{MR}(\rho, k)}{\partial k} = -kf\left(\rho + k\right) - 2\left[F(\rho + k) - F(2k)\right] < 0 \tag{7}$$

for $(\rho, k) \in B$, and

$$\frac{\partial \overline{MR}(\rho, k)}{\partial k} = -2\left[1 - F(2k)\right] < 0 \tag{8}$$

for $(\rho, k) \in C$. Hence the expected marginal revenue function $\overline{MR}$ is decreasing, and therefore the inequality (2) holds on $A \cup B \cup C$. Moreover, since (6) and (7) coincide for $k = \rho$, then $\overline{MR}$ is differentiable on $A \cup B \cup C$, except perhaps in the boundary of B and C.

Thus, for all $\rho \in [0, 1]$ the monopolist's equilibrium capacity $k^*(\rho)$ is the unique solution of the equation (1). Moreover, the Maximum Theorem implies that k^* is a continuous function. We summarize these results in Proposition 1.

Proposition 1. *The monopoly equilibrium capacity k^* is a well defined continuous function of the price cap all $\rho \in [0, 1]$.*

We calculate the equilibrium capacity $k^*(\rho)$. Denote by h the hazard rate of X, i.e., $h(x) = f(x) / [1 - F(x)]$ for all $x \in (0, 1)$. Let us consider first price caps $\rho \in [0, 1/2]$. Then $\bar{\Pi}(\rho, \cdot)$ takes values in regions A and B.

If the capacity that maximizes $\bar{\Pi}(\rho, \cdot)$ is such that $(\rho, k) \in A$, then solving the equation (1) for $\overline{MR}$ given by (6) yields

$$k_A(\rho) = F^{-1}(1 - \frac{c}{\rho}) - \rho.$$

Hence

$$k_A(\rho) + \rho = F^{-1}(1 - \frac{c}{\rho}) < 1,$$

and therefore $k_A(\rho) < 1 - \rho$. If $(\rho, k_A(\rho)) \in A$, then $\rho \leq k_A(\rho)$. This inequality is equivalent to

$$c \leq \rho\,(1 - F(2\rho)) = \overline{MR}(\rho, \rho).$$

Write $M(\rho) := \overline{MR}(\rho, \rho)$. We have

$$\frac{dM(\rho)}{d\rho} = (1 - F(2\rho)) - 2\rho f(2\rho) = (1 - F(2\rho))\,(1 - 2\rho h(2\rho))\,,$$

which is positive for values of ρ close to zero and negative for values of ρ close to $1/2$. Assume that the hazard rate h is increasing. Then the function $M(\rho)$ is strictly concave and reaches its maximum value M^* on $(0, 1/2)$. If $c < M^*$, then the equation $\overline{MR}(\rho, \rho) = c$ has two solutions on $(0, 1/2)$, which we denote by $\rho_-(c)$ and $\rho_+(c)$

with $\rho_-(c) < \rho_+(c)$. In this case, for $\rho \in [\rho_-(c), \rho_+(c)]$, we have $(\rho, k_A^*(\rho)) \in A$. If $\rho \notin [\rho_-(c), \rho_+(c)]$, i.e., $c > \overline{MR}(\rho, \rho)$, then $\bar{\Pi}(\rho, \cdot)$ decreases with k in region A, and reaches its maximum in region B.

Assume that the capacity that maximizes $\bar{\Pi}(\rho, \cdot)$ is such that $(\rho, k) \in B$. Denote by $k_B(\rho)$ the solution to equation (1) for $\overline{MR}$ given by (4). We should not attempt to solve equation (1) explicitly and we will content ourselves for now identifying the subset of price caps $\rho \in [0, 1/2]$ for which the equilibrium capacity is $k_B(\rho)$. If $(\rho, k_B(\rho)) \in B$, then

$$0 < k_B(\rho) < \rho.$$

(Recall that we are identifying the monopolist capacity for $\rho < 1/2$, and therefore $k_B(\rho) < \rho$ implies $k_B(\rho) < 1 - \rho$.) The inequality $k_B(\rho) < \rho$ is equivalent to

$$c > \overline{MR}(\rho, \rho).$$

If $c \leq \overline{MR}(\rho, \rho)$, i.e., $\rho \in [\rho_-(c), \rho_+(c)]$, then $\bar{\Pi}(\rho, \cdot)$ increases with k in region B, and reaches its maximum in region A. The inequality $k_B(\rho) > 0$ is equivalent to

$$c < \int_0^\rho x f(x) dx + \rho \left(1 - F(\rho)\right) = \overline{MR}(\rho, 0),$$

i.e., the expected marginal revenue when output is zero $\overline{MR}(\rho, 0)$ must be greater than the unit cost of capacity c. If this inequality does not hold, then $\bar{\Pi}(\rho, \cdot)$ decreases with k in region B and reaches its maximum at $k^* = 0$. Since $d\overline{MR}(\rho, 0)/d\rho = 1 - F(\rho) > 0$ on $(0, 1)$, then the function $\overline{MR}(\cdot, 0)$ has an inverse, which we denote by $\underline{\rho}$. Then the condition $c < \overline{MR}(\rho, 0)$ above may be written as $\rho > \underline{\rho}(c)$. Since

$$\overline{MR}(\rho, 0) < \int_0^\rho \rho f(x) dx + \rho \left(1 - F(\rho)\right) = \rho,$$

then

$$c = \overline{MR}(\underline{\rho}(c), 0) < \underline{\rho}(c).$$

Therefore the equilibrium capacity is $k^* = 0$ for a range of price caps above the cost of capacity, $\rho \in (c, \underline{\rho}(c)]$. Also, since

$$\overline{MR}(\rho, 0) > \rho \left(1 - F(\rho)\right) > \rho \left(1 - F(2\rho)\right) = \overline{MR}(\rho, \rho),$$

then $\rho < \underline{\rho}(c)$ (i.e., $c > \overline{MR}(\rho, 0)$) implies $\rho < \rho_-(c)$.

Let us now consider price caps $\rho \in (1/2, 1]$. Then $\bar{\Pi}(\rho, \cdot)$ takes values in regions B and C.

Assume that the capacity that maximizes $\bar{\Pi}(\rho, \cdot)$ is such that $(\rho, k) \in B$. If $\rho \leq \underline{\rho}(c)$, then $\bar{\Pi}(\rho, \cdot)$ decreases with k and reaches its maximum at $k = 0$. If $\rho > \underline{\rho}(c)$, then $\bar{\Pi}(\rho, \cdot)$ reaches its maximum in region B if the solution to condition (1), $k_B(\rho)$, satisfies

$$k_B(\rho) < 1 - \rho.$$

This condition is equivalent to

$$c > \int_{2(1-\rho)}^{1} x f(x) dx - 2(1-\rho)[1 - F(2(1-\rho))] = \overline{MR}(\rho, 1-\rho).$$

Note that

$$\frac{d\overline{MR}(\rho, 1-\rho)}{d\rho} = 2(1 - F(2(1-\rho))) > 0.$$

Hence the function $\overline{MR}(\rho, 1-\rho)$ has an inverse on $(1/2, 1)$, which we denote by $\bar{\rho}$, and therefore we may write the above inequality as $\rho < \bar{\rho}(c)$. If $\rho \geq \bar{\rho}(c)$, then $\bar{\Pi}(\rho, \cdot)$ increases with k in region B and reaches its maximum in region C. Note that for $\rho = 1$ we have $\overline{MR}(\rho, 1-\rho) = \overline{MR}(1, 0) = E(X)$. Hence, since $c < E(X)$ by assumption, we have $\bar{\rho}(c) < 1$.

Finally, assume that the capacity that maximizes $\bar{\Pi}(\rho, \cdot)$ is such that $(\rho, k) \in C$. Denote by k_C the solution to the condition (1) for $\overline{MR}$ given by equation (5). Clearly k_C is independent of the price cap ρ. Also, since $\overline{MR}(\rho, 1/2) = 0$, we have $k_C < 1/2$ for all $c \in (0, E(X))$. Since the expected marginal revenue decreases with k, $k_C > 1 - \rho$ implies $c < \overline{MR}(\rho, 1-\rho)$. Moreover, since $\rho > 1/2$ and $\overline{MR}$ is decreasing, then $\overline{MR}(\rho, 1-\rho) < \overline{MR}(\rho, \rho)$. Hence k_C solves the monopolist problem if $\rho \geq \bar{\rho}(c)$. Otherwise, i.e., if $\rho < \bar{\rho}(c)$, then $\bar{\Pi}(\rho, \cdot)$ decreases with k in region C and reaches its maximum in region B.

As shown above $c < \underline{\rho}(c)$. If $c < M^*$, then we have $\underline{\rho}(c) < \rho_-(c) < \rho_+(c) < 1/2$. Since $1/2 < \bar{\rho}(c) < 1$, these inequalities imply

$$c < \underline{\rho}(c) < \rho_-(c) < \rho_+(c) < \bar{\rho}(c) < 1.$$

If $c \geq M^*$, then $c \geq \overline{MR}(\rho, \rho)$ for all $\rho \in [0, 1/2]$, and the equilibrium capacity lies in region B for all $\rho \in [0, 1/2]$.

We summarize these results in Proposition 2.

Proposition 2. *(2.1) The equilibrium capacity is $k^*(\rho) = 0$ whenever $\rho \in [0, \underline{\rho}(c)]$, and it is $k^*(\rho) = k_C$, independently of ρ, whenever $\rho \in [\bar{\rho}(c), 1]$, where $c < \underline{\rho}(c) < \bar{\rho}(c) < 1$ for all $c \in (0, E(X))$.*
(2.2) Assume that the hazard rate of X is increasing. If $c \in (0, M^*)$, then the equilibrium capacity is $k^*(\rho) = k_A(\rho)$ whenever $\rho \in [\rho_-(c), \rho_+(c)]$, and it is $k^*(\rho) = k_B(\rho)$ whenever $\rho \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_-(c), \rho_+(c)]$, where $\underline{\rho}(c) < \rho_-(c) < \rho_+(c) < \bar{\rho}(c)$. If $c \in (M^*, E(X))$, then the equilibrium capacity is $k^*(\rho) = k_B(\rho)$ for all $\rho \in (\underline{\rho}(c), \bar{\rho}(c))$.

The equilibrium capacity is zero for price caps above the unit cost of capacity – specifically, for $\rho \in (c, \underline{\rho}(c)]$. This is easy to understand: if the price cap is near c, because the probability of demand realizations $X < c$ is positive, then the expected marginal revenue near $k = 0$ is below c, and therefore installing capacity entails losses. Thus, the equilibrium capacity is zero unless the price cap is sufficiently high that expected marginal revenue for levels of capacity near cero is greater than c. Hence, unlike in the case of a deterministic demand, price caps close to the unit cost of capacity are suboptimal. (Of course, for demand distributions such that the maximum willingness to pay for the good is above c with probability one the equilibrium capacity is positive even for price caps close to c. Grimm and Zoettl (2010)'s Theorem 2 considers this possibility in their model of full capacity utilization.)

As in the case of a determinist demand, sufficiently large price caps are non-binding. Of course, the upper bound on the interval of binding price caps is determined by the distribution of the demand parameter X; specifically this bound is $\bar{\rho}(c)$ given by the solution to $c = \overline{MR}(\rho, 1 - \rho)$.

Intermediate price caps $\rho \in [\underline{\rho}(c), \bar{\rho}(c))$ affect the monopoly equilibrium capacity is ways that are not as simple to describe as in the case of a deterministic demand. In particular, as we shall see in the next section the level of equilibrium capacity is not monotonically decreasing with the price cap in this interval.

Using the results in tables 1A, 1B and 1C, and the description on the equilibrium capacity given in Proposition 2, one can calculate the expected output and market price as well as the expected (consumer and total) surplus, thus providing a complete description of the monopoly equilibrium. We study in the next section the effect of changes in the price cap on these values.

4 Comparative Statics

In this section we study the comparative static properties of price caps. We show that under the assumptions of Proposition 2 there is a price cap that maximizes the equilibrium capacity $\rho^*(c) \in (\underline{\rho}(c), \bar{\rho}(c))$. Moreover, we show that the equilibrium capacity increases with the price cap on the interval $(\underline{\rho}(c), \rho^*(c))$, and decreases with the price cap on the interval $(\rho^*(c), \bar{\rho}(c))$. Thus, relative to the capacity maximizing price cap $\rho^*(c)$ the impact of price caps on the equilibrium capacity when demand is uncertain is analogous to their impact when demand is deterministic. (Recall that with a deterministic demand the price cap that maximizes capacity is $\rho^*(c) = c$.) Of course, for very low price caps $\rho < \underline{\rho}(c)$ or very high price caps $\rho > \bar{\rho}(c)$, a marginal change of the price cap has no impact on the monopoly equilibrium.

Let $\rho \in (\underline{\rho}(c), \bar{\rho}(c))$. Since the expected marginal revenue $\overline{MR}(\rho, k)$ is differentiable in regions $A \cup B$, we can differentiate equation (1) to get

$$\frac{\partial \overline{MR}(\rho, k)}{\partial k} dk + \frac{\partial \overline{MR}(\rho, k)}{\partial \rho} d\rho = 0.$$

And since $\overline{MR}$ is decreasing, i.e.,

$$\frac{\partial \overline{MR}(\rho, k)}{\partial k} < 0,$$

then

$$\frac{dk^*}{d\rho} = -\frac{\partial \overline{MR}(\rho, k)}{\partial \rho} \left(\frac{\partial \overline{MR}(\rho, k)}{\partial k} \right)^{-1},$$

and

$$\frac{dk^*}{d\rho} \gtreqless 0 \Leftrightarrow \frac{\partial \overline{MR}(\rho, k)}{\partial \rho} \gtreqless 0.$$

Assume that f is continuously differentiable. Then $\overline{MR}$ is twice continuously differentiable, and

$$\begin{aligned}
\frac{d^2 k^*}{d\rho^2} &= -\left(\frac{\partial \overline{MR}(\rho, k)}{\partial k} \right)^{-1} \frac{d}{d\rho} \left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial \rho} \right) \\
&\quad + \frac{\partial \overline{MR}(\rho, k)}{\partial \rho} \left(\frac{\partial \overline{MR}(\rho, k)}{\partial k} \right)^{-2} \frac{d}{d\rho} \left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial k} \right) \\
&= -\left(\frac{\partial \overline{MR}(\rho, k)}{\partial k} \right)^{-1} \left(\frac{d}{d\rho} \left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial \rho} \right) + \frac{dk^*}{d\rho} \frac{d}{d\rho} \left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial k} \right) \right).
\end{aligned}$$

Hence, for ρ such that $dk^*/d\rho = 0$, we have

$$\frac{d^2k^*}{d\rho^2} \gtreqless 0 \Leftrightarrow \frac{d}{d\rho}\left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial \rho}\right) \gtreqless 0.$$

If $(\rho, k^*(\rho)) \in A$, then differentiating $\overline{MR}$ given in (3) yields

$$\frac{\partial \overline{MR}(\rho, k)}{\partial \rho} = 1 - F(\rho + k) - \rho f(\rho + k) = (1 - F(\rho + k))\,(1 - \rho h\,(\rho + k))\,,$$

and

$$\begin{aligned}\frac{d}{d\rho}\left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial \rho}\right) &= -f(\rho + k)\left(1 + \frac{dk_A}{d\rho}\right)(1 - \rho h\,(\rho + k)) \\ &\quad - (1 - F(\rho + k))\,(h\,(\rho + k) + \rho h'\,(\rho + k))\left(1 + \frac{dk_A}{d\rho}\right).\end{aligned}$$

Assume that $dk_A/d\rho = 0$. Then $1 - \rho h\,(\rho + k^*(\rho)) = 0$, and

$$\frac{d}{d\rho}\left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial \rho}\right) = -\,(1 - F(\rho + k^*(\rho)))\,(h\,(\rho + k^*(\rho)) + \rho h'\,(\rho + k^*(\rho)))\,.$$

If the hazard rate is increasing (i.e., $h' > 0$), then we have

$$\frac{d^2k_A}{d\rho^2} < 0,$$

and therefore every critical point of k_A is a local maximum.

If $(\rho, k_B(\rho)) \in B$, then differentiating $\overline{MR}$ given in (4) yields

$$\frac{\partial \overline{MR}(\rho, k)}{\partial \rho} = 1 - F(\rho + k) - kf(\rho + k) = (1 - F(\rho + k))\,(1 - kh(\rho + k))\,,$$

and

$$\begin{aligned}\frac{d}{d\rho}\left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial \rho}\right) &= -f(\rho + k^*(\rho))\,(1 - k^*(\rho)h(\rho + k^*(\rho)))\left(1 + \frac{dk_B}{d\rho}\right) \\ &\quad - (1 - F(\rho + k^*(\rho)))\,k^*(\rho)h'(\rho + k^*(\rho))\left(1 + \frac{dk_B}{d\rho}\right) \\ &\quad - (1 - F(\rho + k^*(\rho)))\,h(\rho + k^*(\rho))\frac{dk_B}{d\rho}.\end{aligned}$$

Assume that $dk_B/d\rho = 0$. Then $1 - k^*(\rho)h\,(\rho + k^*(\rho)) = 0$, and

$$\frac{d}{d\rho}\left(\frac{\partial \overline{MR}(\rho, k^*(\rho))}{\partial \rho}\right) = -\,(1 - F(\rho + k^*(\rho)))\,k^*(\rho)h'(\rho + k^*(\rho)).$$

If the hazard rate is increasing (i.e., $h' > 0$) we have

$$\frac{d^2k_B}{d\rho^2} < 0,$$

and therefore every critical point of k_B is a local maximum. Moreover, we have $k_B(\bar{\rho}(c)) = 1 - \bar{\rho}(c)$, and

$$\begin{aligned}
\frac{\partial \overline{MR}(\bar{\rho}(c), 1-\bar{\rho}(c))}{\partial \rho} &= 1 - F\left(\bar{\rho}(c) + (1-\bar{\rho}(c))\right) - (1-\bar{\rho}(c))\, f\left(\bar{\rho}(c) + (1-\bar{\rho}(c))\right) \\
&= -\left(1-\bar{\rho}(c)\right) f(1) \\
&< 0,
\end{aligned}$$

and therefore $dk_B(\bar{\rho}(c))/d\rho < 0$. Also we have $k_B(\underline{\rho}(c)) = 0$, and

$$\frac{\partial \overline{MR}(\underline{\rho}(c), 0)}{\partial \rho} = 1 - F(\underline{\rho}(c)) < 0,$$

and therefore $dk_B(\underline{\rho}(c))/d\rho > 0$.

Then k^* has a global maximizer $\rho^*(c) \in \left(\underline{\rho}(c), \bar{\rho}(c)\right)$, and satisfies $dk^*/d\rho > 0$ on $(\underline{\rho}(c), \rho^*(c))$ and $dk^*/d\rho < 0$ on $(\rho^*(c), \bar{\rho}(c))$, as we show in the following remark.

Remark. *Let g be a real valued function on some open interval $(a, b) \subset \mathbb{R}$ such that $g'(a) > 0 > g'(b)$, and $g''(y) < 0$ for all $y \in (a, b)$ such that $g'(y) = 0$. Then g has a unique global maximizer $y^* \in (a, b)$, and g' is positive on (a, y^*) and negative on (y^*, b).*

Proof. Since g' is continuous on (a, b) and $g'(a) > 0 > g'(b)$, then there is $y^* \in (a, b)$ such that $g'(y^*) = 0$. We show that g' is positive on (a, y^*). Assume by way of contradiction that there is $\bar{y} < y^*$ such that $g'(\bar{y}) \leq 0$. Then there is $\hat{y} \in [\bar{y}, y^*]$ such that $g'(\hat{y} - \varepsilon) \geq 0 = g'(\hat{y})$ for all $\varepsilon > 0$ sufficiently small. Hence $g''(\hat{y}) \geq 0$, which contradicts our assumption that $g''(\hat{y}) < 0$. The proof that g' is negative on (y^*, b) is analogous. Therefore y^* is the unique global maximizer of g on (a, b).

We summarize these results in Proposition 3.

Proposition 3. *Assume that the hazard rate of X is increasing and its p.d.f. f is continuously differentiable. Then k^* has a global maximum at some $\rho^*(c) \in (\underline{\rho}(c), \bar{\rho}(c))$. Moreover, $dk^*(\rho)/d\rho$ is positive on $(\underline{\rho}(c), \rho^*(c))$, and $dk^*(\rho)/d\rho$ is negative on $(\rho^*(c), \bar{\rho}(c))$.*

It is also useful to calculate the expected output and the expected price using the results described in tables 1A, 1B and 1C, and to examine how they are affected by changes of the price cap. For $\rho \in [\rho_-(c), \rho_+(c)]$ the expected output is

$$E(Q(\rho, k^*(\rho)) = \int_0^{2\rho} \frac{x}{2} f(x)dx + \int_{2\rho}^{\rho+k^*(\rho)} (x-\rho) f(x)dx + \int_{\rho+k^*(\rho)}^{1} k f(x)dx,$$

whereas for $\rho \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_-(c), \rho_+(c)]$ it is

$$E(Q(\rho, k^*(\rho)) = \int_0^{2k^*(\rho)} \frac{x}{2} f(x)dx + \int_{2k^*(\rho)}^{1} k f(x)dx.$$

Thus, for $\rho \in [\rho_-(c), \rho_+(c)]$ we have

$$\frac{dE(Q(\rho, k^*(\rho))}{d\rho} = -[F(\rho + k^*(\rho)) - F(2\rho)] + \frac{dk^*}{d\rho} \left(1 - F(\rho + k^*(\rho))\right),$$

whereas for $\rho \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_-, \rho_+])$ we have

$$\frac{dE(Q(\rho, k^*(\rho))}{d\rho} = \frac{dk^*}{d\rho} \left(1 - F(2k^*(\rho))\right).$$

Hence for $\rho \in [\rho_-(c), \rho_+(c)]$ we have

$$\frac{dk^*}{d\rho} \leq 0 \Rightarrow \frac{dE(Q(\rho, k^*(\rho))}{d\rho} < 0.$$

Thus, the expected output decreases with the price cap beyond the price cap that maximizes capacity; that is, the price cap that maximizes output is below $\rho^*(c)$.

For $\rho \in [\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_-, \rho_+])$ we have

$$\frac{dE(Q(\rho, k^*(\rho))}{d\rho} \gtreqless 0 \Longleftrightarrow \frac{dk^*}{d\rho} \gtreqless 0.$$

That is, the expected output increases with the price cap for $\rho \in (\underline{\rho}(c), \rho^*)$, and decreases for $\rho \in (\rho^*, \bar{\rho}(c))$.

Likewise for $\rho \in [\rho_-(c), \rho_+(c)]$ the expected price is

$$E(P(\rho, k^*(\rho)) = \int_0^{2\rho} \frac{x}{2} f(x)dx + \int_{2\rho}^{1} \rho f(x)dx,$$

and for $\rho \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_-(c), \rho_+(c)]$ it is

$$E(P(\rho, k^*(\rho)) = \int_0^{2k^*(\rho)} \frac{x}{2} f(x)dx + \int_{2k^*(\rho)}^{\rho+k^*(\rho)} (x - k^*(\rho)) f(x)dx + \int_{\rho+k^*(\rho)}^{1} \rho f(x)dx.$$

Hence for $\rho \in [\rho_-(c), \rho_+(c)]$ we have

$$\frac{dE(P(\rho, k^*(\rho))}{d\rho} = 1 - F(2\rho) > 0,$$

for $\rho \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_{-}(c), \rho_{+}(c)]$ we have

$$\frac{dE(P(\rho, k^*(\rho))}{d\rho} = 1 - F(\rho + k) > 0.$$

Thus, the expected price increases with the price cap on $(\underline{\rho}(c), \bar{\rho}(c))$.

We summarize these results in Proposition 4.

Proposition 4. *The expected price increases with the price cap on* $(\underline{\rho}(c), \bar{\rho}(c))$*. Further, if the hazard rate of* X *is increasing and its p.d.f.* f *is continuously differentiable, then:*

(4.1) If $\rho^*(c) \in (\rho_{-}(c), \rho_{+}(c))$*, then the expected output decreases with the price cap for price cap above and around* $\rho^*(c)$*.*

(4.2) If $\rho^*(c) \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_{-}(c), \rho_{+}(c)]$*, then the expected output increases with the price cap on* $(\underline{\rho}(c), \rho^*(c))$ *and decreases on* $(\rho^*(c), \bar{\rho}(c))$ *.*

Thus, the comparative static properties of price caps under demand uncertainty are analogous to those of the benchmark case of a deterministic demand, with an important qualification: when c is sufficiently small that $\rho^*(c) \in (\rho_{-}(c), \rho_{+}(c))$, maximizing capacity does not warrant maximizing the expected output: despite the fact that capacity increases with the price cap below $\rho^*(c)$, the expected output decreases with the price cap even around $\rho^*(c)$. Of course, this fact has direct implications on the price cap that maximizes the expected surplus, as we show in the next section.

5 Optimal Price Caps

A regulator who wants to maximize the expected surplus using a price cap as its single instrument, and cannot force the monopolist to serve its full capacity, must trade off the incentives for capacity investment and capacity withholding, and must account for the cost of installing capacity that is seldom utilized. Thus, the optimal price cap may differ from the price cap that maximizes capacity investment $\rho^*(c)$. (In contrast, in the model of full capacity utilization studied by Earle et al. (2007) and Grimm and Zoettl (2010), maximizing the expected surplus simply amounts to maximizing capacity – see Appendix B.) Indeed, we show that when the unit cost of capacity is small this is the case: the optimal price cap is below $\rho^*(c)$. When the unit cost of capacity is high, however, providing appropriate incentives for capacity investment becomes the dominant objective, and thus the optimal price cap is $\rho^*(c)$.

Following the literature, we simplify somewhat the problem by assuming efficient rationing, i.e., when the price cap is binding the consumers with the largest willingness to pay receive priority to buy the good. Tables 2A describes the surplus $S(\rho, k)$ for each realization of the demand parameter when $(\rho, k) \in A$.

X	$[0, 2\rho)$	$[2\rho, \rho + k)$	$[\rho + k, 1]$
$S(\rho, k)$	$\frac{3}{8}x^2$	$\frac{1}{2}(x^2 - \rho^2)$	$\frac{1}{2}(2x - k)k$

Table 2A: Social Surplus in Region A.

Recall that the monopolist withholds capacity for realizations of the demand parameter $x \in [0, 2\rho)$. Hence the expected surplus depends directly on the price cap, as well as indirectly through its effect on the monopolist capacity decision. The expected surplus for $(\rho, k) \in A$ is

$$\begin{aligned} E(S(\rho, k)) &= \frac{3}{8}\int_0^{2\rho} x^2 f(x)dx + \frac{1}{2}\int_{2\rho}^{\rho+k} (x^2 - \rho^2) f(x)dx \\ &+ \frac{1}{2}\int_{\rho+k}^{1} (2x - k)kf(x)dx - ck. \end{aligned} \tag{9}$$

Table 2BC below describes the surplus $S(\rho, k)$ for each realization of the demand parameter when $(\rho, k) \in B \cup C$.

X	$[0, 2k)$	$[2k, 1]$
$S(\rho, k)$	$\frac{3}{8}x^2$	$\frac{1}{2}(2x - k)k$

Table 2BC: Social Surplus in Regions B and C.

In $B \cup C$ a price cap has no direct effect on the expected surplus, but only has an indirect effect via its influence on the monopolist capacity choice. (Of course, the price cap also determines the distribution of surplus.) The expected surplus for $(\rho, k) \in B \cup C$ is

$$E(S(\rho, k)) = \frac{3}{8}\int_0^{2k} x^2 f(x)dx + \frac{1}{2}\int_{2k}^{1} (2x - k)kf(x)dx - ck. \tag{10}$$

The optimal price cap maximizes $\bar{S}(\rho) = E(S(\rho, k^*(\rho))$.

For price caps $\rho \in [\rho_-(c), \rho_+(c)]$ the price cap-equilibrium capacity pair $(\rho, k^*(\rho))$ is in region A. Differentiating $\bar{S}$ given in (9) yields

$$\frac{d\bar{S}(\rho)}{d\rho} = -[\rho F(\rho + k^*(\rho)) - F(2\rho)] + \frac{dk^*(\rho)}{d\rho}\left(\int_{\rho+k^*(\rho)}^1 (x - k^*(\rho))f(x)dx - c\right),$$

Recall that $\rho^*(c)$ is the capacity maximizing price cap identified in Proposition 3. If $\rho^*(c) \in [\rho_-(c), \rho_+(c)]$, then $dk^*(\rho^*(c))/d\rho = 0$ and $k^*(\rho^*(c)) = k_A(\rho^*(c)) > \rho^*(c)$ imply

$$\frac{d\bar{S}(\rho^*(c))}{d\rho} = -\rho^*(c)[F(\rho^*(c) + k^*(\rho^*(c))) - F(2\rho^*(c))] < 0. \quad (11)$$

Hence the expected surplus decreases with the price cap at $\rho^*(c)$. Decreasing the price cap below $\rho^*(c)$ increases surplus, even though it decreases capacity, because it encourages capacity utilization. Hence the optimal price cap is below $\rho^*(c)$.

For price caps $\rho \in [0,1]\backslash[\rho_-(c), \rho_+(c)]$ we have $(\rho, k^*(\rho)) \in B \cup C$. Differentiating $\bar{S}$ given in (10) yields

$$\frac{d\bar{S}(\rho)}{d\rho} = \frac{dk^*(\rho)}{d\rho}\left(\int_{2k^*(\rho)}^1 (x - k^*(\rho))f(x)dx - c\right). \quad (12)$$

For $\rho \in (\underline{\rho}(c), \bar{\rho}(c))\backslash[\rho_-(c), \rho_+(c)]$, we have $(\rho, k^*(\rho)) \in B$, $k^*(\rho) < \rho$, and

$$\overline{MR}(\rho, k^*(\rho)) = \int_{2k^*(\rho)}^{\rho+k^*(\rho)} (x - 2k^*(\rho))\, f(x)dx + \int_{\rho+k^*(\rho)}^1 \rho f(x)dx = c.$$

Hence

$$\int_{2k^*(\rho)}^1 (x - k^*(\rho))f(x)dx - c = \int_{2k^*(\rho)}^{\rho+k^*(\rho)} k^*(\rho)f(x)dx + \int_{\rho+k^*(\rho)}^1 (x - k^*(\rho) - \rho)f(x)dx > 0,$$

and therefore

$$\frac{d\bar{S}(\rho)}{d\rho} = 0 \Leftrightarrow \frac{dk^*(\rho)}{d\rho} = 0.$$

Differentiating $d\bar{S}(\rho)/d\rho$ we get

$$\begin{aligned}\frac{d^2\bar{S}(\rho)}{d\rho^2} &= \frac{d^2k^*(\rho)}{d\rho^2}\left(\int_{2k^*(\rho)}^1 (x - k^*(\rho))f(x)dx - c\right) \\ &\quad - \left(\frac{dk^*(\rho)}{d\rho}\right)^2 [1 - F(2k^*(\rho)) + 2k^*(\rho)f(2k^*(\rho))],\end{aligned}$$

If $d\bar{S}(\rho)/d\rho = 0$, then $dk^*(\rho)/d\rho = 0$, which as shown above implies $d^2k^*(\rho)/d\rho^2 < 0$. Hence $d^2\bar{S}(\rho)/d\rho^2 < 0$. Thus, by the remark above if $\rho^*(c) \in (\underline{\rho}(c), \bar{\rho}(c))\backslash[\rho_-(c), \rho_+(c)]$, then $\rho^*(c)$ is the unique global maximizer of $\bar{S}$ on $(\underline{\rho}(c), \bar{\rho}(c))$.

Note that since in the boundary of regions A and $B \cup C$ the equilibrium capacity is $k^*(\rho) = \rho$, then the expression for $d\bar{S}(\rho)/d\rho$ in equations (11) and (12) coincide, and therefore $\bar{S}$ is differentiable on $[0, 1]$. Proposition 5 summarizes these results.

Proposition 5. *Assume that hazard rate of X is increasing and its p.d.f. f is continuously differentiable, and let $\rho^*(c)$ be the capacity maximizing price cap identified in Proposition 3. If $\rho^*(c) \in [\rho_-(c), \rho_+(c)]$ then the expected surplus decreases with the price cap above and around $\rho^*(c)$, whereas if $\rho^*(c) \in [0, 1] \backslash (\rho_-(c), \rho_+(c))$, then $\rho^*(c)$ maximizes the expected surplus.*

When the demand is known with certainty the optimal price cap $\rho^*(c) = c$ eliminates all inefficiencies. Under demand uncertainty this is not the case: there is an obvious source of inefficiency resulting from the monopolist withholding capacity for low realizations of the demand. This inefficiency is large enough that when the unit cost of capacity is sufficiently small that $\rho^*(c) \in [\rho_-(c), \rho_+(c)]$, it is socially optimal to decrease the price cap below $\rho^*(c)$ even at the cost of reducing capacity. Nonetheless, even if the optimal price cap is $\rho^*(c)$, the level of capacity installed by the monopolist $k^*(\rho^*(c))$ is below the level of capacity that will be socially optimal if the entire capacity was served for each demand realization. Thus, a price cap alone is a poor regulatory instrument as it cannot provide the appropriate incentives to install the optimal level of capacity and simultaneously eliminate the inefficiencies arising from capacity withholding.

For the sake of discussion, it is useful to consider the artificial scenario in which the regulator chooses the level of capacity, and although it does not control the use of capacity by the monopolist (i.e., the monopolist may withhold capacity) it sets a price cap in order to alleviate capacity withholding. Taking derivatives with respect to ρ in (9) and (10) we have

$$\frac{\partial E(S(\rho, k))}{\partial \rho} = -\rho \left(F(\rho + k) - F(2\rho) \right) < 0,$$

for $(\rho, k) \in A$, and $\partial E(S(\rho, k))/\partial \rho = 0$ for $(\rho, k) \in B \cup C$. Hence the optimal price cap is $\rho = 0$, and the surplus is

$$S^*(k) = E(S(0, k)) = \frac{1}{2} \int_0^k x^2 f(x) dx + \frac{1}{2} \int_k^1 (2x - k) k f(x) dx - ck,$$

which is the entire surplus that can be realized given k; i.e., an optimal price cap effectively eliminates capacity withholding. The socially level of capacity maximizes $S^*(k)$; i.e., k^W solves the equation

$$\frac{dS^*(k)}{dk} = \int_k^1 (x-k)\, f(x)dx - c = 0$$

(Note that $d^2 S^*(k)/dk^2 = -[1-F(k)] < 0$.)

It is easy to show that $k^W > k^*(\rho^*(c)) \geq k^*(\rho)$ for all $\rho \in [0,1]$. Let us fix c and reduce notation by writing k^* and ρ^* for $k^*(\rho^*(c))$ and $\rho^*(c)$, respectively. If $\rho^* \in [\rho_-(c), \rho_+(c)]$, then $k^* \geq \rho^*$ and

$$\overline{MR}(\rho^*, k^*) = \int_{\rho^*+k^*}^1 \rho^* f(x)dx = c$$

imply

$$\begin{aligned}
\frac{dS^*(k^*)}{dk} &= \int_{k^*}^1 (x-k^*)\, f(x)dx - \int_{\rho^*+k^*}^1 \rho^* f(x)dx \\
&= \int_{k^*}^{\rho^*+k^*} (x-k^*) f(x)dx + \int_{\rho^*+k^*}^1 (x-\rho^*-k^*) f(x)dx \\
&> 0.
\end{aligned}$$

Hence $k^W > k^*$. If $\rho^* \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_-(c), \rho_+(c)]$, then $k^* \leq \rho^*$ and

$$\overline{MR}(\rho^*, k^*) = \int_{2k^*}^{\rho^*+k^*} (x-2k^*) f(x)dx + \int_{\rho^*+k^*}^1 \rho^* f(x)dx = c$$

imply

$$\begin{aligned}
\frac{dS^*(k^*)}{dk} &= \int_{k^*}^1 (x-k^*) f(x)dx - \left(\int_{2k^*}^{\rho^*+k^*} (x-2k^*) f(x)dx + \int_{\rho^*+k^*}^1 \rho f(x)dx \right) \\
&= \int_{k^*}^{2k^*} (x-k^*)\, f(x)dx + \int_{2k^*}^{\rho^*+k^*} k^* f(x)dx + \int_{\rho^*+k^*}^1 (x-\rho^*-k^*) f(x)dx \\
&> 0.
\end{aligned}$$

Hence $k^W > k^*$ as well.

Thus, a price cap alone cannot provide appropriate incentives to install the optimal level of capacity and simultaneously eliminate the inefficiencies arising from capacity withholding. It is worth noticing that even when the monopolist cannot withhold capacity a price cap is not able to induce the monopolist to install the optimal level of capacity; in fact, the maximum level of capacity installed by a monopolist that cannot withhold capacity tends to be even lower than that of a monopolist that withholds capacity – see Appendix B.

6 An Example

Assume that X is uniformly distributed on $[0,1]$, i.e., $f(x)=1$. Thus, X has an increasing hazard rate $h(x)=(1-x)^{-1}$, and its *p.d.f.* f is continuously differentiable. Since $E(X)=1/2$, we consider values of the unit costs of capacity $c\in(0,1/2)$.

Let us calculate the equilibrium capacity in this setting. We have

$$k_A(\rho)=F^{-1}(1-\frac{c}{\rho})-\rho=1-\frac{c}{\rho}-\rho.$$

The marginal revenue given in (4) is in this setting

$$\overline{MR}(\rho,k)=\frac{k^2}{2}+\frac{\rho}{2}[2\,(1-2k)-\rho].$$

Solving equation (1) yields

$$k_B(\rho)=2\rho-\sqrt{2c-\rho\,(2-5\rho)}.$$

The marginal revenue given in (5) is in this setting

$$\overline{MR}(\rho,k)=\frac{1}{2}\,(1-2k)^2\,,$$

Solving equation (1) yields

$$k_C=\frac{1-\sqrt{2c}}{2}.$$

Let us calculate the functions $\underline{\rho}$, ρ_-, ρ_+ and $\bar{\rho}$. The function $\underline{\rho}$ is the solution to the equation

$$c=\overline{MR}(\rho,0)=\int_0^\rho xf(x)dx+\rho\,(1-F(\rho))=\frac{\rho\,(2-\rho)}{2},$$

i.e.,

$$\underline{\rho}(c)=1-\sqrt{1-2c}.$$

The functions ρ_- and ρ_+ are the smaller and larger solutions to the equation

$$c=\overline{MR}(\rho,\rho)=\rho\,(1-F(2\rho))=\rho(1-2\rho),$$

which are readily calculated as

$$\rho_-(c)=\frac{1}{4}\left(1-\sqrt{1-8c}\right),\ \rho_+(c)=\frac{1}{4}\left(1+\sqrt{1-8c}\right).$$

These functions are well defined on $(0,1/8)$, where $M^*=1/8$ is the maximum value of $M(\rho)=\overline{MR}(\rho,\rho)$. For $c>1/8$ the above equation has no solution on $[0,1]$. The function $\bar{\rho}$ solves the equation

$$c=\overline{MR}(\rho,1-\rho)=\int_{2(1-\rho)}^{1}xf(x)dx-2(1-\rho)\left[1-F(2(1-\rho))\right]=\frac{(1-2\rho)^2}{2},$$

i.e.,

$$\bar{\rho}(c)=\frac{1+\sqrt{2c}}{2}.$$

It is easy to check that for $c\in(0,1/2)$ we have

$$c<\underline{\rho}(c)<\bar{\rho}(c)<1.$$

Further, for $c\in(0,1/8)$ we have

$$\underline{\rho}(c)<\rho_-(c)<\rho_+(c)<\frac{1}{2}<\bar{\rho}(c).$$

Figure 4 provides a description of the function k^* for value of $c\in(0,1/2)$. For $c\leq 1/9$ the equilibrium capacity $k^*(\rho)$ reaches its maximum at the price cap $\rho_A^*=\sqrt{c}\in[\rho_-(c),\rho_+(c)]$. For $c>1/9$, the equilibrium capacity $k^*(\rho)$ reaches its maximum at $\rho_B^*=\left(1+2\sqrt{10c-1}\right)/5\in(\underline{\rho}(c),\bar{\rho}(c))\backslash[\rho_-(c),\rho_+(c)]$. (For $c>1/8$ the interval $[\rho_-(c),\rho_+(c)]$ is empty.) Interestingly, for $c\in(1/9,1/8)$ the equilibrium capacity $k^*(\rho)$ is increasing in the interval $(\rho_-(c),\rho_+(c))$, and reaches its maximum at $\rho^*(c)\in(\rho_+(c),\bar{\rho}(c))$.

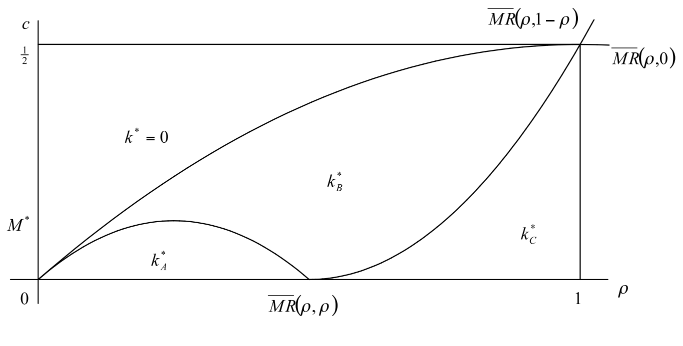

Figure 4: Equilibrium Capacity.

We calculate the expected surplus. If $\rho<\underline{\rho}(c)$, then the expected surplus is $\bar{S}(\rho)=0$. If $\rho\in[\rho_-(c),\rho_+(c)]$, which requires $c<1/8$, then the expected surplus is

$$\bar{S}(\rho)=\frac{\rho^3\left(1+4\rho^3\right)+3\rho^2\left(c\left(c-2\rho\left(1-\rho\right)\right)-\rho^3\right)-c^3}{6\rho^3}.$$

If $\rho \in (\underline{\rho}(c), \bar{\rho}(c)) \backslash [\rho_-(c), \rho_+(c)]$, then the expected surplus is

$$\bar{S}(\rho) = \frac{\rho}{2}(4 - 9\rho) - c(1 + 2\rho) + \left(c + 2\rho - \frac{1}{2}\right)\sqrt{2c - \rho(2 - 5\rho)}.$$

And if $\rho \in [\bar{\rho}(c), 1]$ then

$$\bar{S}_{BC}(\rho) = \frac{1 - 6c}{8} + \frac{\sqrt{2c^3}}{2}.$$

Figure 5 displays the equilibrium capacity and surplus as functions of the price cap when the unit cost of capacity is $c = 1/32$. the price cap that maximizes capacity is $\rho^* = \sqrt{2}/8$ whereas, consistently with Proposition 5, the expected surplus is maximized at $\rho = 1/8 < \rho^*$.

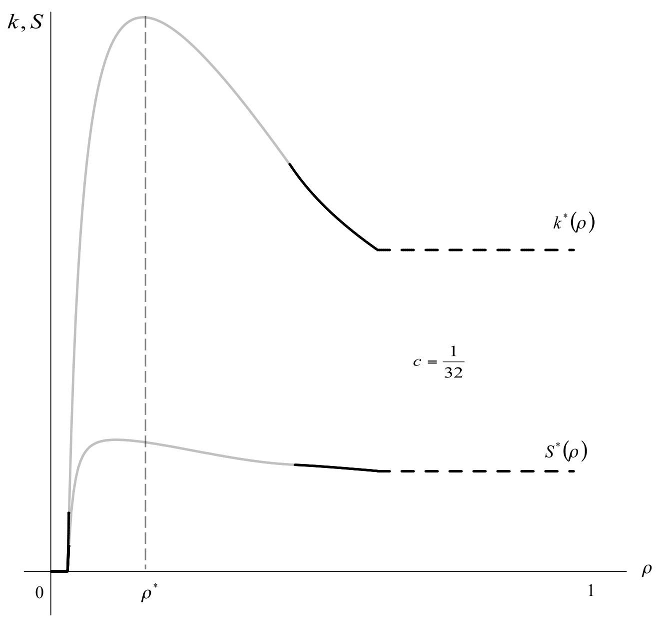

Figure 5: Capacity and Surplus for $c = 1/32$.

Figure 6 shows the graphs of the capacity and the expected surplus for $c = 3/25$. For this unit cost of capacity we have $[\rho_-(c), \rho_+(c)] = [2/10, 3/10]$. (Note that $c = 3/25 < 1/8$.) The price cap that maximizes both capacity and expected surplus is $\rho_B^* = (2\sqrt{5}+5)/25 \in (\underline{\rho}(c), \bar{\rho}(c))$, i.e., the maximum capacity is reached at a price cap-capacity pair in region B, and consistently with Proposition 5, the expected surplus is maximal at this price cap.

Suppose that the regulator sets the optimal price cap $\rho = 0$, and independently choose the level of capacity. Using the results obtained in Section 5 we calculate the expected surplus as a function of the capacity as

$$S^{*}(0,k) = \frac{k^{2}(k-3)}{6} + \frac{k(1-2c)}{2},$$

which is maximized at $k^{W} = 1 - \sqrt{2c}$.

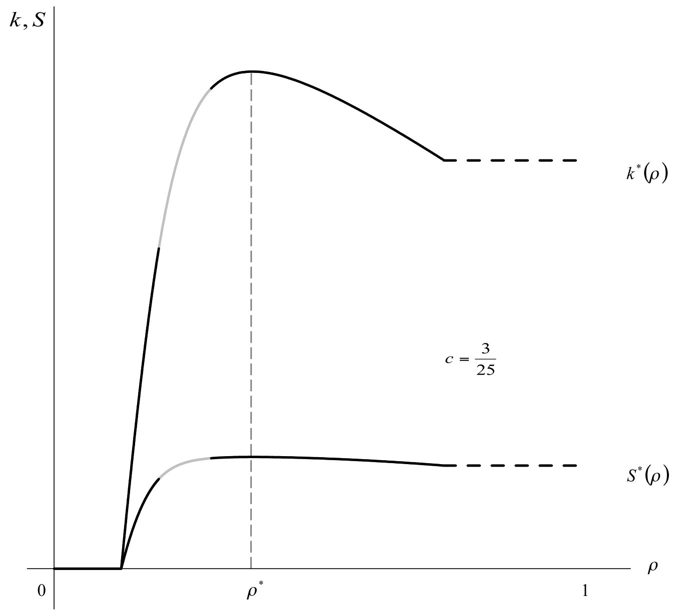

Figure 6: Capacity and Surplus for $c = 3/25$.

With capacity withholding, for $c = 1/32$ the optimal capacity is $k^{*}(\rho^{*}) = (0.86)\,k^{W}$ and the expected surplus is $\bar{S}(k^{*}(\rho^{*})) = (0.93)S^{*}(0,k^{W})$. For $c = 3/25$ these numbers are considerably lower, $k^{*}(\rho^{*}) \simeq (0.61)k^{W}$ and $\bar{S}(k^{*}(\rho^{*})) = (0.81)S^{*}(0,k^{W})$. These numbers suggest that with capacity withholding price caps are more effective when unit cost of capacity is small than when it is large.

7 Conclusions

Under demand uncertainty price cap regulation has to deal with a trade off involving the incentives for capacity investment and capacity withholding: decreasing the price cap alleviates capacity withholding but disincentives capacity investment. As a consequence, an optimal price cap may not maximize capacity investment: when the cost of capacity is low, maximizing the expected surplus calls for a low price cap that discourages capacity withholding, even at the cost of reducing capacity investment. Price cap regulation cannot restore efficiency. (It is noteworthy that even if capacity withholding is not an issue, i.e., even if the regulator may enforce full capacity utilization, price cap regulation does not provide appropriate incentives for capacity investment either. In fact, both capacity investment and surplus may be smaller with full capacity utilization than with capacity withholding. See the example discuss in Appendix B.)

Nonetheless, price cap regulation provides useful instrument to enhance market efficiency. Moreover, under standard regularity assumptions on the demand, the comparative static properties of price caps relative to the price cap that maximizes capacity are analogous to those obtained in the case of a deterministic demand.

8 Appendix A

Earle et al. (2007)'s Theorem 6 seemingly establishes that our propositions 3 to 5 fail for an open and dense subset of probability distributions of the demand parameter X. Considering that Earle et al. (2007) seem to have in mind a large set of probability distributions (their proof involves a discontinuous *c.d.f.*), this result is hardly surprising, and is not inconsistent with propositions 3 to 5. (A generic continuous *p.d.f.* on $[0, 1]$ is nowhere differentiable by Banach-Mazurkiewicz Theorem. Thus, the set continuously differentiable *p.d.f.*'s with an increasing hazard rate is a meagre subset of this set.)

Nonetheless, their claim that the proof of their Theorem 4, which establishes this result in the model of full capacity utilization, also applies to the model with capacity withholding that we study here is incorrect. In this section we show in the example discussed in Section 6 perturbing the distribution of the demand parameter X as in the proof of Earle et al. (2007)'s Theorem 4 does not produce the desired results. Of course, this does not prevent the existence of *p.d.f.*'s on $[0, 1]$ for which the conclusions of propositions 3 to 5 do not hold.

Earle et al. (2007)'s proof of Theorem 4 shows that given a *c.d.f.* F and a *binding* price cap $\bar{p}$ (i.e., $\bar{p}$ satisfies $\Pr(X-\bar{p} > k^*(\bar{p})) > 0$, which in our setting amounts to $\bar{p} \in (\underline{\rho}(c), \bar{\rho}(c)))$, and such that $dk^*(\bar{p})/d\rho < 0$, then by perturbing F in a certain way one can obtain another c.d.f. $\tilde{F}$ arbitrarily close to F and such that equilibrium capacity when the demand parameter is distributed according to $\tilde{F}$, $\tilde{k}^*$ satisfies $d\hat{k}^*(\bar{p})/d\rho > 0$. We show that the perturbation used in the proof of their Theorem 4 does not produce this result when the monopolist can withhold capacity.

Assume that X is uniformly distributed, and that the unit cost of capacity is $c = 1/32$. Consider the price cap $\bar{p} = 2/5 \in [\rho_-(1/32), \rho_+(1/32)] = \left[\frac{1}{4} - \frac{1}{8}\sqrt{3}, \frac{1}{4} + \frac{1}{8}\sqrt{3}\right]$. As shown in Section 6 we have $k^*(\rho) = 1 - \frac{c}{\rho} - \rho$. Hence

$$\frac{dk^*(\bar{p})}{d\rho} = \frac{c}{\bar{p}^2} - 1 = -\frac{103}{128}.$$

i.e., capacity decreases with the price cap near $\bar{p}$. (In the language of Earle et al. (2007), the comparative static properties near $\bar{p}$ are standard.)

Using the results of table 1A, we see that for demand realizations such that $X - \bar{p} < k^*(\bar{p})$, i.e., $X \in (\tilde{x}, 1]$ where $\tilde{x} = \frac{59}{64}$, the monopolist withholds capacity. Let us study the comparative static properties for a new perturbed distribution of X, denoted by $\tilde{F}$ which assigns probability uniformly on $[0, 1]$ except on the interval $[\tilde{x} - \varepsilon, \tilde{x} + \varepsilon]$, on which the probability is shifted to the end points, thus creating two atoms at $\tilde{x} - \varepsilon$ and $\tilde{x} + \varepsilon$. The probabilities assigned to these atoms are $2\alpha\varepsilon$ and $2(1 - \alpha)\varepsilon$, where ε and α are such that the optimal capacity when the price cap $\bar{p} = 2/5$ remains $k^*(\bar{p}) = 167/320$; that is, ε and α are chosen in such a way that

$$\frac{\partial}{\partial k}\left(\int_{\tilde{x}-\varepsilon}^{\tilde{x}} (x - \bar{p})\,\bar{p}dF(x) + \int_{\tilde{x}}^{\tilde{x}+\varepsilon} \bar{p}k^*(\bar{p})dF(x)\right) = \varepsilon\bar{p}$$

equals

$$\frac{\partial}{\partial k}\left(\int_{\tilde{x}-\varepsilon}^{\tilde{x}} (x - \bar{p})\,\bar{p}d\hat{F}(x) + \int_{\tilde{x}}^{\tilde{x}+\varepsilon} \bar{p}k^*(\bar{p})d\hat{F}(x)\right) = 2\,(1-\alpha)\,\varepsilon\bar{p}.$$

Solving this equation yields $\alpha = 1/2$, independently of ε. Therefore let $\alpha = 1/2$.

When the demand parameter is distributed according to $\tilde{F}$ the expected profit is

$$\begin{aligned}\tilde{\Pi}(\rho, k) &= \int_0^{2\rho} \left(\frac{x}{2}\right)^2 dx + \int_{2\rho}^{k+\rho} (x - \rho)\rho dx + \int_{k+\rho}^{1} \rho k dx - ck \\ &= -\frac{\rho}{2}k^2 + [\rho\,(1-\rho) - c]k + \frac{\rho^3}{6}\end{aligned}$$

if $\rho + k \in [0, \tilde{x} - \varepsilon)$, it is

$$\begin{aligned}
\tilde{\Pi}(\rho, k) &= \int_0^{2\rho} \left(\frac{x}{2}\right)^2 dx + \int_{2\rho}^{\tilde{x}-\varepsilon} (x-\rho)\rho dx + \varepsilon\left((\tilde{x}-\varepsilon)-\rho\right)\rho \\
&\quad + \rho\varepsilon k + \int_{\tilde{x}+\varepsilon}^{1} \rho k dx - ck \\
&= [\rho(1-\tilde{x}) - c]k + \frac{\rho}{6}(3\hat{x}^2 - 6\hat{x}\rho - 3\varepsilon^2 + 4\rho^2)
\end{aligned}$$

if $\rho + k \in [\tilde{x} - \varepsilon, \tilde{x} + \varepsilon]$, and it is

$$\begin{aligned}
\tilde{\Pi}(\rho, k) &= \int_0^{2\rho} \left(\frac{x}{2}\right)^2 dx + \int_{2\rho}^{\tilde{x}-\varepsilon} (x-\rho)\rho dx + \varepsilon\left((\tilde{x}-\varepsilon)-\rho\right)\rho \\
&\quad + \rho\varepsilon k + \int_{\tilde{x}+\varepsilon}^{k+\rho} (x-\rho)\,\rho dx + \int_{k+\rho}^{1} \rho k dx - ck \\
&= -\frac{\rho}{2}k^2 + \left(\rho\left(1+\varepsilon-\rho\right) - c\right)k + \frac{\rho^3}{6} - \rho\varepsilon^2 + \rho^2\varepsilon - \rho\hat{x}\varepsilon
\end{aligned}$$

if $\rho + k > \tilde{x} + \varepsilon$. Figure 7 displays the graphs of the expected profit for ρ near $\bar{\rho}$. If $\rho > \bar{\rho}$, then $\tilde{\Pi}(\bar{\rho}, \cdot)$ is increasing in capacity. If $\rho < \bar{\rho}$, then $\tilde{\Pi}(\bar{\rho}, \cdot)$ is decreasing in capacity. Hence $\tilde{k}^*(\rho) = \tilde{x} - \varepsilon - \rho$ if $\rho > \bar{\rho}$, and $\tilde{k}^*(\rho) = \tilde{x} + \varepsilon - \rho$ if $\rho < \bar{\rho}$ for ρ near $\bar{\rho}$. That is, the equilibrium capacity is decreasing in the price cap. If $\rho = \bar{\rho}$, then $\tilde{\Pi}(\bar{\rho}, \cdot)$ is constant and maximal for $k \in [\tilde{x} - \varepsilon - \bar{\rho}, \tilde{x} + \varepsilon - \bar{\rho}]$.

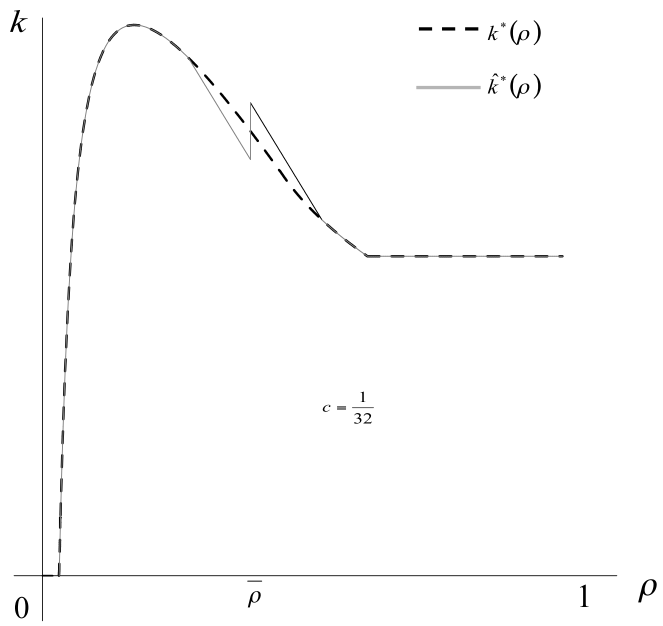

Figure 7: Profits near $\bar{\rho}$.

Figure 8 provides the graphs of k^* and $\tilde{k}^*$ for $\varepsilon = \frac{1}{30}$. Although the mapping $\tilde{k}^*(\rho)$ becomes a correspondence for $\bar{\rho}$, comparative statics for ρ near $\bar{\rho}$ remain standard, i.e., $\partial\tilde{k}^*(\rho)/\partial\rho = -1$ near $\rho = \bar{\rho}$. (Except on $\bar{\rho}$ itself, where the derivative is not defined since mapping providing the equilibrium capacity becomes a correspondence.) If the monopolist withholds capacity, after this perturbation capacity continues to decrease with the price cap for all price caps in a neighborhood of $\bar{\rho}$.

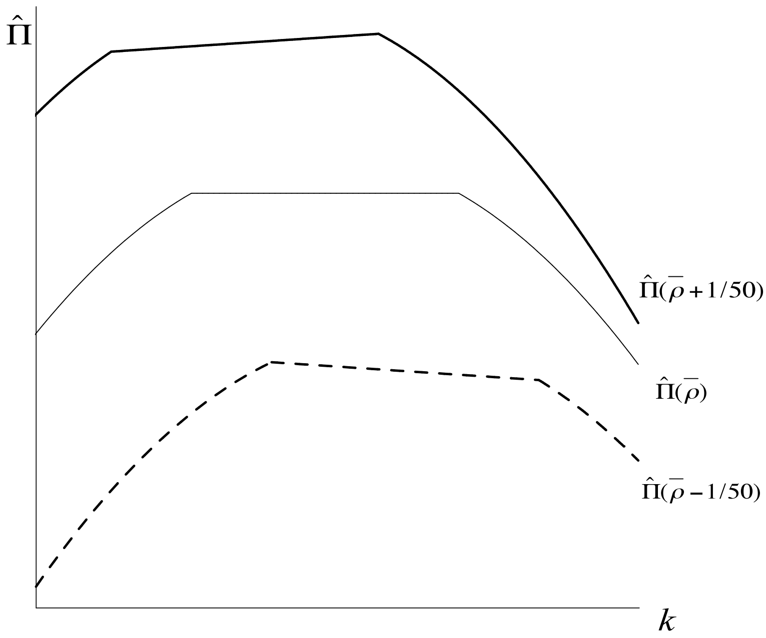

Figure 8: the Functions $k^*(\rho)$ and $\tilde{k}^*(\rho)$.

Thus, Earle et al. (2007)'s proof, which relies on this perturbation, does not apply to a model where the monopolist may withhold capacity. In fact, this perturbation has an effect on the monopolist profit and the profit maximizing level of capacity akin to that of creating a flat spot on the demand when the demand is known with certainty.

9 Appendix B: Full Capacity Utilization

Assume that the monopolist cannot withhold capacity, i.e., must supply its entire capacity for each demand realization. One may interpret this setting as one where the monopolist delivers its output to the market before the demand is realized. This model is studied by Earle et al. (2007) and Grim and Zoettl (2010). We show that the equilibrium and the comparative static properties of price caps in this model are significant different from those of our model where the monopolist may withhold capacity.

Monopoly Equilibrium with a Price Cap

Assume that a regulatory agency imposes a price cap $\rho \in [0, 1]$. Table 3A identifies the market equilibrium price for each demand realization if the monopolist installs a capacity $k < 1 - \rho$ (and supplies it inelastically to the market).

X	$[0, k)$	$[k, \rho + k)$	$[\rho + k, 1]$
$\hat{P}(\rho, k)$	0	$x - k$	ρ

Table 3A: Equilibrium Price for $k \in [0, 1 - \rho)$.

Table 3B identifies the market equilibrium price for each demand realization when the monopolist installs a capacity $k \geq 1 - \rho$.

X	$[0, k)$	$[k, 1]$
$\hat{P}(\rho, k)$	0	$x - k$

Table 3B: Equilibrium Price for $k \in [1 - \rho, 1]$.

Note that if $k \geq 1 - \rho$ the price cap is non-binding.

For $k < 1 - \rho$ the expected price is

$$E(\hat{P}(\rho, k)) = \int_k^{\rho+k} (x - k) f(x) dx + \int_{\rho+k}^1 \rho f(x) dx.$$

Hence

$$\frac{\partial E(\hat{P}(\rho, k))}{\partial k} = - \int_k^{\rho+k} f(x) dx,$$

and

$$\frac{\partial^2 E(\hat{P}(\rho, k))}{\partial k^2} = f(k) - f(\rho + k).$$

For $k \geq 1 - \rho$ the expected price is

$$E(\hat{P}(\rho, k)) = \int_k^1 (x - k) f(x) dx.$$

Hence

$$\frac{\partial E(\hat{P}(\rho,k))}{\partial k} = -\int_k^1 f(x)dx,$$

and

$$\frac{\partial^2 E(\hat{P}(\rho,k))}{\partial k^2} = f(k).$$

The monopolist chooses the level of capacity k in order to maximize its expected profit

$$\hat{\Pi}(\rho,k) = E\left([\hat{P}(\rho,k) - c]k\right) = [E(\hat{P}(\rho,k)) - c]k,$$

Clearly $\hat{\Pi}$ is continuous on $[0,1]^2$. In an interior equilibrium k solves

$$\frac{\partial E(\hat{P}(\rho,k))}{\partial k}k + E(\hat{P}(\rho,k)) = c, \tag{13}$$

and satisfies

$$\frac{\partial^2 \hat{\Pi}(\rho,k))}{\partial k^2} = \frac{\partial^2 E(\hat{P}(\rho,k))}{\partial k^2}k + 2\frac{\partial E(\hat{P}(\rho,k))}{\partial k} < 0. \tag{14}$$

We have

$$\frac{\partial^2 \hat{\Pi}(\rho,k))}{\partial k^2} = -k(f(\rho+k) - f(k)) - 2(F(\rho+k) - F(k)).$$

for $k < 1-\rho$, and

$$\frac{\partial^2 \hat{\Pi}(\rho,k))}{\partial k^2} = kf(k) - 2\left(1 - F(k)\right).$$

for $k \geq 1-\rho$. The sign of these expressions is ambiguous. In fact, it is not difficult to find examples for which the profit function $\hat{\Pi}(\rho,\cdot)$ is not concave for some values of ρ. (E.g., take $f(x) = 2(1-x)$ and $\rho = 1/4$.) This property of this model of full capacity utilization stands in contrast with that of our model of capacity withholding, in which the expected profit is a concave function.

An Example: The Uniform Distribution

Assume that X is uniformly distributed and $c \in (0, 1/2)$. For $k < 1-\rho$ we have

$$E(\hat{P}(\rho,k)) = \frac{1}{2}\rho\left(2 - 2k - \rho\right),$$

and for $k \geq 1-\rho$, we have

$$E(\hat{P}(\rho,k)) = \frac{1}{2}\left(1-k\right)^2.$$

Hence for $k < 1-\rho$, we have

$$\frac{\partial^2 \hat{\Pi}(\rho,k))}{\partial k^2} = -2\rho.$$

and for $k \geq 1-\rho$, we have

$$\frac{\partial^2 \hat{\Pi}(\rho,k))}{\partial k^2} = -2+3k.$$

If the equilibrium capacity is $k < 1-\rho$, then equation (13) is

$$-\rho k + \frac{1}{2}\rho\left(2-2k-\rho\right) = c.$$

Solving this equation we get

$$k_1(\rho) = \frac{1}{2}\left(1-\frac{c}{\rho}-\frac{\rho}{2}\right).$$

Hence $k_1(\rho)$ is the solution to the monopolist problem provided $0 < k_1(\rho) < 1-\rho$, i.e.,

$$\underline{\hat{\rho}}(c) := 1-\sqrt{1-2c} < \rho < \frac{1}{3}\sqrt{6c+1}+\frac{1}{3} := \overline{\hat{\rho}}(c).$$

If $\rho < \underline{\hat{\rho}}(c)$, then expected profit decreases with k and the equilibrium capacity is $k^* = 0$. If $\rho > \overline{\hat{\rho}}(\rho)$, then expected profit increases with k at $k = 1-\rho$.

If the equilibrium capacity is $k \geq 1-\rho$, then equation (13) is

$$-\left(1-k\right)k + \frac{1}{2}\left(1-k\right)^2 = c.$$

Solving this equation we get

$$k_2 = \frac{2-\sqrt{1+6c}}{3}.$$

Note that $k_2 > 0$ for all $c \in (0,1/2)$. Hence k_2 is the solution to the monopolist problem provided $k_2 \geq 1-\rho$, i.e.,

$$\rho \geq \overline{\hat{\rho}}(c).$$

If $\rho < h(c)$ the expected profit decreases with k at $k = 1-\rho$.

The equilibrium capacity is therefore given by

$$\hat{k}^*(\rho) = \begin{cases} 0 & \text{if } \rho \leq [0, \underline{\hat{\rho}}(c)], \\ k_1(\rho) & \text{if } \rho \in (\underline{\hat{\rho}}(c), \overline{\hat{\rho}}(c)), \\ k_2 & \text{if } \rho > [\overline{\hat{\rho}}(c), 1]. \end{cases}$$

The maximum capacity is installed for $\hat{\rho}^*$ solving

$$\frac{dk_1(\rho)}{d\rho} = \frac{1}{2}\left(\frac{c}{\rho^2} - \frac{1}{2}\right) = 0;$$

i.e., $\hat{\rho}^* = \sqrt{2c}$. (Note that $d^2k_1(\rho)/d\rho^2 = -c/\rho^3 < 0$.) The maximum capacity is

$$k_1(\hat{\rho}^*) = \frac{1}{2} - \sqrt{\frac{c}{2}} > k_2.$$

As shown in Section 6 the optimal capacity is $k^W = 1 - \sqrt{2c} = 2k_1(\hat{\rho}^*)$. Hence ρ^* is indeed the optimal price cap. Moreover, since $\hat{k}^*(\rho^*) > k_2$, then a binding price increases expected surplus, but is unable to provide incentives for the monopolist to install the optimal level of capacity. Thus, a price cap is a poor regulatory instrument also in this framework: price caps provide even worse incentives for capacity investment than when the monopolist can withhold capacity – see Figure 9.

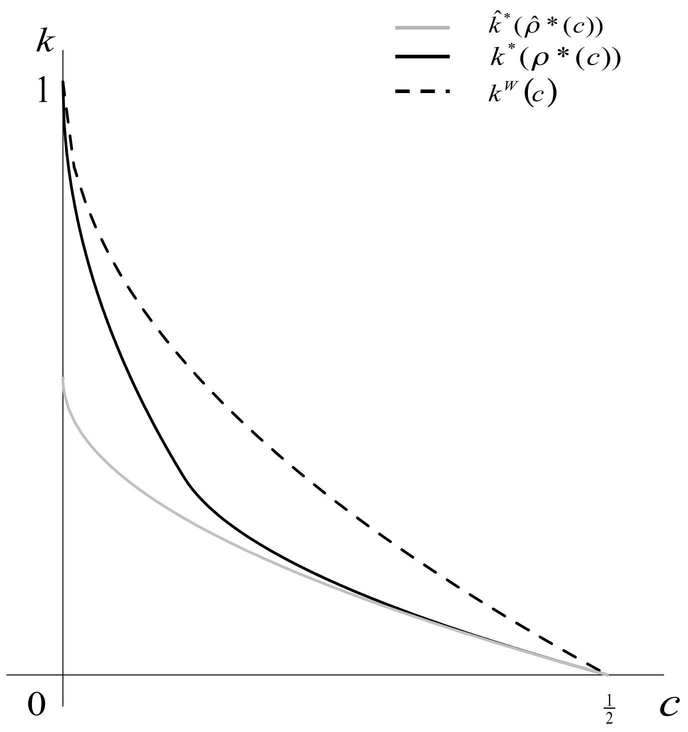

Figure 9: Capacity Investment with and without Withholding.

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